THE METHOD OF CUT VOLTAGES FOR NETWORKS CONTAINING NULLATORS AND NORATORS

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The application of nullators and norators in network models is justified by the possibility they offer to reduce the analysis of linear networks containing coupled two-poles and two-ports (e.g. controlled generator, ideal transformer, gyrator) to the calculation of networks consisting of two-poles without coupling: impedances, independent sources, nullators and norators. Several methods are known for the analysis of such networks [1, 2, 3, 4]. It seems expedient to elaborate the method of cut voltages applicable for linear networks consisting of admittances and independent sources for networks containing nullators and norators. This is the aim of the present paper.

The network analysis problem under discussion is known to be solvable if the numbers of nullators and norators coincide in the model. This will be presumed in the following.

For linear networks formed by independent generators and admittances (impedances) the method of cut voltages [1] means the determination of the column matrix $\mathbf{V}_Q$ of cut voltages on the basis of the equation

$$\mathbf{Y}_Q\mathbf{V}_Q=\mathbf{Q}(\mathbf{Y}\mathbf{U}_g-\mathbf{I}_g)\,. \tag{1}$$

In Eq. (1)

$$\mathbf{Y}_Q=\mathbf{Q}\mathbf{Y}\mathbf{Q}^+ \tag{2}$$

is the cut admittance matrix, $\mathbf{Q}$ is the basis cut matrix of the network, $\mathbf{Q}^+$ is its transpose, $\mathbf{Y}$ is the branch admittance matrix of the network, $\mathbf{U}_g$ and $\mathbf{I}_g$ are the column matrices of source voltages and source currents, respectively. The matrices are arranged according to the same sequence of the branches. The column matrix $\mathbf{U}$ of branch voltages can be obtained from cut voltages $\mathbf{V}_Q$ according to

$$\mathbf{U}=\mathbf{Q}^+\mathbf{V}_Q\,, \tag{3}$$

i.e. by the superposition of cut voltages. Thereupon the branch currents can be calculated from the branch voltages in the knowledge of the admittances.

Since no admittance can be attributed to either a nullator (Fig. 1a: $U=0$, $I=0$) or a norator (Fig. 1b: U, I of arbitrary value), the branch admittance

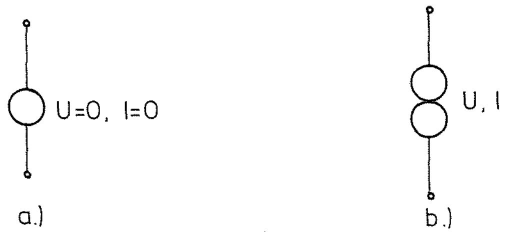

Fig. 1

matrix $\mathbf{Y}$ and cut admittance matrix $\mathbf{Y}_Q$ can not be written for networks containing nullators and norators, i.e. Eq. (1) is not applicable. The method of cut voltages can be employed in the analysis of such networks as follows.

Each branch of the network is considered to be formed by one of the following: an admittance, a Thevenin generator, a Norton generator, a voltage source (with zero inner impedance), a current source (with zero inner admittance), a nullator, a norator. It is naturally expedient to decrease the number of branches taking into account that e.g. a series connection of a norator and a current source is equivalent to a current source, the series connection of a nullator and norator to an open-circuit etc. [1], taking care to maintain the coincidence of the numbers of nullators and norators.

Before the branches of the network were given order numbers for writing the equations, let a tree of the network's graph be chosen with each nullator and each voltage source corresponding to a tree branch and each norator to a chord. Let thereupon the branches of the network be classified into four groups. The first group is formed by the nullators with order numbers $1, 2, \ldots, b_1$. Voltage sources are assigned to the second group with order numbers $b_1+1, b_1+2, \ldots, b_1+b_2$. The third group consists of the norators numbered $b_1+b_2+1, b_1+b_2+2, \ldots, 2b_1+b_2$. The further b_3 branches are included in the fourth group.

The set of cutsets generated by the tree selected will be used for the calculation. The cutsets are numbered to have cutsets $1, 2, \ldots, b_1+b_2$ contain the branches with the corresponding order numbers. Cutsets $1, 2, \ldots, b_1$ are of arbitrary orientation, and cutsets $b_1+1, b_1+2, \ldots, b_1+b_2$ have orientations coinciding with the reference directions of source voltages along the tree branches. Thus in the column matrix of cut voltages the first b_1 elements are zero and the following b_2 elements yield the source voltages of the voltage sources, i.e.

$$\mathbf{V}_Q = \begin{bmatrix} \mathbf{0} \\ \mathbf{U}_{g2} \\ \mathbf{V}_e \end{bmatrix} \tag{4}$$

where $\mathbf{V}_e$ includes the $n-b_1-b_2-1$ cut voltages to be determined, n being the number of nodes in the model.

The left side in Eq. (1), i.e. $\mathbf{Y}_Q\mathbf{V}_Q$ is well known to yield the currents of the admittance in the linearly independent cutsets of the network. Let this be written for our network with the norators taken into account by zero admittances, the nullators and voltage sources by arbitrary admittances. Let the matrix $\mathbf{Y}_Q$ thus written be partitioned according to (4):

$$\mathbf{Y}_Q\mathbf{V}_Q=\begin{bmatrix}\mathbf{Y}_{11} & \mathbf{Y}_{12} & \mathbf{Y}_{13}\\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \mathbf{Y}_{23}\\ \mathbf{Y}_{31} & \mathbf{Y}_{32} & \mathbf{Y}_{33}\end{bmatrix}\begin{bmatrix}\mathbf{0}\\ \mathbf{U}_{g2}\\ \mathbf{V}_e\end{bmatrix} \tag{5}$$

The hypothetical admittances of nullators and voltage sources occur in blocks $\mathbf{Y}_{11}$ of order b_1 and $\mathbf{Y}_{22}$ of order b_2. These play no part in our calculations, and this is why $\mathbf{U}_{g2}$ can be written as the second block of $\mathbf{V}_Q$. Let the diagonal matrix $\mathbf{Y}_2$ be formed by the above hypothetical arbitrary internal admittances of voltage sources. Thus among the matrices on the right side of (1)

$$\mathbf{Y}\mathbf{U}_g=\begin{bmatrix}\mathbf{0}\\ \mathbf{Y}_2\mathbf{U}_{g2}\\ \mathbf{0}\\ \mathbf{Y}_3\mathbf{U}_{g3}\end{bmatrix} \tag{6}$$

where the number of elements in $\mathbf{0}$ is b_1, $\mathbf{Y}_3$ is the admittance matrix of the branches belonging to the fourth group, $\mathbf{U}_{g3}$ is the column matrix formed by the source voltages of the branches in group 4. Column matrix $\mathbf{I}_g$ comprises the source currents:

$$\mathbf{I}_g=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{I}_{g3}\end{bmatrix} \tag{7}$$

where the first and third $\mathbf{0}$ matrices consist of b_1 and the second one of b_2 elements. Let the column matrix formed by the so far unknown currents of the norators be denoted by $\mathbf{I}_n$. Thus, the linearly independent cut equations of the network can be written as

$$\begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \mathbf{Y}_{13} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \mathbf{Y}_{23} \\ \mathbf{Y}_{31} & \mathbf{Y}_{32} & \mathbf{Y}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{U}_{g2} \\ \mathbf{V}_e \end{bmatrix} =$$

$$= \begin{bmatrix} \mathbf{1}_{b1} & \mathbf{0} & \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{0} & \mathbf{1}_{b2} & \mathbf{Q}_{21} & \mathbf{Q}_{22} \\ \mathbf{0} & \mathbf{0} & \mathbf{Q}_{31} & \mathbf{Q}_{32} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{Y}_2\mathbf{U}_{g2} \\ -\mathbf{I}_n \\ \mathbf{Y}_3\mathbf{U}_{g3}-\mathbf{I}_{g3} \end{bmatrix} \tag{8}$$

where the cut matrix has been partitioned in accordance with the three groups of cutsets and four groups of branches, i.e. $\mathbf{1}_{b1}$ and $\mathbf{1}_{b2}$ are unit matrices of order b_1 and b_2, respectively and $\mathbf{Q}_{11}$ is a quadratic block of order b_1. From Eq. (8):

$$\mathbf{Y}_{12}\mathbf{U}_{g2}+\mathbf{Y}_{13}\mathbf{V}_e=-\mathbf{Q}_{11}\mathbf{I}_n+\mathbf{Q}_{12}(\mathbf{Y}_3\mathbf{U}_{g3}-\mathbf{I}_{g3}) \tag{9}$$

$$\mathbf{Y}_{32}\mathbf{U}_{g2}+\mathbf{Y}_{33}\mathbf{V}_e=-\mathbf{Q}_{31}\mathbf{I}_n+\mathbf{Q}_{32}(\mathbf{Y}_3\mathbf{U}_{g3}-\mathbf{I}_{g3}) \tag{10}$$

In case $\mathbf{Q}_{11}$ is nonsingular, (9) yields:

$$\mathbf{I}_n=\mathbf{Q}_{11}^{-1}[\mathbf{Q}_{12}(\mathbf{Y}_3\mathbf{U}_{g3}-\mathbf{I}_{g3})-\mathbf{Y}_{12}\mathbf{U}_{g2}-\mathbf{Y}_{13}\mathbf{V}_e] \tag{11}$$

Substituting this into Eq. (10) the column matrix of the cut voltages sought is

$$\begin{aligned}\mathbf{V}_e=(\mathbf{Y}_{33}-\mathbf{Q}_{31}\mathbf{Q}_{11}^{-1}\mathbf{Y}_{13})^{-1}[(\mathbf{Q}_{32}-\mathbf{Q}_{31}\mathbf{Q}_{11}^{-1}\mathbf{Q}_{12})(\mathbf{Y}_3\mathbf{U}_{g3}-\mathbf{I}_{g3})+\\ +(\mathbf{Q}_{31}\mathbf{Q}_{11}^{-1}\mathbf{Y}_{12}-\mathbf{Y}_{32})\mathbf{U}_{g2}]\end{aligned} \tag{12}$$

In the knowledge of $\mathbf{V}_e$ Eq. (11) yields the current of the norators, and the voltages, currents of the network can be determined. The result has been obtained by the inversion of a matrix of order b_3.

The method of cut voltages is illustrated by the determination of the voltages on the resistances of the simple network shown in Fig. 2. The values of the resistances as well as the hybrid parameters h_{11}, $h_{12}\approx 0$, h_{21}, h_{22} of the transistor are given. Substituting the transistor by a current-controlled current generator (Fig. 3) a model containing nullators and norators can be constructed for the connection (Fig. 4) [1, 5, 6]. The denotations $G=1/R$, $G_{12}=1/R_1+1/R_2$, $G_{11}=1/h_{11}$, $G_e=1/R_e$, $G_{ct}=1/R_c+1/R_t$ have been employed in Figs 3 and 4 out of G_a and G_b one can be chosen at will and $h_{21}=G_b/G_a$. The graph of the model is shown in Fig. 5 with the branches

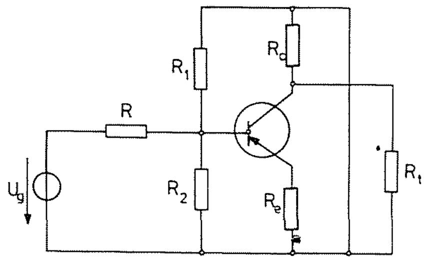

Fig. 2

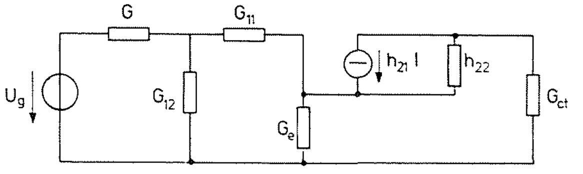

Fig. 3

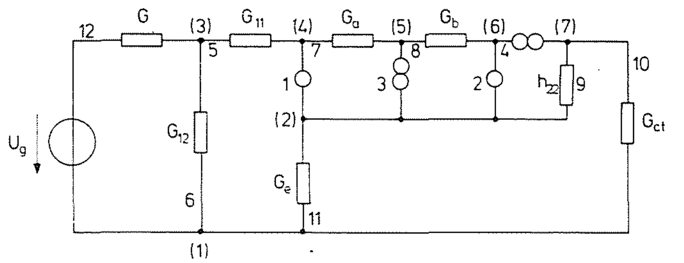

Fig. 4

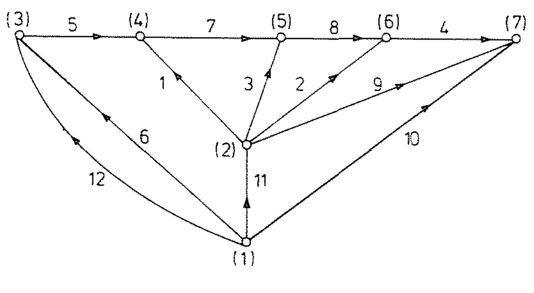

Fig. 5

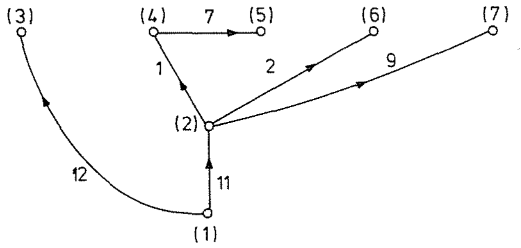

Fig. 6

oriented arbitrarily. The tree of the graph shown in Fig. 6 has been chosen for the calculation. The branches are thereupon given order numbers (Figs 4, 5, 6). The matrix of the set of cutsets generated by the tree is

$$\mathbf{Q}=\left[\begin{array}{cc|cc|ccccccccc}1&0&1&0&1&0&0&-1&0&0&0&0\\0&1&0&-1&0&0&0&1&0&0&0&0\\\hline 0&0&1&0&0&0&1&-1&0&0&0&0\\0&0&0&1&0&0&0&0&1&1&0&0\\0&0&0&0&1&0&0&0&0&1&1&0\\0&0&0&0&-1&1&0&0&0&0&0&1\end{array}\right]$$

where the partitioning of Eq. (8) has been indicated. The number of blocks is less here due to the fact that there is no voltage source in the model. The cut admittance matrix partitioned according to (5) is

$$\mathbf{Y}_Q=\left[\begin{array}{cc|cccc}X&-G_b&G_b&0&G_{11}&-G_{11}\\-G_b&X&-G_b&0&0&0\\\hline G_b&-G_b&G_a+G_b&0&0&0\\0&0&0&G_{ct}+h_{22}&G_{ct}&0\\G_{11}&0&0&G_{ct}&G_{11}+G_{ct}+G_e&-G_{11}\\-G_{11}&0&0&0&-G_{11}&G_{11}+G_{12}+G\end{array}\right]$$

X denotes the elements of the matrix that can not be interpreted. Since no voltage or current source or Norton generator is present in the network examined, $\mathbf{U}_{g2}$ and $\mathbf{I}_{g3}$ do not exist, the excitation is taken into account by column matrix

$$\mathbf{YU}_g = \begin{bmatrix} \mathbf{0} \\ -GU_g \end{bmatrix}$$

where the number of elements in $\mathbf{0}$ is 11. Hence according to Eq. (12):

$$\mathbf{V}_e = \begin{bmatrix} U_7 \\ U_9 \\ U_{11} \\ U_{12} \end{bmatrix} = D^{-1} \begin{bmatrix} G_{11}[(G_{ct}+h_{22})(G_{11}+G_{ct}+G_e)-G_{ct}^2-G_{11}(G_{ct}+h_{22})]GU_g \\ G_{11}[G_a G_{ct}+G_b(G_{11}+G_{ct}+G_e)-G_{11}G_b]GU_g \\ -G_{11}[G_a(G_{ct}+h_{22})+G_b G_{ct}]GU_g \\ -[G_a(G_{ct}+h_{22})(G_{11}+G_{ct}+G_e)+G_{ct}(G_{11}G_b-G_a G_{ct})]GU_g \end{bmatrix}$$

$$D=(G_{11}+G_{12}+G)\,[G_a(G_{ct}+h_{22})(G_{11}+G_{ct}+G_e)+G_{ct}(G_{11}G_b-G_a G_{ct})]-$$

$$-G_{11}^2[G_a(G_{ct}+h_{22})+G_b G_{ct}]\,.$$

Thus, the cut voltages have been determined. The cut voltages associated with the nullators are zero ($U_1=0$, $U_2=0$).

Summary

The method of cut voltages is applicable for the analysis of linear networks consisting of admittances and independent sources. The introduction of nullators and norators permits the construction of models for networks including coupled two-poles, and two-ports that consist of noncoupled admittances, independent sources, nullators and norators. The paper presents a possibility for the application of the method of cut voltages for such models.

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