MODELLING FIRST- AND SECOND-ORDER SYSTEMS WITH SIGNAL-DEPENDENT PARAMETERS

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Introduction

Nonlinear dynamic systems are generally described by complicated differential equation systems. The determination—identification—of the model parameters needs long computations. Several algorithms have been elaborated for the identification of linear dynamic systems. These identification programs are usually parts of a controller-aided design program package.. The claim arises to extend these identification methods to nonlinear systems.

On the other hand, every real process is known to be more or less nonlinear, but can be considered linear in the close vicinity of a working point. The linear models—valid only for small changes—are likely to have different parameters in different working points. This fact has led to the idea to describe nonlinear dynamic systems by "linear" models with working point- or signal-dependent parameters. The signal—the parameters depend on—may be

— the input signal,
— the output signal,
— any other (external) signal,
— an observable state variable,
— any measurable variable signal.

The signal may be

— constant,
— time function,
— the change of a time function or state variable,
— the sign (direction) of the change, etc.

Dependence of any parameter of linear systems may be written in this form:

— the gain,
— the time constant,
— the damping coefficient,
— the coefficients of the transfer function,
— the dead time,
— the poles and zeros of the transfer function, etc.

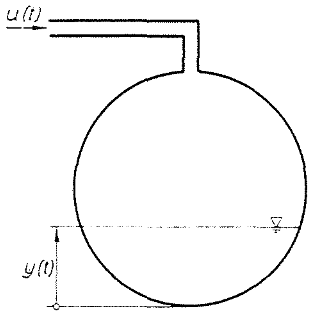

Fig. 1. Horizontal drum boiler

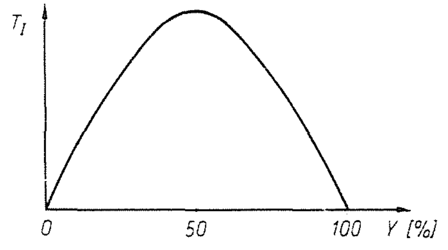

Fig. 2. The integrating time constant as a function of the level

As an example let us consider the level control of a horizontal drum boiler seen in Fig. 1. The relation between the water flow rate ($u(t)$) and the change of the level ($\Delta y(t)$) is

$$\Delta y(t) = \frac{1}{A} u(t) . \tag{1}$$

Transforming Eq. (1) into the Laplace domain, we get

$$Y(s) = \frac{1}{sT_I} U(s) . \tag{2}$$

Here the time constant T_I is proportional to the surface area and therefore is function of the level:

$$T_I = A = A(y(t)) . \tag{3}$$

The dependence is shown in Fig. 2.

Graphical relation between continuous and discrete descriptions

Up-to-date identification methods are based upon the estimation of the parameters of the difference equation describing the system. Thus the identification of a process having signal-dependent parameters requires the knowledge of the parameter-dependence in the discrete model. Namely, knowing the dependence on the signal $x(t)$ of the parameters of the discrete transfer function:

$$\frac{Y(z)}{U(z)}=\frac{\sum_{i=0}^{n} b_i z^{-i}}{\sum_{j=0}^{n} a_j z^{-j}} \tag{4}$$

$$b_i=\sum_{q=0}^{q_i} f_{iq}(x(k))b_{iq}, \tag{5}$$

$$a_j=\sum_{q=0}^{q_j} f_{jq}(x(k))a_{jq}. \tag{6}$$

The difference equation is linear in the parameters

$$\sum_{j=0}^{n}\sum_{q=0}^{q_j} f_{jq}(x(k-j))a_{jq}y(k-j)=$$
$$=\sum_{i=0}^{n}\sum_{q=0}^{q_i} f_{iq}(x(k-i))b_{iq}u(k-i), \tag{7}$$

that can be determined by regression measuring the input-output—and the $x(t)$ signals.

Fig. 3 shows the effect of changing of the gain and the time constant of a first-order lag term

$$W(s)=\frac{K_0}{1+sT_0}=\frac{2}{1+s10} \tag{8}$$

on the coefficients of the equivalent transfer function impulse

$$W(z^{-1})=\frac{b_0+b_1z^{-1}}{1+a_1z^{-1}}. \tag{9}$$

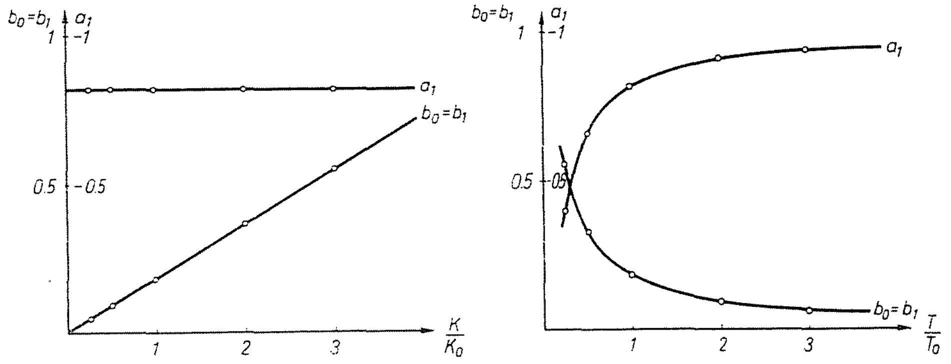

Fig. 3. First-order system

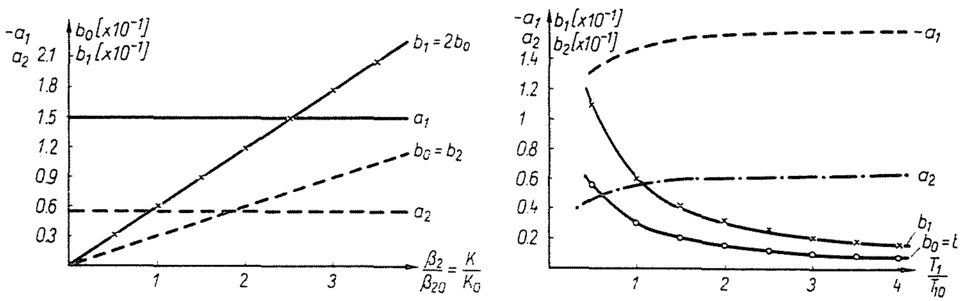

Fig. 4. Second-order aperiodic system

Fig. 4 presents the effect of changing the gain and one of the time constants of a second-order aperiodic system

$$W(s) = \frac{K_0}{(1+sT_{10})(1+sT_{20})} = \frac{2}{(1+10s)(1+5s)} \tag{10}$$

on the coefficients of the equivalent impulse transfer function

$$W(z^{-1}) = \frac{b_0+b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}. \tag{11}$$

Fig. 5 gives the effect of changing the gain, the time constant and the damping factor of a second-order periodic term

$$W(s) = \frac{K_0}{1+2\zeta_0 sT_0+s^2T_0^2} = \frac{2}{1+2\cdot 0.5\cdot 10s+10^2s^2} \tag{12}$$

on the coefficients of the equivalent impulse transfer function.

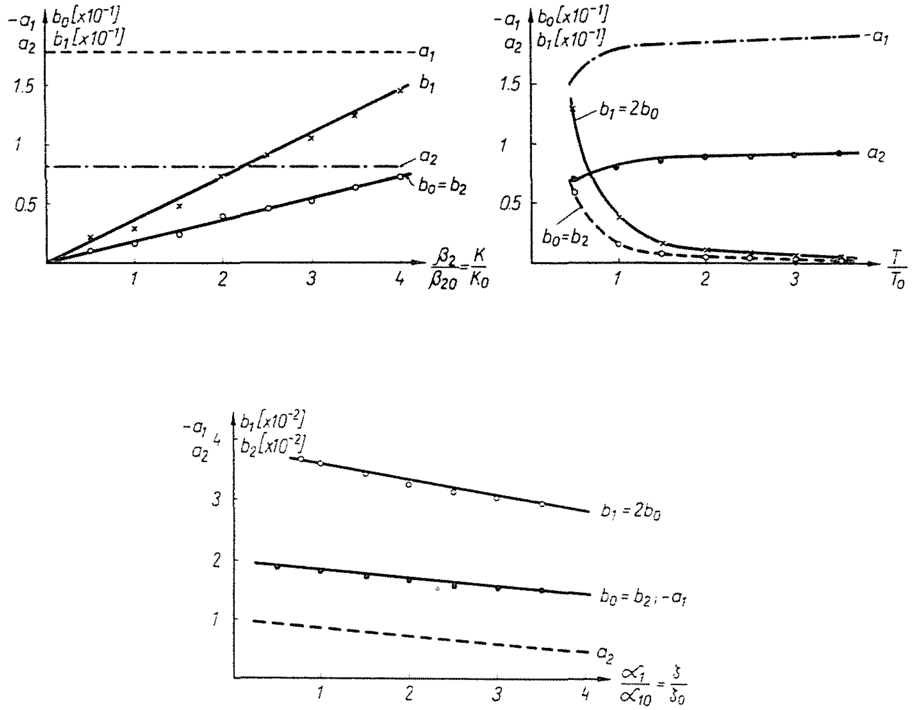

Fig. 5. Second-order periodic system

Here the bilinear transformation [1] was used and the sampling period was

$$h=2.$$

The dependence of the discrete parameters can be read from the diagrams, i.e. it can be determined about what degree Taylor polynomials the discrete parameters of such terms are.

Analytical relation between continuous and discrete descriptions

Let us write the differential equations of first- and second-order processes:

$$T\dot{y}(t)+y(t)=Ku(t)\ ; \tag{13}$$

$$T_1T_2\ddot{y}(t)+(T_1+T_2)\dot{y}(t)+y(t)=Ku(t)\ ; \tag{14}$$

$$T^2\ddot{y}(t)+2\zeta T\dot{y}(t)+y(t)=Ku(t)\,. \tag{15}$$

Using bilinear transformation, the equivalent difference equations have the form

$$(h+2T)y(k)=Khu(k)+Khu(k-1)-(h-2T)y(k-1)\,; \tag{16}$$

$$[4T_1T_2+2(T_1+T_2)h+h^2]y(k)=Kh^2u(k)+$$
$$+2Kh^2u(k-1)+Kh^2u(k-2)-(2h^2-8T_1T_2)y(k-1)-$$
$$-[4T_1T_2-2(T_1+T_2)h+h^2]y(k-2)\,; \tag{17}$$

$$(4T^2+4T\zeta h+h^2)y(k)=Kh^2u(k)+2Kh^2u(k-1)+$$
$$+Kh^2u(k-2)-(2h^2-8T^2)y(k-1)-(4T^2-4T\zeta h+h^2)y(k-2)\,. \tag{18}$$

The discrete models can be proven to be linear in the parameters if the parameters are linear functions of the signal $x(t)$ [2]:

$$K=K_0+K_1x_k(t)\,; \qquad T=T_0+T_1x_T(t)\,; \qquad \zeta=\zeta_0+\zeta_1x_\zeta(t)\,.$$

Formally the models are of multi-input single-output type with transfer functions having common denominators. The quasiinput signals are nonlinear functions of the measured input and output signals. Thus the discrete models suit parameter estimation. They are seen in Fig. 6 for the first-order case, in Fig. 7 and Fig. 8 for second-order aperiodic and periodic cases, respectively.

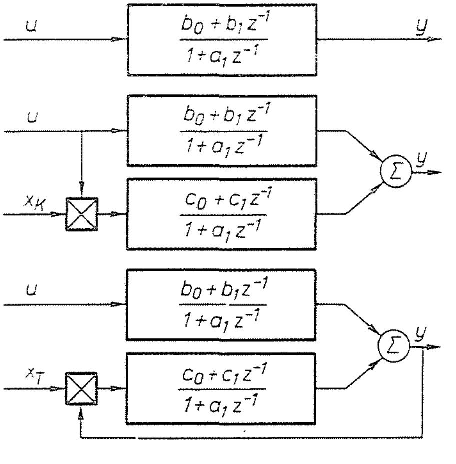

Fig. 6. First-order system

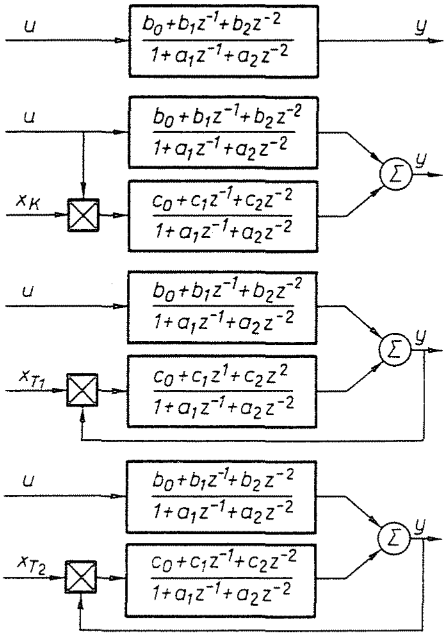

Fig. 7. Second-order aperiodic system

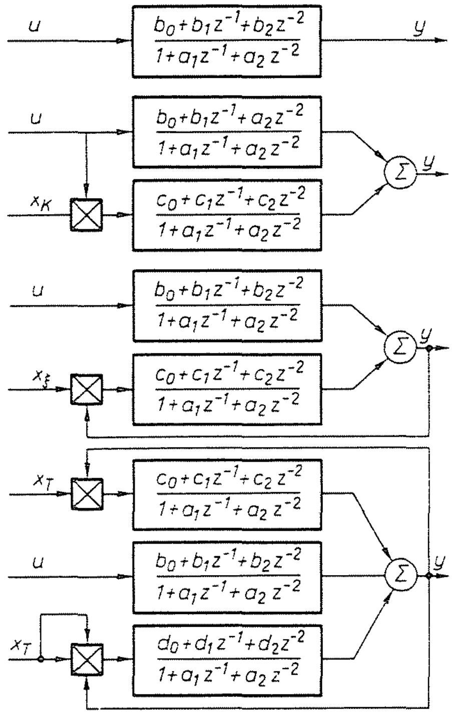

Fig. 8. Second-order periodic system

Conclusions

The aim of the present study was to establish an illustrative model of complicated industrial processes, and to present an identification method for this model class. Several further practical problems still await elaboration, e.g. determination of the signal-dependence of unknown structures etc. They are given in [3] in details. We do hope to have some practical applications soon.

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Summary

A specific class of nonlinear dynamic systems, those with signal-dependent parameters will be discussed. Applying the bilinear transformation the signal-dependent "linear" systems with constant parameters lead to multi-input single-output quasilinear discrete models linear in parameters. These models can be identified by the conventional parameter estimation algorithms.

Parameter dependence of the equivalent discrete models of first- and second-order "linear" lag systems having signal-dependent parameters, is presented both analytically and in graphically. The equivalent difference equations and the block-oriented models are given, too.

References

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