TWO-PORT MODELS CONTAINING NULLATORS AND NORATORS

By

E. Hollós

Department of Theoretical Electricity, Technical University Budapest

Received: March 3, 1981 Presented by Prof. Dr. I. VÁGÓ

Two-ports in active networks treatable as linear are commonly characterized by a system of parameters. Models of a two-port characterized by its impedance, admittance, hybrid or inverse hybrid parameters, containing nullators, norators and impedances can be given [1, 2]. The current and voltage of a nullator (Fig. 1) are known to be zero, the norator (Fig. 2) is a twopole with both its current and voltage of arbitrary value.

Two-port models are derived by applying equivalent circuits containing nullators and norators for controlled sources. Models of controlled sources are known [1, 2], with one pole of the primary and secondary sides being common (Fig. 3). By their application two-ports characterized by impedance, admittance, hybrid or inverse hybrid parameters can be modelled by equivalent circuits with one pole of the primary and secondary sides being on the same potential (Fig. 4). This circumstance reduces the applicability of these connections. In this paper, models are presented, eliminating this short-circuit between the two poles.

Models of controlled sources are given in Fig. 5. Equivalent circuits for further two-ports are hence derived as follows (Fig. 6). Selecting the models of the two controlled sources corresponding to the relationships described by the elements outside the main diagonal of the parameter matrix characterizing the two-port, a norator or nullator is connected to one pole at their primary or secondary sides. Nullator is connected to the pole at the primary or secondary side if the controlled source serves to ensure the prescription relating to the primary or secondary voltage, and a norator if the prescription concerns the current. The models thus supplemented are connected as shown in Fig. 7.



1*

E. HOLLÓS

Denomination	Characteristic equation	Equivalent circuits
current-controlled voltage-source	$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ Z & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$	$\begin{array}{c c} \underline{I}_1 & \underline{I}_2 & \underline{I}_1 & \underline{Z} & \underline{I}_2 \\ \underline{I}_1 & \underline{O} & \underline{O} & \underline{I}_2 & \underline{I}_2 \\ \underline{I}_2 & \underline{O} & \underline{O} & \underline{I}_2 \\ \underline{I}_2 & \underline{O} & \underline{O} & \underline{O} & \underline{O} \\ \underline{I}_2 & \underline{I}_2 & \underline{O} & \underline{O} & \underline{O} \\ \underline{I}_2 & \underline{I}_2 & \underline{I}_2 & \underline{I}_2 \\ \underline{I}_2 & \underline{I}_2 & \underline{I}_2 & \underline{I}_2 \\ \underline{I}_2 & \underline{I}_2 & \underline{I}_2 & \underline{I}_2 & \underline{I}_2 \\ \underline{I}_2 & \underline{I}_2 & \underline{I}_2 & \underline{I}_2 & \underline{I}_2 \\ \underline{I}_2 & $
voltage-controlled current-source	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ Y & 0 \end{bmatrix} \begin{bmatrix} U_2 \end{bmatrix}$	$\begin{array}{c c} I_{\pm 0} & I_{2} & I_{\pm 0} & Z_{\pm 1/Y} & I_{2} \\ \hline \downarrow & & & & & & & \\ U_{1} & & & & & & \\ \hline U_{2} & & & & & & & \\ \end{array}$
current-controlled current-source	$\begin{bmatrix} U_1 & 0 & 0 \\ I_2 & \mu & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
voltage-controlled voltage-source	$\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$	

Fig. 3

Characteristic equation	Equivalent circuit
$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$	$U_{1} \qquad \underbrace{\begin{matrix} I_{1} & z_{11} \\ \bullet & \bullet \\ & \downarrow \end{matrix}}_{z_{12}} \qquad \underbrace{\begin{matrix} 0 \\ \bullet \\ & \downarrow \end{matrix}}_{z_{21}} \qquad \underbrace{\begin{matrix} z_{22} & I_{2} \\ & \downarrow \end{matrix}}_{z_{21}} \qquad \underbrace{\begin{matrix} U_{2} \\ & U_{2} \end{matrix}}_{z_{21}} \end{matrix} \end{matrix}_{z_{21}} \qquad \underbrace{\begin{matrix} U_{2} \\ & U_{2} \end{matrix}}_{z_{21}} \end{matrix} \end{matrix}_{z_{21}} \end{matrix} \end{matrix}_{z_{21}} \end{matrix} \end{matrix}_{z_{21}} \end{matrix} \end{matrix}_{z_{21}} \end{matrix} \end{matrix}_{z_{21}} \end{matrix}$
$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$	$\begin{array}{c c} I_1 & & & & I_2 \\ \hline & & & & & \\ U_1 & & & & \\ U_1 & & & & \\ \end{array} \begin{array}{c} I_1 & & & & \\ \hline & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & & \\ \end{array} \begin{array}{c} & & & & \\ & & & \\ \end{array} \begin{array}{c} & & & \\ & & & \\ & & & & \\ \end{array} \end{array}$
$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix}^{=} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

Fig. 4

It is indicated by the equivalent circuits drawn in Figs 6 and 8 that the desired aim can be attained by the insertion of a resistance or a nullator and a norator. The models of an ideal transformer, a negative impedance converter and a gyrator are drawn in Fig. 8.

In case of the application of the models mentioned, network analysis is possible by the method of node potentials or by other methods known from literature [1, 3, 4]. The number of nodes and branches is naturally greater in the

MODELS CONTAINING NULLATORS AND NORATORS

Denomination	Equivalent circuits	
current-controlled voltage-source	$\begin{array}{c c} & & & I_1 \\ & & & & \\ U_1 = 0 \\ & & & \\ & & & \\ & & & \\ & & & \\ \end{array} \begin{array}{c} & & & \\ & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & & \\ & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & & \\ & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & & \\ & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & & \\ & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & & \\ & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & \\ & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \begin{array}{c} & & \\ & & \\ & & \\ & & \\ \end{array} \end{array}$	
voltage-controlled current-source	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
current-controlled current-source	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	
voltage-controlied voltage-source	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	

F	iσ	5
	· ~ ·	~

Characteristic equation	Equivalent circuit
$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$	$\begin{array}{c c} & & & & \\ \hline I_1 & Z_{11} & & & \\ \hline & & & & \\ U_1 & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & &$
$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

Fig. 6

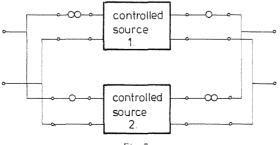


Fig. 7

Denomination	Characteristic equation	Equivalent circuit
ideal transformer	$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$	$\begin{array}{c c} I_1 & & & R_1 \\ \hline & & & & \\ \downarrow & \downarrow & 0 \\ \downarrow & & & R_{1,2} \\ \hline & & & & & R_{2} \\ \hline & R_{2} \\$
negative impedance converter	$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & k \\ k & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
gyrator	$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 0 & -R_1 \\ R_2 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$	$\begin{array}{c c} I_1 & & & & & \\ \hline I_1 & & & & \\ \hline U_1 & & & & \\ \hline \end{array} \\ \begin{array}{c} & & & \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} & & \\ \end{array} \\ \end{array}$

Fig. 8

models shown in Figs 6 and 8 than in those seen in Fig. 4. Therefore, if it is possible, the application of the latter is more advantageous.

Since the equivalent circuits in Figs 5, 6 and 8 contain nullator-norator pairs, the models constructed with their aid permit to reduce the network synthesis to two-pole synthesis.

Summary

Models containing nullators, norators and impedances are known for two-ports characterized by impedance, admittance, hybrid or inverse hybrid parameters with one pole of the primary and secondary sides on the same potential. The paper presents equivalent circuits with no short circuit between the two poles. The application of such models permits the performance of analysis for a wider range of networks than do the models known earlier.

References

- Vágó, I.: Application of graph theory in the calculation of electrical networks. (In Hungarian) Műszaki Könyvkiadó, Budapest, 1976.
- VAGÓ, I.—HOLLÓS, E.: Two-port models with nullators and norators. Periodica Polytechnica El. Eng. 17 (1973) p. 301—309.
- VAGÓ, I.: Calculation of network models containing nullators and norators. Periodica Polytechnica El. Eng. 17 (1973) p. 311–319.
- DAVIES, A. C.: Matrix analysis of networks containing nullators and norators. Electronics Letters 1966, p. 48–49.

Edit Hollós H-1521 Budapest