PARAMETER ESTIMATION OF CONTINUOUS DYNAMIC SYSTEMS FROM SAMPLED SIGNALS

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Received October 5, 1979
Presented by: Prof. Dr. R. TUSCHÁK

1. Introduction

The usual applied identification method of continuous dynamic systems consists of two steps. First the discrete model of the continuous system is determined from sampled signals by discrete identification technique. The applied algorithm depends on the structure of the system and the noise. The second step of the model construction is the determination of the equivalent linear continuous system from the obtained discrete model.

The paper presents a method by means of which the parameters of the continuous process can be directly obtained if the Laplace transform of the input signal is known. The used technique is the generalization of Prony's method. The paper also deals with the connection of Prony's and Stieglitz—McBride's algorithms.

2. Statement of problem

The studied system is shown in Fig. 1. The continuous input and output signal of the process are $u(t)$ and $y(t)$, respectively. The sampled values of these signals are used for the model construction. They are denoted by

$$u_k = u(t=kh)$$

$$y_k = y(t=kh)$$

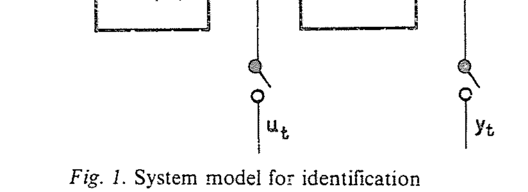

Fig. 1. System model for identification

where h is the sampling period. The paper assumes that the input of the system is continuous, and its Laplace transform is known. The purpose is the determination of the transfer function $H(s)$, the dynamics of the process.

The simplest solution of the above task is the determination of the discrete transfer function $Y(z)$ from the sampled output signal. We can use, for example, Prony's method by means of which the parameters of the numerator and the denominator can be estimated separately. Since the input signal $\delta(t)$ is known, the equivalence continuous form of the discrete transfer function $Y(z)$ can be determined. Using the Laplace transform of the input yields the continuous transfer function of the process:

$$H(s)=\frac{Y(s)}{U(s)}$$

Since the transfer function $Y(s)$ is estimated from the measured output signal, the exact values of the poles and zeros of the input cannot be obtained. From $Y(s)$ the zeros and the poles of the input cannot be cancelled.

In model construction it is advisable to take into consideration the effects which don't correlate with the input signal (Fig. 2). In the figure $N(s)$ represents the dynamics of noise, and it is assumed that the source noise is white. Another problem of the above suggested method is that the algorithm indirectly estimates the parameters of the input signal. The method presented in this paper eliminates the above mentioned disadvantages.

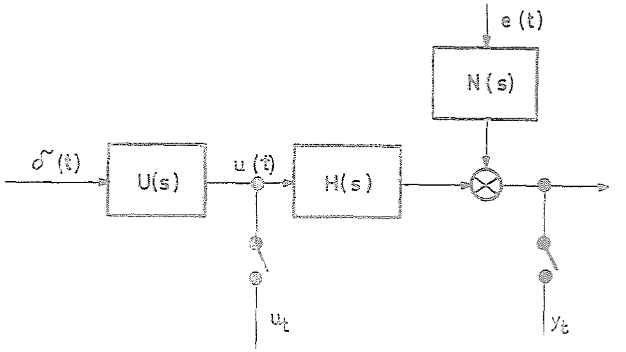

Fig. 2. Investigated continuous system

3. Process model for identifying the sampled system

Let us consider that it is desired to identify the discrete process from the z transform of the input signal and the sampled output values. In case of a given input the discrete equivalent form of the process in Fig. 2 is shown in Fig. 3. The model used for identification is

$$y_t = z^{-d}\frac{B(z^{-1})}{A(z^{-1})}u_t + \frac{C(z^{-1})}{A(z^{-1})}e_t \tag{1}$$

where the dynamics of the process is represented by the first term in the right-hand side of the equation. The second term is the noise model by means of which the effect of the noise on the output signal is taken into consideration.

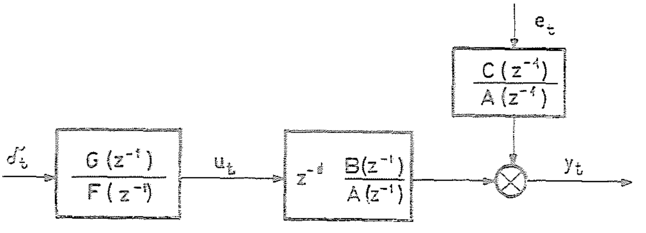

Fig. 3. Discrete representation of a continuous system

The source noise e_t is assumed to be of normal distribution with finite variance and zero mean value. The z transform of the input signal is considered to exist, and to be known (for example: step function, sine function). The sampled input can be written in the following form:

$$u_t = \frac{G(z^{-1})}{F(z^{-1})}\delta_t \tag{2}$$

where

$$\delta_t = \begin{cases} 1 & \text{if} \quad t=0 \\ 0 & \phantom{\text{if}} \quad t\neq 0. \end{cases}$$

The structures of the polynomials in the system equation and in the impulse function of the input are

$$A(z^{-1}) = 1 + a_1 z^{-1} \ldots + a_n z^{-n}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} \ldots + b_m z^{-m} \qquad m \leq n$$

$$C(z^{-1}) = 1 + c_1 z^{-1} \ldots + c_k z^{-k}$$

$$G(z^{-1}) = g_0 + g_1 z^{-1} \ldots + g_p z^{-p}$$

$$F(z^{-1}) = 1 + f_1 z^{-1} \ldots + f_q z^{-q} \tag{3}$$

If the step function has no jump at $t=0$, then $b_0=0$. We remark here that the coefficients of the polynomial $B(z^{-1})$ depend on the type of the continuous input signal. The coefficients of the denominator are, however, not modified by the applied input signal.

4. Determination of system parameters

The parameters of the system can be determined from

$$y_t = z^{-d}\frac{B(z^{-1})}{A(z^{-1})}\frac{G(z^{-1})}{F(z^{-1})}\delta_t + \frac{C(z^{-1})}{A(z^{-1})}e_t \tag{4}$$

where the z transform of the input is assumed to exist. Let us rewrite Eq. 4 in the following form

$$A(z^{-1})F(z^{-1})y_t = z^{-d}B(z^{-1})G(z^{-1})\delta_t + C(z^{-1})F(z^{-1})e_t\,. \tag{5}$$

From Eq. 5 it is seen that the first term in its right-hand side is zero if $t>d+m+p$, that is

$$A(z^{-1})F(z^{-1})y_t = C(z^{-1})F(z^{-1})e_t \qquad t>d+m+p \tag{6}$$

This fact gives possibility to determine the parameters of the system in two steps. First the parameters $\mathbf{a}=[a_1, a_2, \dots a_n]^T$ and $\mathbf{c}=[c_1, c_2, \dots c_k]^T$ can be determined then using these vectors the coefficients of the denominator of the process dynamics can be estimated. On the basis of Eq. 6 the parameter vectors ($\mathbf{a}$ and $\mathbf{c}$) can be estimated iteratively, for example by quasi-linearization methods. If these methods are applied, it is useful to introduce the following filtered signals:

$$y_t^F = F(z^{-1})y_t \qquad e_t^F = F(z^{-1})e_t\,. \tag{7}$$

The presented algorithm gives possibility to estimate the poles of the discrete system. Since the transformation of the poles does not depend on the input signal, the poles of the continuous process (α_i) can be obtained by means of the poles of the discrete system z_i:

$$\alpha_i = \frac{1}{h}\ln z_i\,.$$

Here it is assumed that there are no pole multiplicities.

The residua of the continuous system can be estimated on the basis of the following equation

$$\sum_{i=1}^{n} d_i Z_t \left\{ U(s) \frac{1}{\alpha_i + s} \right\} = y_{t+d} + \frac{C(z^{-1})}{A(z^{-1})} e_{t+d} \tag{8}$$

$$t = 1, 2, 3 \ldots$$

where d_i is the residuum of the pole α_i. Symbol Z_t denotes the $s-z-t$ transformation. In this way the dynamics of the process is

$$H(s) = e^{-dhs} \sum_{i=1}^{n} \frac{d_i}{\alpha_i + s}.$$

The continuous representation of the noise model can be determined, as well. The denominator corresponds to that of the process model. In determining the numerator it has to be taken into consideration that the input of the noise model is white noise.

The special case of the presented algorithm is Prony's method.

5. Prony's method

Prony's method constructs the step function of the system from exponential terms:

$$y(t) = \sum_{k=1}^{r} d_k \exp(\alpha_k t) + e(t) \tag{9}$$

where $y(t) =$ step function;
$r =$ number of the terms;
$d_k =$ coefficient of the terms;
$\alpha_k =$ exponential coefficients.

Since Eq. 9 is a step function, one of the exponential terms is constant, that is, its exponential coefficient (for example α_1) is zero. Prony's method finds the parameter vectors

$$\mathbf{d} = [d_1, d_2, \ldots, d_r]^T \quad \text{and} \quad \boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_r]^T$$

by the following three-stage procedure:

1. It solves the matrix equation

$$\mathbf{Yh} + \mathbf{y} = \boldsymbol{\varepsilon}$$

for $\mathbf{h}$, where

$$\mathbf{Y}=\begin{bmatrix} y_0 & y_1 & \dots & y_{r-1} \\ y_1 & y_2 & \dots & y_r \\ \vdots & & & \\ y_{N-r} & y_{N-r+1} & \dots & y_{N-1} \end{bmatrix}$$

$$\mathbf{y}=[y_r\, y_{r+1} \dots y_N]^T$$

$$\boldsymbol{\varepsilon}=[\varepsilon_r\, \varepsilon_{r+1} \dots \varepsilon_N]^T$$

and

$$\mathbf{h}=[h_0\, h_1 \dots h_{r-1}]^T .$$

For the estimation $N+1$ measured values can be used. ε is the equation error.

2. It determines the roots of the algebraic equation

$$h_0+h_1 z+\dots+h_{r-1}z^{r-1}+z^r=0 . \tag{11}$$

3. By means of the roots the parameter vector $\mathbf{d}$ is estimated on the basis of the equation

$$\mathbf{Z}\mathbf{d}=\mathbf{y}^*+\mathbf{e} \tag{12}$$

where

$$\mathbf{Z}=\begin{bmatrix} 1 & 1 & \dots & 1 \\ z_1 & z_2 & \dots & z_r \\ z_1^2 & z_2^2 & \dots & z_r^2 \\ \vdots & & & \\ z_1^N & z_2^N & \dots & z_r^N \end{bmatrix}$$

$$\mathbf{y}^*=[y_0 y_1 \dots y_N]^T$$

$$\mathbf{e}=[e_0 e_1 \dots e_N]^T .$$

Here $z_i=\exp(\alpha_i h)$.

Using the notation of the previous sections, Eq. 9 can be written as

$$y_t=\frac{B(z^{-1})}{A(z^{-1})}u_t+e_t \tag{13}$$

where

$$u_t=\frac{1}{1-z^{-1}}\delta_t$$

It is assumed that $C(z^{-1})=A(z^{-1})$.

In Prony's method, Eq. 6 becomes:

$$A(z^{-1})\,(1-z^{-1})y_t = A(z^{-1})\,(1-z^{-1})e_t \qquad t>m\,. \tag{14}$$

Eq. 10 is obtained from Eq. 14 by applying the following assumption

$$A(z^{-1})\,(1-z^{-1})y_t = A^*(z^{-1})y_t = \varepsilon_t \tag{15}$$

since $h_i = a_i^*$. From this fact it is seen that Prony's method does not minimize the equation error of Eq. 9. It determines the denominator of the system dynamics unnecessarily in a parameter space higher by one dimension. It is a special problem to recover the unit pole.

By the quasilinearized method [4] based on Eq. 14 the norm of the error in Eq. 9 can be minimized by iteration. Using the prefilter $y_t^F = (1-z^{-1})y_t$ it is unnecessary to execute the iteration in a higher parameter space. The third problem doesn't emerge in applying the suggested method since the unit pole is separated.

Let us investigate the similarity of the estimation methods of the parameter vector $\mathbf{d}$. In the above studied case Eq. 8 can be written as

$$\sum_{i=1}^{r-1} d_i Z_t \left\{ \frac{1}{s} \frac{1}{\alpha_i + s} \right\} = y_t + e_t\,. \tag{16}$$

It is a possibility to directly estimate the coefficients of the polynomial $B(z^{-1})$. On the basis of Eq. 13,

$$y_t = B(z^{-1})u_t^F + e_t \tag{17}$$

From this equation the parameter vector $\mathbf{b}$ can be estimated directly, by noniterative way.

The solution on the basis of Eq. 12 corresponds to the parameter estimation based on Eq. 17 and the determination of the residua by root separation, ignoring the filter of the input signal. This statement is seen from the following equation

$$\sum_{i=1}^{r} \frac{d_i}{(1-z_i z^{-1})}\,\delta_t = \frac{B(z^{-1})}{A(z^{-1})}\,\frac{1}{(1-z^{-1})}\,\delta_t$$

where the left-hand side of the equation is the discrete z transform of the first term in Eq. 9. According to Prony's method the poles can be separated before determining the vector $\mathbf{d}$, and the accuracy of the determination of the poles significantly influences the accuracy of the obtained parameter vector $\mathbf{d}$. In estimating the vector $\mathbf{b}$ this problem does not emerge, or if the determination of the residua is desired, the error is not multiplied. By estimating the residua, Prony's method estimates the coefficient b_0 indirectly, as well.

On the above facts it can be declared that the following method is suitable for application instead of Prony's algorithm.

1. The parameter vector $\mathbf{a}$ is estimated from the equation

$$A(z^{-1})y_t^F = A(z^{-1})\,(1-z^{-1})e_t \qquad t>m$$

by quasilinearization method.

2. The parameter b is determined by the least squares method which uses the filtered input signal

$$u_t^F = \frac{1}{A(z^{-1})\,(1-z^{-1})}\,\delta_t\,.$$

3. The parameters of the continuous system can be obtained by using the roots of equation

$$1+a_1 z^{-1}+\ldots+a_n z^{-n}=0$$

and by applying the residua.

Of course, if we don't want to estimate the parameter vector $\mathbf{a}$ and $\mathbf{b}$ separately, the well-known discrete identification methods can also be used. For example, if $A(z^{-1})=C(z^{-1})$ as in Prony's method, the parameters can be obtained quickly by Stieglitz—McBride's method [4]. If the parameters of the discrete model are known the step function equivalence continuous model is easy to obtain.

Summary

The paper presents a method by means of which the parameters of the continuous process can be directly obtained if the Laplace transform of the input signal is known. The used technique is the generalization of Prony's method. The paper also deals with the connection of Prony's and Stieglitz—McBride's algorithms.

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