INFORMATION TRANSMISSION WITH INTERMEDIATE STORAGE UNDER THE SPECIAL CONDITIONS OF MICROCOMPUTERS

By

S. CSIBI, L. GYÖRFI, Z. GYÖRFI and I. VAJDA

Research Group on Informatics and Electronics of the Hungarian Academy of Sciences at the Institute of Communication Electronics, Technical University, Budapest

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Intermediate Storage — Queues

In information transmission it is a characteristic requirement that the message should be transmitted to the user at the same rate as it is produced by the source.Processes of this sort have to work, of course, in real time.

These real-time processes are featured by closely related tasks of transmission, coding, estimation of signals and decisions, to be completed in most cases under very strict conditions of reliability and economy.

In everyday practice devising of procedures of this kind was restricted until recently by the difficulty of intermediate storage of the message to be transmitted, or that of the received signal (for conversion and processing), or of leaving a part of processing to the next processing cycle.

The recent development of microelectronics resulted — among others — in an essential new possibility: Today internal storage and control by a stored program are widely possible not only in data transmission, but also in the digital transmission of analog signals.

All these raise the questions, known earlier in computer technology, in connection with message transmission such as wider application of the procedures of searching, estimation and decision, formulation of their rules via recursive algorithms, imposing constraints on the storage and on the computations and a consideration of the processing time ([1] to [5]).

Naturally, storing, e.g., a given segment of the received signal and processing this, not all the prescribed operations may be performed during a working period, resulting in backlog. In other cases the message parts transmitted together (i.e., the packets), are deliberately accepted to be transmitted with more or less delay, in order to better exploit the channel.

In both cases, in performing the transmission procedure, queues are generated in the actually unbroken procedures of information transfer, resulting in queuing problems.

In this paper two examples will be presented for the occurrence and solution of a simple, essential queuing problem.

Successive Development and Preservation of bit-Synchronism

Let us take a binary data source with the following transmission signal:

$$s(t)=\sum_{k=-\infty}^{\infty} m_{\rho(k)}(t-kT)$$

where $\{\rho(k)\}$ is a sequence of random variables taking values 0 and 1. $(P(\rho(k)=1)=P(\rho(0)=1))$. The functions m_0 and m_1 (waveforms) vanish outside the signalling interval $(0, T)$.

Assume the received noisy signal to be:

$$z(t)=s(t-\vartheta(t))+v(t).$$

The trajectory of ϑ within the time interval T can be regarded as constant. Thus, in the interval $[kT, (k+1)T]: \vartheta(t)=\vartheta(kT)$. Here $\vartheta(t)$ $(-\infty<t<+\infty)$ is a stochastic process with a range $(0, T)$ and v is additive noise.

The detection is intended to make a decision on the sequence $\rho(k)$ $(k=0, \pm 1, \pm 2, \ldots)$ in observing the process z, without the preliminary knowledge of the delay ϑ.

Let $\tilde{\vartheta}_k$ be an estimation of $\vartheta(kT)$ and η_k the vector of samples taken in $[kT+\tilde{v}_k, (k+1)T+\tilde{v}_k]$ from process z. Estimate recursively the parameter vector C of the optimum detector (e.g. [1], [2], [5])

$$C_{k+1}=g_k(C_k, \eta_k), \tag{1}$$

and the delay $\vartheta((k+1)T)$ (e.g. [3], [4])

$$\tilde{\vartheta}_{k+1}=f_k(\tilde{\vartheta}_k, \eta_k) \tag{2}$$

Let us decide with respect to $\rho(k)$. Let the estimate be

$$\tilde{\rho}(k)=h(C_k, \tilde{\vartheta}_k, \eta_k). \tag{3}$$

Let us consider an application where a given duration (or number of operations) is available. In a typical case the required time of operations both for (1) and (2) exceeds the signal time T, while (3) may be realized in a fraction of T.

One obvious solution is to perform corrections (1) and (2) according to a fixed schedule, but less frequently. If there is a criterion of deciding about the necessity of correction, then corrections are made rarely but at random. In this case the number of operations left over is a stochastic process, leading to a traffic problem. The length of the queue is to be examined, for instance its average length or distribution.

Let F be this criterion, viz., in the case of $F(\tilde{\vartheta}_k, c_k, \eta_k) \leqq c$ only (3) is performed, otherwise (1), (2), (3). For instance, F may be the sample variance of a given short series of the samples around the sample mean.

Let us assume that the correction is rare enough, and the sequences of events

$$A_k = \{F(\tilde{\vartheta}_k, c_k, \eta_k) \leqq c\}$$

may be regarded as independent.

$$q = P(A_k).$$

Let N_1 be the number of operations required by (1) and (2) and N_2 the number of operations to be performed during T. In this case the number of operations left over for the $n+1$-th bit period is:

$$\xi_{n+1} = (\xi_n - N_2)^+ + \chi_{\bar{A}_n} N_1 .$$

For practical cases $N_1 \gg N_2$.

For the sake of simplicity, $K = \frac{N_1}{N_2}$ is assumed to be an integer. In this case

$$\tilde{\xi}_{n+2} \triangleq \frac{\xi_{n+1}}{N_2} = (\tilde{\xi}_n - 1)^+ + \chi_{\bar{A}_n} K .$$

The generating function of the stationary distribution of $\tilde{\xi}_n$ can be computed (see the last section). Hence:

$$\mathbf{E}\xi_n = N_2 \mathbf{E}\tilde{\xi}_n \sim N_2 \frac{P(\bar{A}_1)N_1/N_2}{1 - P(\bar{A}_1)N_1/N_2} \cdot \frac{1}{2} \frac{N_1}{N_2} \leqq$$

$$\leqq \frac{N_1}{2} \frac{1}{1 - P(\bar{A}_1)N_1/N_2} .$$

Concentration of Data Packets

As another illustration of the same traffic modell let us consider the concentration of digital channels. More exactly, let the messages of length $K-F$ from any channel be condensed into one packet of length K. Let the collected packets queue up at the input of a common, high-speed digital channel.

In our discrete time, service model the time unit is the signal time of the common channel. Let A_k be the event that in the k-th time unit no new packet arrives. Then the number of bits left over for the time unit $k+1$ is

$$\xi_{k+1}=(\xi_k-1)^+ + \chi_{\bar{A}_k}K$$

for which, according to the last section

$$\mathbf{E}\xi_k \sim F(K)=\frac{P(\bar{A}_1)K}{1-P(\bar{A}_1)K}\cdot\frac{K}{2}.$$

Let us compute $P(A_1)$, using the traffic parameters of the channels to be concentrated. Let V_i be the average data rate of the i-th terminal (in bit/sec unit), W the data rate of the common channel (in bit/sec). A packet of length K consists of a head of length F and of a message of length $K-F$.

In this case the average utilization factor of the channel is

$$C \triangleq \frac{\sum_{i=1}^{M} V_i}{W}.$$

The process is an ergodic one. For this reason

$$P(\bar{A}_1)=\frac{C}{K-F}.$$

Therefore

$$\mathbf{E}\xi_n \sim F(K)=\frac{\frac{CK}{K-F}}{1-\frac{CK}{K-F}}\cdot\frac{K}{2}=\frac{C}{1-\frac{F}{K}-C}\,\frac{K}{2}$$

The function $F(K)$ is interpreted in the range

$$K > K_{\min} \triangleq \frac{F}{1-C}$$

and the minimum of $F(K)$ is (Fig. 1)

$$K_{\text{opt}} = 2K_{\min}$$

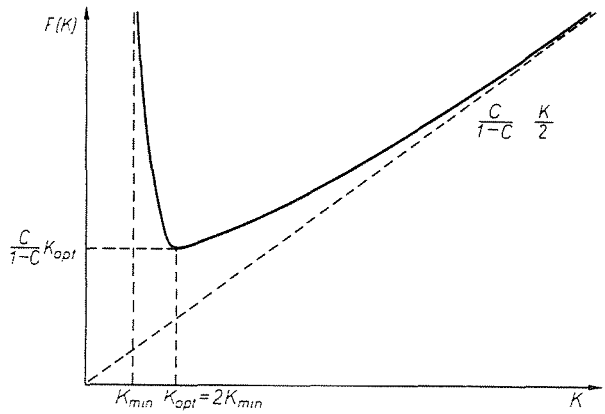

Fig. 1. Expectation of the queue length *vs* packet length

Knowing the head length F and the channel utilisation factor C, the expectation of the queue length can be minimalized by selecting the proper packet length. If, e.g., $C = 0.8$, then $K_{\text{opt}} = 10F$ and $F(K) = 40F$.

Traffic Problem of Constant Response Time and Independent Arrival

Let us examine more closely the traffic problem, described in the previous two sections.

Let

$$\xi_{n+1} = (\xi_n - 1)^+ + \chi_{\bar{A}_n} K \tag{4}$$

Here $A_1, A_2, \ldots$ are independent events and

$$q = P(A_n) \quad (1 - q < 1/K)$$

(for every n).

Let us examine the stationary distribution of ξ_n and the corresponding expectation.

Let F_N be the probability that the length of the queue is not less than N:

$$F_N = P(\xi_n \geqq N) = P(\xi_1 \geqq N)$$

(for every n).

In this case on the one hand

$$F_0 = 1 \tag{5}$$

and on the other hand

$$F_1 = 1 - P(\xi_n = 0) = 1 - p_0 \tag{6}$$

can be computed, using (4), namely

$$\mathbf{E}\xi_{n+1'} = \mathbf{E}\chi_{\{\xi_n \geqq 1\}}(\xi_n - 1) + (1-q)K = \mathbf{E}\xi_n - \mathbf{E}\chi_{\{\xi_n \geqq 1\}} + (1-q)K$$

$$F_1 = (1-q)K \quad \text{and} \quad p_0 = 1 - (1-q)K \tag{7}$$

(4) results also in

$$\{\xi_{n+1} \geqq \mathrm{N}\} = \bar{A}_n \bigcap \{(\xi_n - 1)^+ + K \geqq N\} \bigcup$$

$$\bigcup A_n \bigcap \{(\xi_n - 1)^+ \geqq N\}.$$

Hence, for $N \leqq K$

$$F_N = (1-q) + qF_{N+1} \tag{8}$$

for $N > K$

$$F_N = (1-q)F_{N+1-K} + qF_{N+1}. \tag{9}$$

It can be proven, by induction, that for $1 \leqq N \leqq K+1$, using (6) and (8):

$$F_N = 1 + p_0(1/q)^{N-1}. \tag{10}$$

The generating function of $\{F_N\}$ can be presented in closed form, using (9) and (10). Let G be the generating function of F_N.

That is:

$$G(z) = \sum_{N=1}^{\infty} F_N z^N.$$

In this case:

$$G(1)=\sum_{N=1}^{\infty} F_N=\sum_{N=1}^{\infty} NP(\xi_n=N)=\mathbf{E}\xi_n.$$

From (9) and (10):

$$G(z)=\frac{(z-q)\sum_{N=1}^{K} z^N+qp_0z-pz^{K+1}-(1-q)(1-p_0)z^{K+1}}{z-q-(1-q)z^K}.$$

Using the l'Hospital formula:

$$\mathbf{E}\xi_n=G(1)=\frac{1-p_0}{p_0}\left(\frac{K+1}{2}+p_0\right)\sim F(K)\triangleq$$

$$\triangleq\frac{1-p_0}{p_0}\frac{K}{2}=\frac{(1-q)K}{1-(1-q)K}\frac{K}{2}.$$

Summary

A traffic problem of constant response time is examined in connection with two different information transmission and processing problems and its consequences are shown in an estimation-decision problem in fact, a data transmission problem with no previous synchronisation. The other application is the examination of the queue length within a data concentrator.

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Prof. Dr. Sándor CSIBI
Dr. László GYÖRFI
Dr. Zoltán GYÖRFI
Dr. István VAJDA

H-1521 Budapest