STATISTICAL PROGRAM SIMULATING LC FILTER TUNING

By

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1. Introduction

In case of mass production, before manufacturing, it is always necessary to check the design in order to estimate the expected yield. The reliability of this control depends on the accuracy of the simulation of the production phases.

The most important phase of LC filter production is the tuning of the mounted circuits. This paper deals with the statistical program ISOA* [1], the main task of which is the simulation as well as statistical evaluation of the aforementioned production phase. Tuning is formulated as a nonlinear constrained optimization problem. A new method for solving the above problem and the principle of ladder network analysis is discussed.

2. Circuit and specification

LC filters are practically used in ladder form. In order to compensate the loss of circuit elements an equalizer is often included in the filter unit. Thus the allowable circuit configuration is a two-section ladder network built up from a given branch set (25 different branches of maximum 5 elements), where the filter and equalizer sections are separated. Each branch from the set can be either in series or in parallel position, although the types of position don't have to alternate.

According to the handling in the program, any network element and/or its value can be characterized as follows:

(i) statistical element (usually capacitances): it has a given probability density function in its tolerance range for statistical simulation,

(ii) varying element (usually inductances): its value can vary in a given tuning range during the simulated tuning process,

* The name ISOA is abbreviation of the words input, statistics, optimization and analysis.

(iii) fixed element (usually the terminating resistances or any in the problem unimportant element): its value is constant during both the statistical and the tuning simulation,

(iv) any inductance can be pretuned before the tuning process to a given resonant frequency either with a given capacitor or with a series /parallel combination of two capacitors,

(v) any inductor/capacitor can be ideal or lossy, the Q-factor can be of quadratic frequency dependence,

(vi) any Q-factor can be statistical with uniform distribution in a common tolerance range.

The filter requirements are in the form of the commonly used gradual tolerance scheme. They can specify attenuation and/or reflection. For statistical purposes there can simultaneously be given several specifications. In the statistical checking it is possible to use more frequency points than those in the tuning process. The attenuation requirements either refer to absolute values or give the deviations from a reference attenuation.

3. Program structure and statistical investigations

The structure of the program is shown in the flow chart, Fig. 1. First the input data (circuit description, specifications, statistical informations) are read in, printed out and reorganized, then the program picks up the random values of statistical elements, pretunes inductances, if necessary, for resonant frequencies and calls the optimization section that simulates the tuning process. If the tuning in a given number of iterations isn't successful the picked circuit sample is considered as non-intunable. Otherwise optimization stops when specification is first met. When tuning is finished, all data necessary for statistical evaluation are stored, then a new random value selection follows.

The Monte Carlo cycle having finished the program gives the following statistical results concerning the stage after tuning:

(i) yield estimation (the rate of intunable circuit samples—in case of reflection requirement both for attenuation and reflection),

(ii) histograms of 32 intervals for each frequency containing the characteristics of circuit samples (in the passband the reflection characteristics as well),

(iii) global histogram containing worst case points in the passband.

The program gives the possibility of statistical checking the postproduction behaviour of the filter regarding either temperature dependence or aging. If the input data contain requirements (there may be more than one) and statistical information referring to one of these investigations, the program, starting from the tuned circuit samples, gives the same statistical results for temperature dependence or aging as for tuning phase.

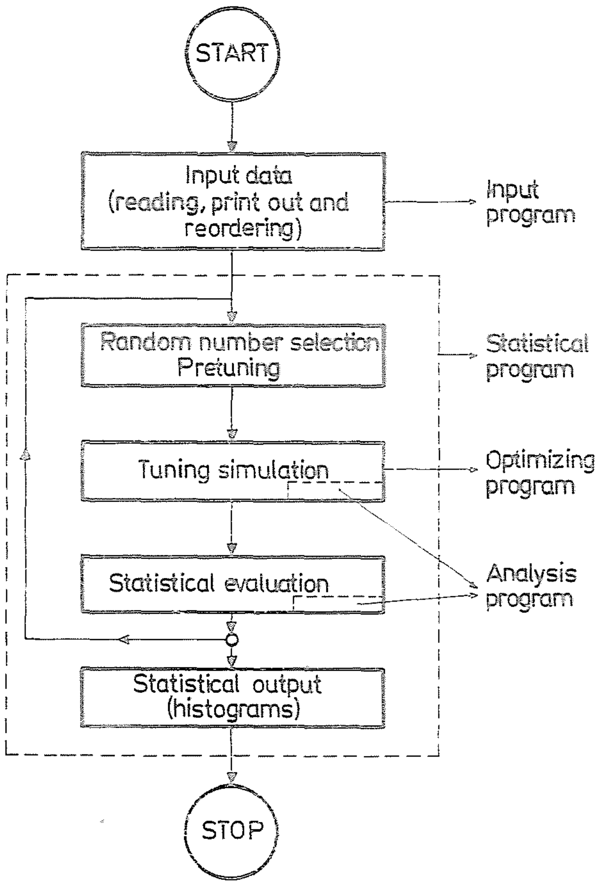

Fig. 1

In addition to its main task the program can be used for special purposes as well:

(i) If there is no element to be tuned, then the optimizing program is out of action, thus a simple Monte Carlo analysis of the circuit is carried out (no tuning).

(ii) If the number of the Monte Carlo samples is zero, then the statistical program is felt out, thus only a single circuit sample is optimized starting with the nominal values. As any network element, just like all ones, can be taken varying, the optimizing program improves the result of an initial design if the requirements are strict enough. Therefore the program can really be used for computer aided design.

4. Tuning as optimization problem

In the tuning process the tunable elements are varied until setting such values that the circuit meets the specification. Simulation of tuning can mathematically formulated as an optimization problem [2].

Let the requirement of Fig. 2 be given. Tuning is necessary if the filter characteristic $a(\omega, \mathbf{x})$ doesn't meet the specifications $\alpha_l(\omega)$ and $\alpha_u(\omega)$. Tuning leads to decreasing the maximum error $E(\mathbf{x})$ and in case of successful tuning $E(\mathbf{x}) \leqq 0$.

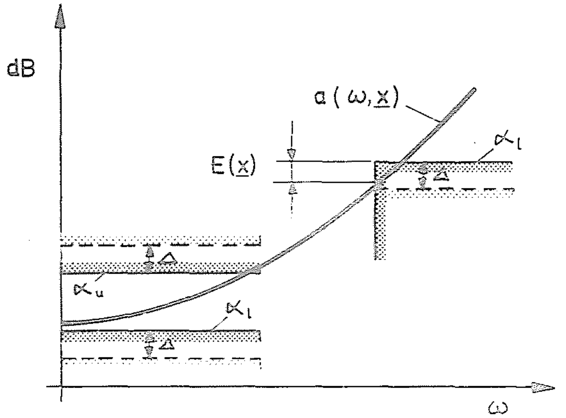

Fig. 2

Thus the following optimization problem is to be solved:
Minimize

$$E(\mathbf{x}) = \max_{\Omega_l,\, \Omega_u} \{[\alpha_l(\omega_k) - a(\omega_k, \mathbf{x})]_{\omega_k \in \Omega_l},\ [a(\omega_k, \mathbf{x}) - \alpha_u(\omega_k)]_{\omega_k \in \Omega_u}\} \tag{1}$$

At the end of tuning the following inequalities must hold:

$$\begin{aligned} a(\omega_k, \mathbf{x}) &\geqq \alpha_l(\omega_k) \qquad \omega_k \in \Omega_l \\ a(\omega_k, \mathbf{x}) &\leqq \alpha_u(\omega_k) \qquad \omega_k \in \Omega_u \end{aligned} \tag{2}$$

In the above expressions $\mathbf{x}$ denotes the vector of tunable elements, i.e. the variables of optimization; $a(\omega_k, \mathbf{x})$ is the insertion loss of the filter at frequency ω_k; Ω_l and Ω_u are the frequency sets having the lower and upper bound specifications α_l and α_u, respectively. Since $a(\omega_k, \mathbf{x})$ is a nonlinear function of $\mathbf{x}$, the task formulated in expressions (1) and (2) is a nonlinear constrained optimization problem.

The constrained problem can be reduced to unconstrained one by means of, for example, the Sequential Unconstrained Minimization Technique, SUMT, of Fiacco—McCormick [3]. In this way the original problem

$$\begin{aligned} &\text{minimize} && f(\mathbf{x}) \\ &\text{subject to} && g_k(\mathbf{x}) \geqq 0 \qquad k=1,2,\ldots \end{aligned} \tag{3}$$

is converted to the following sequence of unconstrained problems

$$\begin{aligned} &\text{minimize} && F(\mathbf{x}, r_j) = f(\mathbf{x}) + r_j P(\mathbf{x}) \\ & && P(\mathbf{x}) = \sum \frac{1}{g_k(\mathbf{x})} \qquad r_j > r_{j+1} > 0 \qquad j=1,2,\ldots \end{aligned} \tag{4}$$

The penalty term $P(\mathbf{x})$ holds the consequences of the constraints and its influence is decreased step by step by r_j, hence the minimum of F converges to that of f.

The SUMT supposes an initial value $\mathbf{x}_0$ taking place in the feasible region, i.e. $g_k(\mathbf{x}_0) \geqq 0$ holds for all k. The convergence of the solution namely requires all g_k not to alter their sign, otherwise P would have a high negative or positive value near to the zero-crossing of any g_k. Since at the beginning of the tuning at least one of the inequalities (2) does evidently not hold, the original problem of Eqs (1) and (2) has to be rewritten.

If the specifications are modified by $\Delta > E(\mathbf{x}_0)$, as shown in Fig. 2 in dashed line, but $E(\mathbf{x})$ is calculated from the original specifications, then the inequalities

$$\begin{aligned} g_k(\mathbf{x}) &= a(\omega_k, \mathbf{x}) - [\alpha_l(\omega_k) - \Delta] > 0 \qquad \omega_k \in \Omega_l \\ g_k(\mathbf{x}) &= [\alpha_u(\omega_k) + \Delta] - a(\omega_k, \mathbf{x}) > 0 \qquad \omega_k \in \Omega_u \end{aligned} \tag{5}$$

together with Eq. (1) are suitable for applying the SUMT. Thus in the unconstrained problem

$$\text{minimize} \qquad F(\mathbf{x}, r_j) = E(\mathbf{x}) + r_j P(\mathbf{x}) \tag{6}$$

$$P(\mathbf{x}) = \sum_{\omega_k \in \Omega_l} \frac{1}{a(\omega_k, \mathbf{x}) - \alpha_l(\omega_k) + \Delta} + \sum_{\omega_k \in \Omega_u} \frac{1}{-a(\omega_k, \mathbf{x}) + \alpha_u(\omega_k) + \Delta} \tag{7}$$

$P(\mathbf{x}) > 0$ is always valid. The solution $\hat{\mathbf{x}}$ giving the minimum corresponds to the solution of tuning if $E(\hat{\mathbf{x}}) \leqq 0$, otherwise the given circuit sample is non-intunable.

To get the solution of the problem in Eq. (6) it is applied a quadratic method similar to that in [4], which uses only first order derivatives of the objective function $F(\mathbf{x})$. This method consists of two steps in every iterations. First a direction, in which $F(\mathbf{x})$ decreases, is selected, then the minimum of $F(\mathbf{x})$ along the given direction is searched. Since in our case the objective function $F(\mathbf{x})$ is not differentiable because of the term $E(\mathbf{x})$, i.e. the maximum error may take place at different frequencies, we have made the following modifications. To select the direction only the differentiable term $P(\mathbf{x})$ is used and in the second step the minimum of the actual error $E(\mathbf{x})$ is looked for by a Fibonacci search. These modifications resulted in significantly less calculations and better convergency [2].

5. Ladder network analysis

The analysis of the filter is necessary for both optimization, i.e. tuning simulation, and statistical investigation. In order to have as quick run as possible, a special ladder network analysis program is used that is based on the continuants [5] and is written in Usercode.

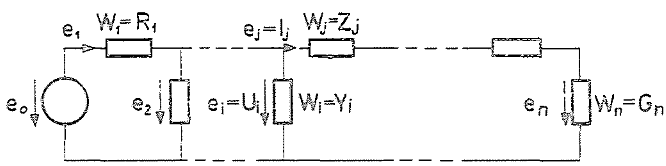

Fig. 3

Continuants are in connection with the current-voltage relationships of the ladder branches. Using the notations in Fig. 3, any branch quantity e_{i-1} can be expressed by those of the next two branches, i.e. by e_i and e_{i+1} and, therefore, after step by step substitution, by that of the last branch, i.e. by e_n, as follows

$$e_{i-1} = W_i e_i + e_{i+1} = \dots = K_i^n e_n$$
$$K_n^n = W_n \qquad K_{n+1}^n = 1 \tag{8}$$

Substituting in Eq. (8) for e_i and e_{i+1} the appropriate forms expressed by e_n we get

$$K_i^n = W_i K_{i+1}^n + K_{i+2}^n \tag{9}$$

The K's are the continuants, their definition is given in Eq. (8), and Eq. (9) contains the recursion formula for computing them.

Taking the voltage source to the output and changing the direction of currents we get the analogue expressions

$$e_{i+1} = K_1^i e_1 \qquad K_1^1 = W_1 \qquad K_1^0 = 1 \tag{10}$$

$$K_1^i = W_i K_1^{i-1} + K_1^{i-2} \tag{11}$$

Applying Eqs (8) to (10), for transmission factor, for input impedance and for reflection coefficient we get, respectively,

$$\Gamma \equiv \frac{e_0}{2e_n}\sqrt{\frac{R_n}{R_1}} = \frac{K_1^n}{2\sqrt{W_1 W_n}} \qquad Z_{in} \equiv \frac{e_2}{e_1} = \frac{K_3^n}{K_2^n}$$

$$r \equiv \frac{Z_{in}-R_1}{Z_{in}+R_1} = 2\frac{K_3^n}{K_1^n} - 1 \tag{12}$$

The first order sensitivities necessary for optimization can be calculated using the expressions [5]

$$S_i^a \equiv \frac{\partial a}{\partial W_i} = Re\frac{\partial \ln \Gamma}{\partial W_i} = Re\frac{K_1^{i-1}K_{i+1}^n}{K_1^n}$$

$$S_i^{a_r} \equiv \frac{\partial a_r}{\partial W_i} = -Re\frac{\partial \ln r}{\partial W_i} = 2W_1(-1)^i Re\left(\frac{K_{i+1}^n}{K_1^n}\right)^2 \tag{13}$$

The structure of the branches in the branch set of the program gives the possibility for calculating the branch immittancies W_i and their first-order sensitivities $\partial W_i/\partial x_j$, regarding the elements x_j, using continuants as well, that are related to the element immittancies.

The possibility for recursive calculation of continuants as well as the simple expressions of Eqs (12) and (13) gave a very efficient analysis program.

6. Conclusion

In the paper the main feature of the statistical program ISOA has been presented. The program runs on an ICL System 4–70 computer, the storage requirement is 130 kByte with a three-segment overlay structure. It is mainly written in FORTRAN IV, but the analysis part and some little subroutines are in Usercode.

Some filters have been tested by the program and useful information has been reached about the quality of the design as well as about the necessary tolerance assignment to the circuit elements.

The program has also been used by Telefongyár with great success.

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Summary

A statistical program is presented that simulates tuning and temperature dependence or aging of LC filters. The program gives the yield of the simulated tuning process as well as detailed statistical information about the characteristics of the tuned filters. Tuning is handled as a nonlinear optimization problem for which a new solution method is given. The outline of the ladder network analysis is presented, too.

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