MODELLING OF BINARY RADIO TRANSMISSION CHANNELS

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The joint modelling of noisy radio channels and decoding algorithms is very profitable in the evaluation of error control procedures. The real behaviour of the decoders can namely be characterized only approximately. In addition, the various codes respond to the same error structure differently. The characteristic features of the codes can be judged or compared more reliably on the basis of their modelling.

Some elementary signals (bits) fail in the radio channel during transmission. Experience shows that the existence of errors in certain channels is independent when errors have occurred formerly. Such errors are called independent and the channel is termed symmetrical.

A symmetrical channel can be characterized by one parameter p:

$$\mathrm{Pr}(0/0)=p \tag{1}$$

$$Pr(1/0)=q=1-p \tag{2}$$

In some channels errors will appear more frequently when other errors occur in their neighbourhood. In such cases most of the errors are concentrated to relatively short intervals. Such errors are called burst-errors and the channel is a bursty channel. To characterize such channels a further parameter is needed:

$$Pr(1/1)=P \tag{3}$$

$$Pr(0/1)=Q=1-P \tag{4}$$

The mathematical model of a symmetrical or bursty channel can easily be constructed by the use of parameters p, q or p, q, P, Q.

In real channels the independent and burst-type errors appear mixed. The ratio of independent errors to burst errors may be very different. Sometimes there are long error-free intervals, but also long bursts or bursts close to each other can be observed. The behaviour of real channels cannot generally be characterized by a simple model.

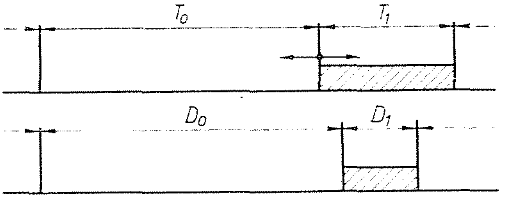

Fig. 1. Comparison of the parameters T_0, T_1 and D_0, D_1

A better model well adaptable to real channels can be constructed by proper combination of the simple models mentioned above. A practicable mathematical model, the Fritchman model forms an $n \times n$ Markov matrix. The elements of this matrix are probabilities p_{ii}, P, q_{in} and Q_{ni}. In the main diagonal and in the last row of the matrix there are the probabilities $p_{11}, p_{22}, \ldots, P$ and $Q_{n1}, Q_{n2}, \ldots, P$, respectively. With a proper choice of the probabilities good channel models can be realized. It is simple to write a computer program for simulating various noisy channels.

The error-correction ability of certain convolutional codes can be characterized by

— the length of a correctable burst D_1,
— the length of the error-free interval D_0 between errors or bursts (so-called guard space) which is necessary for correction.

These parameters are, with some restriction, constant for a coding system.

If the error structure of a transmission channel is characterized similarly by

— the length distribution of the bursts (the burst length is T_1)
— the length distribution of the error-free intervals (the error-free interval being T_0),

then the properties of the error correcting procedure can simply be examined.

Figures 1a and 1b show these parameters of the channel and of the code system, respectively. The error-free and erroneous intervals T_0 and T_1 can fluctuate between wide limits, while the parameters D_0 and D_1 are, with certain simplifications, constant.

The decoder will make an error,

— when an error-free portion is shorter than the guard-space D_0,
— when the burst length T_1 is longer than the length of the correctable burst D_1.

In our experiments a variety of transmission channels and several decoding procedures were examined. The examinations were aimed at determining the reason of the error in decoding. It was established that

— in symmetrical channels the reason of an erroneous decoding was, in 70 to 85 per cent of the cases, the occurrence of an error in the guard-space,

— in bursty channels the plurality of the erroneous decoding occurred because the burst length T_1 was longer than D_1.

In real channels, opposed conditions must be satisfied by the construction of an optimum decoding procedure. Error correction in such channels requires

— decoders with more complicated structure,

— a coding system containing several cascaded simple coding procedures as proposed in the literature. [1]

In further investigations decoding procedures were examined in which not adjacent bits but farther (n-th) bits were considered. These elected bits formed a new series.

The statistical behaviour of this series will markedly differ from the original one. Let the probability of the initial state of a stationary Markov chain be denoted by $P_0(0)$ and $P_0(1)$. After n transitions, the probability of the chain characterized by parameters p, q, P and Q will be

$$P_n(1) = \frac{Q}{Q+q} + (1-q-Q)^n \left(P_0(1) - \frac{Q}{Q+q} \right) \tag{5}$$

$$P_n(0) = \frac{q}{Q+q} + (1-q-Q)^n \left(P_0(0) - \frac{q}{Q+q} \right) \tag{6}$$

Equations (5) and (6) show that when the second, third and n-th element is elected and posed successively, the error distribution will change. When $n \to \infty$, the factor with the exponent n will tend to zero. As a consequence, the state value after n steps will less and less depend on whether the initial state was zero or one.

In the sampled series the burst character of the errors vanishes more and more and approaches the value of independent distribution.

Figure 2 shows the results of a computer simulation. $P(n)$ gives the probability of the error-free intervals being longer than the abscissa. The parameter of the curves indicates the sampling value n. The parameter $n=1$ corresponds to the original series. As can be seen, the probability of error-free portions is more and more decreasing in the cases of $n = 14, 64, 512$. In such a channel, thus, it is not necessary to employ a decoder for the correction of burst errors.

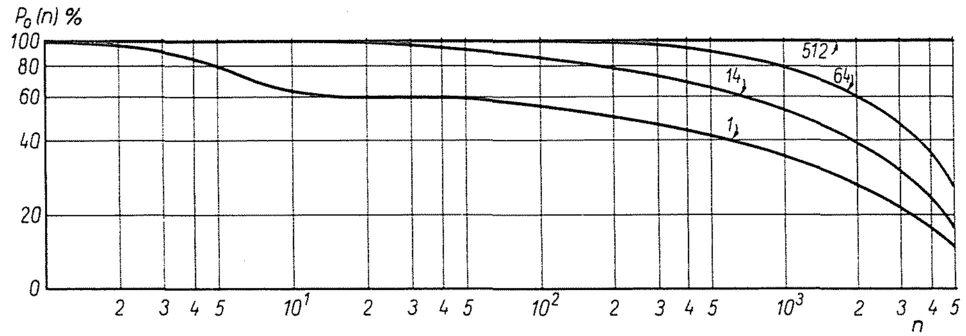

Fig. 2. The effect of the sampling on the distribution of error free intervals

The two methods discussed above were compared in the course of the investigations at the Chair of Microwave Communication. To examine the coding system, a channel model with a computer was constructed by which radio channels of various design could be simulated. Error corrections were performed by means of various convolution codes. In the experiments algebraic codes as well as orthogonal and orthogonized codes were applied. The procedures were realized by means of a computer, but also an experimental equipment system was constructed. During the simulation, and also under real conditions, the ability of error-correction, the error propagation and the possibility of synchronizing of the codes were examined. The simultaneous application of two codes with differing correction abilities was also tested. In addition to high efficiency, a simple design was also an important point of view. Taking also this condition into consideration, it was established that in HF systems the optimum solution is a joint application of a relatively simple code for correcting independent errors and of scrambling in the transmission channel.

Figure 3 shows the results of some simulations carried out on a computer simulating three different transmission channels and two decoding procedures, a simple self-orthogonal one (A) and an orthogonizable one (B) with sign-mixing. The overall length of the code could be varied as a function of the mixing factor. After decoding, the codes without sign-mixing exhibited approximately identical error probabilities of $h_e \cong 10^{-2}$; $7 \cdot 10^{-3}$; $4 \cdot 10^{-4}$ in each of the channel models. As can be seen from the Figure 3, the simple codes gave a better error probability with increasing overall length. In addition, it can be seen that the improvement does not change significantly beyond a given value of the length.

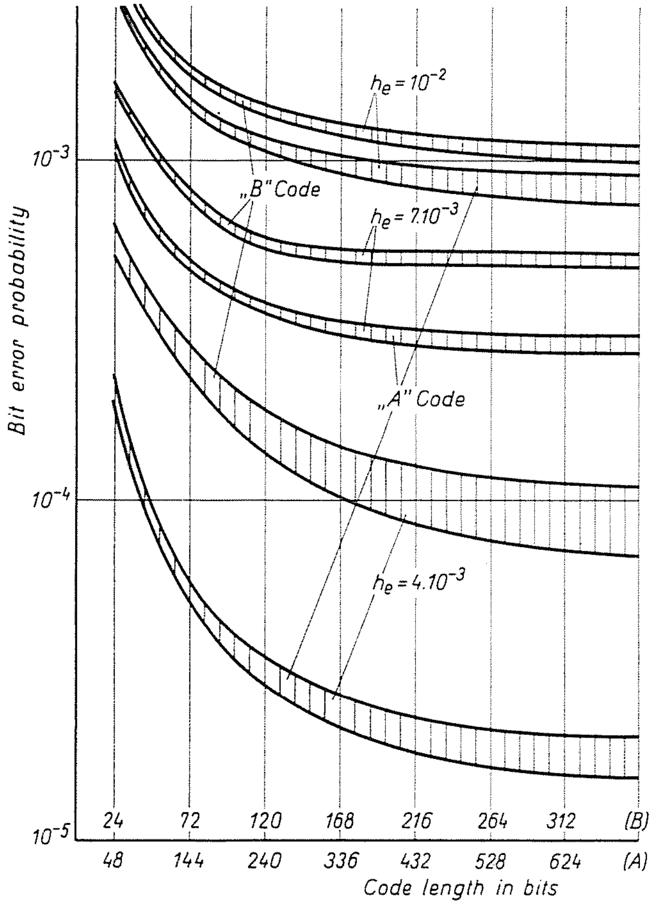

Fig. 3. The effect of scrambling on the bit error probability

Summary

The error-correcting encoding of binary radio channels requires partly the knowledge of the error structure of the transmission channel, partly the knowledge of the properties of the encoding system. A simultaneous modelling of the channel and of the encoding procedure is one of the possible and efficient methods to examine the transmission system. For modelling the binary channels the authors propose the use of the Markov matrix. The computer-aided modelling of different encoding procedures gave useful experience concerning the conditions to be expected.

Reference

1. LIN-NAN LEE, "Concatenated Coding Systems..." IEEE Transactions on Comm. Vol. COM-25 No. 10 Oct. 1977. page 1064—1074.

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