THE METHOD OF LOOP-CURRENTS FOR NETWORKS CONTAINING NULLATORS AND NORATORS

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The application of nullators and norators in network models permits the calculation of networks containing coupled two-poles and two-ports (e.g. controlled generator, gyrator, ideal transformer, negative impedance converter) on the basis of connections formed by two-poles without coupling [1, 2, 3, 4]. Several methods are known for the calculation of such linear networks consisting of impedances, independent sources, independent generators, nullators, norators [5, 6]. One of the well-known procedures for the analysis of linear networks formed by impedances and independent generators is the method of loop-currents [1]. This method is applied on the presumption that every branch of the network has a finite or infinite impedance. Since no impedance can be attributed to either a nullator (Fig. 1a: $U=0$, $I=0$) or a norator (Fig. 1b: U, I of arbitrary value), the equations for loop-currents cannot be written for networks containing such two-poles in the way known from the literature. In this paper the application of loop-currents is presented for networks containing nullators and norators. Since the network analysis problem can only be solved uniquely if the number of nullators and norators coincides in the model, the procedure to be presented can solely be applied in such cases.

The method of loop-currents means the determination of the column matrix $\mathbf{J}$ of loop-currents on the basis of the equation

$$\mathbf{Z}_B\mathbf{J} = \mathbf{B}(\mathbf{Z}\mathbf{I}_g - \mathbf{U}_g)\,, \tag{1}$$

where

$$\mathbf{Z}_B = \mathbf{B}\mathbf{Z}\mathbf{B}^+ \tag{2}$$

is the loop-impedance matrix, $\mathbf{B}$ is the basis loop matrix of the network, $\mathbf{B}^+$ is its transpose, $\mathbf{Z}$ is the branch-impedance matrix of the network, $\mathbf{I}_g$ and $\mathbf{U}_g$ are the column matrices of source-currents and source-voltages, respectively. The matrices are arranged according to a given sequence of the branches. The column matrix $\mathbf{I}$ of branch-currents is known to be obtainable from loop-currents $\mathbf{J}$ according to

$$\mathbf{I} = \mathbf{B}^+\mathbf{J}\,, \tag{3}$$

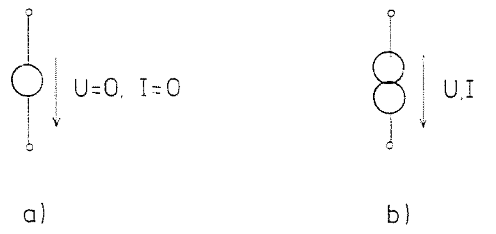

Fig. 1

i.e. by the superposition of loop-currents. In the knowledge of the impedances the voltages can be calculated from the branch-currents.

In case of a network containing nullators and norators, the branch-impedance matrix $\mathbf{Z}$, and thus the loop-impedance matrix $\mathbf{Z}_B$ cannot be defined. Our task in the following is to overcome this difficulty.

The following elements are considered to form a branch of the network: a Thevenin-generator, a Norton-generator, a voltage source (with zero inner impedance), a current source (with zero inner admittance), a nullator, a norator. It is naturally expedient to decrease the number of edges if possible by taking into account that e.g. series connection of a nullator and a norator is equivalent to an open circuit, their parallel connection to a short-circuit, etc. [1].

Before numbering the branches of the network, let a tree of the network's graph be selected, with each norator corresponding to a tree-branch, each nullator and each current source to a chord. Let thereupon the branches of the network be classified into four groups and numbered. The first group of branches is formed by the nullators numbered $1, 2, \ldots, b_1$. Norators are assigned to the second group numbered $b_1+1, b_1+2, \ldots, 2b_1$. The third group consists of the current-sources numbered $2b_1+1, 2b_1+2, \ldots, 2b_1+b_2$. The further branches are included in the fourth group, their number is denoted by b_3.

The set of loops generated by the tree above will be used for the calculation. This means that each nullator and current-source is included in one loop only. The loops are numbered to have loops $1, 2, \ldots, b_1$ with arbitrary orientation, contain branches $1, 2, \ldots b_1$, and loops $b_1+1, b_1+2, \ldots, b_1+b_2$ include branches $2b_1+1, 2b_1+2, \ldots, 2b_1+b_2$ with their orientation along these branches coincident with the reference directions of the source currents in the branches. Numbering and orientation of further loops are arbitrary. Thus, in the column matrix $\mathbf{J}$ of loop currents the first b_1 elements are zero while the following b_2 elements yield the source currents of the current sources expressed as:

$$\mathbf{J} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_{g2} \\ \mathbf{J}_e \end{bmatrix}, \tag{4}$$

where $\mathbf{J}_e$ includes the $m-b_1-b_2$ loop currents to be determined, m is the number of linearly independent loops in the network.

$\mathbf{Z}_B\mathbf{J}$ in (1) is known to yield the voltages on the impedances in the linearly independent loops of the network. Let this be written for the network examined with the norators taken into account with zero impedances, the nullators and current-sources with arbitrary impedances, and let the matrix $\mathbf{Z}_B$ thus formed be partitioned in accordance with (4):

$$\mathbf{Z}_B\mathbf{J}=\begin{bmatrix}\mathbf{Z}_{11} & \mathbf{Z}_{12} & \mathbf{Z}_{13}\\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \mathbf{Z}_{23}\\ \mathbf{Z}_{31} & \mathbf{Z}_{32} & \mathbf{Z}_{33}\end{bmatrix}\begin{bmatrix}\mathbf{0}\\ \mathbf{I}_{g2}\\ \mathbf{J}_e\end{bmatrix}. \tag{5}$$

Here, the hypothetical impedances of nullators and current sources appear in blocks $\mathbf{Z}_{11}$ of order b_1 and $\mathbf{Z}_{22}$ of order b_2. These will not occur in our calculations, so that $\mathbf{I}_{g2}$ can be written as the second block of $\mathbf{J}$. Let the diagonal matrix $\mathbf{Z}_2$ be formed by the above hypothetical arbitrary impedances of the current sources and thus among the matrices in the right-hand side of (1):

$$\mathbf{Z}\mathbf{I}_g=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{Z}_2\mathbf{I}_{g2}\\ \mathbf{Z}_3\mathbf{I}_{g3}\end{bmatrix}, \tag{6}$$

where $\mathbf{0}$ is zero matrix with b_1 elements, $\mathbf{Z}_3$ is the impedance matrix of the branches belonging to the fourth group, $\mathbf{I}_{g3}$ is the column matrix formed by source currents of the branches in group 4. Column matrix $\mathbf{U}_g$ comprises the source voltages:

$$\mathbf{U}_g=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{U}_{g3}\end{bmatrix}, \tag{7}$$

where the first two matrices $\mathbf{0}$ consist of b_1 and the third one of b_2 elements. Let the column matrix formed by the so far unknown voltages of the norators be $\mathbf{U}_n$. Thus the independent loop equations of the network can be written as

$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \mathbf{Z}_{13} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \mathbf{Z}_{23} \\ \mathbf{Z}_{31} & \mathbf{Z}_{32} & \mathbf{Z}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_{g2} \\ \mathbf{J}_e \end{bmatrix} = \begin{bmatrix} \mathbf{1}_{b1} & \mathbf{B}_{11} & \mathbf{0} & \mathbf{B}_{12} \\ \mathbf{0} & \mathbf{B}_{21} & \mathbf{1}_{b2} & \mathbf{B}_{22} \\ \mathbf{0} & \mathbf{B}_{31} & \mathbf{0} & \mathbf{B}_{23} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ -\mathbf{U}_n \\ \mathbf{Z}_2\mathbf{I}_{g2} \\ \mathbf{Z}_3\mathbf{I}_{g3} - \mathbf{U}_{g3} \end{bmatrix} \tag{8}$$

where the loop matrix has been partitioned in accordance with the three groups of loops and four groups of branches, i.e. $\mathbf{1}_{b1}$ and $\mathbf{1}_{b2}$ are unit matrices of order b_1 and b_2, respectively, and $\mathbf{B}_{11}$ is also of order b_1. The following two matrix equations can be derived from (8):

$$\mathbf{Z}_{12}\mathbf{I}_{g2} + \mathbf{Z}_{13}\mathbf{J}_e = -\mathbf{B}_{11}\mathbf{U}_n + \mathbf{B}_{12}(\mathbf{Z}_3\mathbf{I}_{g3} - \mathbf{U}_{g3}), \tag{9}$$

$$\mathbf{Z}_{32}\mathbf{I}_{g2} + \mathbf{Z}_{33}\mathbf{J}_e = -\mathbf{B}_{31}\mathbf{U}_n + \mathbf{B}_{32}(\mathbf{Z}_3\mathbf{I}_{g3} - \mathbf{U}_{g3}). \tag{10}$$

In case $\mathbf{B}_{11}$ is nonsingular, (9) yields

$$\mathbf{U}_n = \mathbf{B}_{11}^{-1}\mathbf{B}_{12}(\mathbf{Z}_3\mathbf{I}_{g3} - \mathbf{U}_{g3}) - \mathbf{B}_{11}^{-1}\mathbf{Z}_{12}\mathbf{I}_{g2} - \mathbf{B}_{11}^{-1}\mathbf{Z}_{13}\mathbf{J}_e \tag{11}$$

Substituting this into (10) the column matrix of the loop-currents sought is obtained:

$$\begin{aligned} \mathbf{J}_e = (\mathbf{Z}_{33} - \mathbf{B}_{31}\mathbf{B}_{11}^{-1}\mathbf{Z}_{13})^{-1}\,[&(\mathbf{B}_{32} - \mathbf{B}_{31}\mathbf{B}_{11}^{-1}\mathbf{B}_{12})\,(\mathbf{Z}_3\mathbf{I}_{g3} - \mathbf{U}_{g3}) + \\ &+ (\mathbf{B}_{31}\mathbf{B}_{11}^{-1}\mathbf{Z}_{12} - \mathbf{Z}_{32})\mathbf{I}_{g2}]\,. \end{aligned} \tag{12}$$

In the knowledge of $\mathbf{J}_e$, (11) yields the voltages of the norators, and thus the currents and voltages of the network can be determined. The result has been obtained by the inversion of a matrix whose order is less than the number of independent loops by the number of nullator-norator pairs and current-sources.

It is noted that a sufficient condition of the invertibility of $\mathbf{B}_{11}$ is if branch 1 containing a nullator is in one loop with branch b_1+1 containing a norator, branch 2 with branch $b_1+2, \ldots$, and finally branch b_1 with branch $2b_1$, and none of these norators is in a common loop with any other nullator. In case of the application of the models presented in [3, 4], this can be ensured.

The application of the method is illustrated by a simple example. In the network shown in Fig. 2 the sinusoidal voltage U_g, the values of the resistances as well as the hybrid parameters h_{11}, $h_{12} \approx 0$, h_{21}, h_{22} of the transistor are given. In determining the currents of the resistors the direct-current source U_0 can be regarded as a short-circuit. Substituting the transistor by a current-controlled

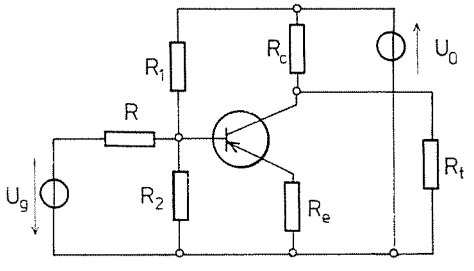

Fig. 2

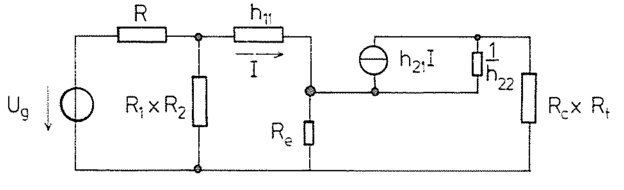

Fig. 3

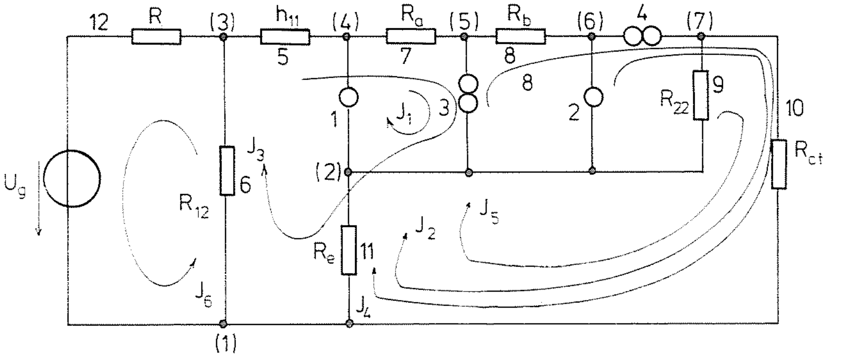

Fig. 4

current generator (Fig. 3) a model containing nullators and norators can be constructed for the connection (Fig. 4) [1, 3], where notations $R_1 \times R_2 = R_{12}$, $R_c \times R_t = R_{ct}$, $R_{22} = 1/h_{22}$ have been employed. Out of R_a and R_b one can be chosen at will and $h_{21} = R_a/R_b$. The graph of the model has been drawn in Fig. 5 with the branches oriented arbitrarily. The tree of the graph shown in Fig. 6 has been chosen for the calculation. The branches are thereupon assigned order

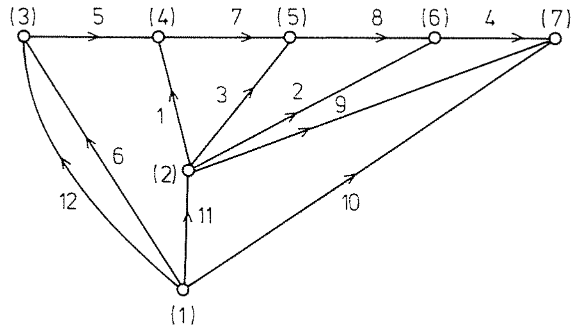

Fig. 5

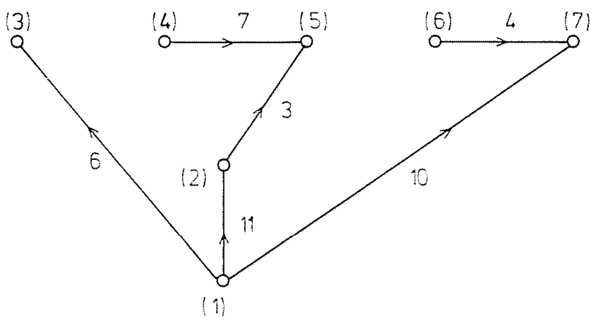

Fig. 6

numbers which have been indicated in Figs 4, 5 and 6. The loop matrix of the set of loops generated by the tree in Fig. 6 is

$$\mathbf{B}=\left[\begin{array}{cc|cc|cccccccc}
1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
\hline
0 & 0 & -1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1
\end{array}\right],$$

where the partitioning according to (8) has been indicated. The number of blocks is less here due to the fact that there is no current source in the model. The loop-impedance matrix is:

$$\mathbf{Z}_B = \left[\begin{array}{cc|cccc} X & 0 & R_a & 0 & 0 & 0 \\ 0 & X & -R_e & R_{ct}+R_e & R_{ct}+R_e & 0 \\ \hline R_a & -R_e & h_{11}+R_{12}+R_a+R_e & -R_e & -R_e & R_{12} \\ 0 & R_{ct}+R_e & -R_e & R_b+R_{ct}+R_e & R_{ct}+R_e & 0 \\ 0 & R_{ct}+R_e & -R_e & R_{ct}+R_e & R_{22}+R_{ct}+R_e & 0 \\ 0 & 0 & R_{12} & 0 & 0 & R_{12}+R \end{array}\right]$$

partitioned according to (5). X denotes the elements of the matrix which cannot be interpreted. Since there is no current source or Norton-generator in the network examined, $\mathbf{I}_{g2}$ and $\mathbf{I}_{g3}$ do not exist. Excitation is taken into account by column matrix

$$\mathbf{U}_{g3} = \begin{bmatrix} \mathbf{0} \\ -U_g \end{bmatrix},$$

where the number of elements in $\mathbf{0}$ is 7. From the above matrices, according to (12):

$$\mathbf{J}_e = \begin{bmatrix} J_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix} = \begin{bmatrix} h_{11}+R_{12}+R_e & -R_e & -R_e & R_{12} \\ R_a & R_b & 0 & 0 \\ -R_e & R_{ct}+R_e & R_{22}+R_{ct}+R_e & 0 \\ R_{12} & 0 & 0 & R_{12}+R \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -U_g \end{bmatrix} =$$

$$= D^{-1} \begin{bmatrix} R_{12}R_b(R_{22}+R_{ct}+R_e)U_g \\ -R_{12}R_a(R_{22}+R_{ct}+R_e)U_g \\ R_{12}(R_aR_{ct}+R_aR_e+R_eR_b)U_g \\ -\{R_b[(h_{11}+R_{12}+R_e)(R_{22}+R_{ct}+R_e)-R_e^2]-R_aR_eR_{22}\}U_g \end{bmatrix}$$

$$D = (R_{12}+R)\{R_b(h_{11}+R_{12}+R_e)(R_{22}+R_{ct}+R_e)-R_bR_e^2-R_aR_eR_{22}-$$

$$-R_{12}R_b(R_{22}+R_{ct}+R_e)\}\,.$$

Thus the loop currents have been determined since the loop currents flowing through the nullators are zero ($J_1=0$, $J_2=0$).

Summary

The method of loop-currents is applicable for the analysis of linear networks consisting of impedances and independent sources. The introduction of nullators and norators permits the construction of models for networks including coupled two-poles and two-ports that consist of non-coupled impedances, independent sources, nullators and norators. The paper presents a possibility for the application of the method of loop currents for such models.

References

1. VÁGÓ, I.: The application of graph theory in the calculation of electrical networks. (In Hungarian) Műszaki Könyvkiadó, Budapest, 1976.
2. DAVIES A. C.: Nullator-norator equivalent networks for controlled sources. Proc. of IEEE, 1957, p. 722—723.
3. VÁGÓ I.—HOLLÓS E.: Two-port models with nullators and norators. Per. Polytech. El. Eng. Vol. 17 (1973), p. 301—309.
4. HOLLÓS E.: Two-port models containing nullators and norators Per. Polytech. El. Eng. Vol. 25 (1981) No. 3. p. 167—170.
5. VÁGÓ I.: Calculation of network models containing nullators and norators. Per. Polytech. El. Eng. Vol. 17 (1973) p. 311—319.
6. DAVIES A. C.: Matrix analysis of networks containing nullators and norators. Electronics Letters, 1966. Vol. 2. p. 48—49.

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