DYNAMIC IDENTIFICATION OF INTERMEDIATE PRESSURE GAS DISTRIBUTING SYSTEMS UNDER INDETERMINATE INITIAL CONDITIONS

By

É. LEBOVITS-KÁLMÁN[1], I. SZABÓ[2], J. KOCSIS[3] and F. CSÁKI[4]

Technical University, Budapest

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1. Introduction

In the second half of sixties, gas distributing systems for big cities, were supplied by control systems suiting partly to automatically evaluate the measured parameters of the system and partly to be computer controlled themselves (on-line or off-line control).

Before developing a gas distributing system which suits automated data collection and computer control, several tasks have to be solved. Just as for other types of continuous non-linear controlled processes operating under stochastic conditions, also the design of gas distributing systems is preconditioned by the development *theoretical computation methods* with *acceptable accuracy* to investigate the *static* and *dynamic characteristics* of the processes in the system.

In determining these characteristics, first of all, the parameters of mass and energy transfer, unambiguously determining the state of the system have to be analyzed.

In the following the characteristics of the dynamic behaviour of the intermediate pressure gas distributing system are investigated, in particular, the problems of the dynamic system identification.

Although in the past century, several publications have been concerned with the mathematical description of gas distributing systems and its practical applications (e.g. [1] to [7], [10], [13]), there are still several questions to be answered, including — to our best knowledge — that of dynamic identification. The mathematical background of the quasi-optimal learning algorithms has been developed in the seventies [8], [9] and applied mainly in filtering problems of telecommunication, and in the following it will be used for describing intermediate pressure gas distributing systems.

[1] Department of Precision Mechanics and Optics.
[2] Heat and System Engineering Institut.
[3] Centre for EDP and Computing Application.
[4] Department of Automation.

2. Simplified dynamic model of intermediate pressure gas distributing systems

From the aspect of dynamic behaviour, the gas distributing systems can be considered as non-linear dynamic systems with distributed parameters, where the transient processes generated by the switching effects are rather inaccessible to analytical treatment as a function of the two independent variables (space and time). Therefore, dynamic analyses investigations involve mostly simplifications valid under restrictions determined in the literature [11], [12], [14].

Dynamic analyses of the frictional flow of compressable media start generally from the following partial differential equation (cf. [12]):

$$\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial c}{\partial t}+c\frac{\partial c}{\partial x}=-S \tag{1}$$

where c is the flow velocity at x; ρ the density of the gas at x; p the pressure of the gas at x; S the change in pressure over unit length, due to friction (see e.g. in [11]). The continuity equation for this case is generally of the form

$$\rho w^2\frac{\partial c}{\partial x}+c\frac{\partial p}{\partial x}+\frac{\partial p}{\partial t}=0 \tag{2}$$

(w being the wave propagation velocity in the medium), for a pressure range $(4<p<7)10^5$ Pa [14].

The system of partial differential equations (1) and (2), is the most general mathematical model for frictional flow in long pipelines with ditributed parameters. The solutions $c(x, t)$ and $p(x, t)$ provide the transient processes of dependent variables of velocity and pressure as a function of independent variables space and time. Unfortunately, approximate solution of the system of partial differential equation (1) and (2) without simplifications requires an over complicated mathematical apparatus so in general, simplifications are applied in order to provide practical results [11]. The most common simplifying assumptions are:

a) the model medium is partially compressible (adopted also here, provided $\rho\approx$const., $w\gg c$, taking pressure changes due to changes in density into consideration);

b) the model medium is partially compressible and the friction force vs. velocity is linearized;

c) the model medium is partially compressible, the friction drag is neglected (loss-free flow), as a first approximation, the network is assumed to be a system with concentrated parameters.

Naturally, other assumptions are possible but here c) will be analysed. For small changes:

$$\frac{dv}{dt}=\frac{1}{C}[\Phi_b(t)-\Phi_k(t)] \tag{3}$$

where v refers to the alteration of any intensive characteristic of the outflow of substance or energy, C is the capacity factor, $\Phi_b(t)$ the inflow function, $\Phi_k(t)$ the outflow function.

At the same time, statically, the differential pressure between the two ends of pipe, with constant pipe-diameter and horizontal layout, is of the form:

$$\Delta p=R_1\Phi_m^2 \tag{4}$$

where R_1 is a resistance coefficient, depending on the material and the dimension of the pipeline, and on the flowing substance. Φ_m is assumed to be a mass flow constant along the pipeline.

The system of equation (3) and (4) is suitable for the dynamic network analysis and for its connection with the exact system of partial differential equations (1) and (2) we refer to [11], [12] and [14].

The application of the system of Eqs (3) and (4), for a short length of the Budapest intermediate pressure gas distributing system is presented in the Appendix.

The principles of dynamic modelling of gas distributing systems have been outlined above. In the following, problems of the dynamic identification of such systems, possible if an appropriate gauging and collecting system is available will be considered.

3. Dynamic identification of intermediate pressure gas distributing systems

Demonstration of the basic concepts of the dynamic identification of gas distributing systems starts from the simple subsystem in *Fig. 1* (where the p_i and the Φ_i designate pressure and mass-flow values, resp., for a section *i*). According to considerations under 2, and with assumption under 2.c, hold, the actual network can be approximated by an $R-C$ circuit with concentrated

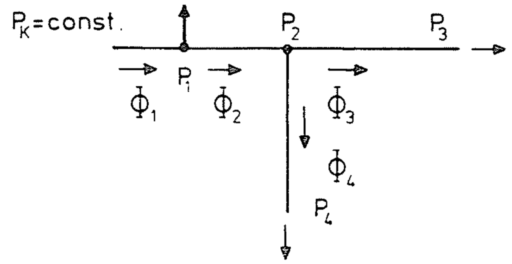


Fig. 1

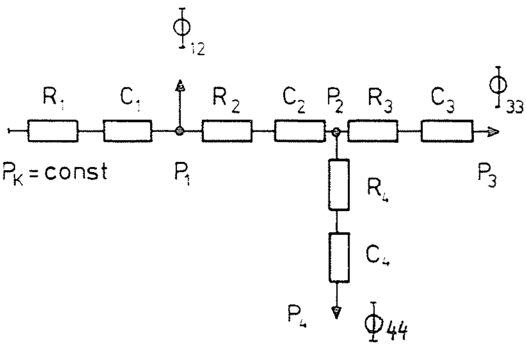


Fig. 2

parameters, for each pipeline segment [14]. (cf. *Fig. 2*). Thus the dynamic system equations take the following form:

$$
\begin{aligned}
\frac{dp_1}{dt} &= \frac{1}{C_1\sqrt{R_1}}\sqrt{p_k - p_1} - \frac{1}{C_1\sqrt{R_2}}\sqrt{p_1 - p_2} - \frac{\Phi_{12}}{C_1} \\
\frac{dp_2}{dt} &= \frac{1}{C_2\sqrt{R_2}}\sqrt{p_1 - p_2} - \frac{1}{C_2\sqrt{R_3}}\sqrt{p_2 - p_3} - \frac{1}{C_2\sqrt{R_4}}\sqrt{p_2 - p_4} \\
\frac{dp_3}{dt} &= \frac{1}{C_3\sqrt{R_3}}\sqrt{p_2 - p_3} - \frac{\Phi_{33}}{C_3} \\
\frac{dp_4}{dt} &= \frac{1}{C_4\sqrt{R_4}}\sqrt{p_2 - p_4} - \frac{\Phi_{44}}{C_4},
\end{aligned}
\tag{5}
$$

where C_i is the capacity of the i-th segment, R_i is the resistance of the i-th segment, Φ_{ij} is the flow to consumers. Using the notations:

$$a_1 = \frac{1}{C_1\sqrt{R_1}}; \quad a_2 = -\frac{1}{C_1\sqrt{R_2}}; \quad a_3 = -\frac{\Phi_{12}}{C_1};$$
$$b_1 = \frac{1}{C_2\sqrt{R_2}}; \quad b_2 = -\frac{1}{C_2\sqrt{R_3}}; \quad b_3 = -\frac{1}{C_2\sqrt{R_4}};$$
$$c_1 = \frac{1}{C_3\sqrt{R_3}}; \quad c_2 = -\frac{\Phi_{33}}{C_3};$$
$$d_1 = \frac{1}{C_4\sqrt{R_4}}; \quad d_2 = -\frac{\Phi_{44}}{C_4};$$
$$p_1 = x_1; \quad p_2 = x_2; \quad p_3 = x_3; \quad p_4 = x_4; \tag{6}$$

and assuming discrete time increments, the system equations (5) can be put in the following form: [8]

$$x_1[n+1] = x_1[n] + (a_1\sqrt{p_k - x_1[n]} + a_2\sqrt{x_1[n] - x_2[n]} + a_3)\,T,$$
$$x_2[n+1] = x_2[n] + (b_1\sqrt{x_1[n] - x_2[n]} + b_2\sqrt{x_2[n] - x_3[n]} +$$
$$+ b_3\sqrt{x_2[n] - x_4[n]})\,T, \tag{7}$$
$$x_3[n+1] = x_3[n] + (c_1\sqrt{x_2[n] - x_3[n]} + c_2)\,T,$$
$$x_4[n+1] = x_4[n] + (d_1\sqrt{x_2[n] - x_4[n]} + d_2)\,T,$$

where n is the discrete time and T is the sampling interval. In this case, the relationships of the dynamic model may take the form:

$$\hat{x}_1[n+1] = x_1[n] + \mathbf{a}^T[n]\varphi_1(\mathbf{x}[n]),$$
$$\hat{x}_2[n+1] = x_2[n] + \mathbf{b}^T[n]\varphi_2(\mathbf{x}[n]),$$
$$\hat{x}_3[n+1] = x_3[n] + \mathbf{c}^T[n]\varphi_3(\mathbf{x}[n]), \tag{8}$$
$$\hat{x}_4[n+1] = x_4[n] + \mathbf{d}^T[n]\varphi_4(\mathbf{x}[n]),$$

where

$$\mathbf{a}=\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix},\qquad \mathbf{b}=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix},\qquad \mathbf{c}=\begin{bmatrix}c_1\\c_2\end{bmatrix},\qquad \mathbf{d}=\begin{bmatrix}d_1\\d_2\end{bmatrix},$$

$$\boldsymbol{\varphi}_1=\begin{bmatrix}\varphi_{11}\\\varphi_{12}\\\varphi_{13}\end{bmatrix}=\begin{bmatrix}T\sqrt{p_k-x_1[n]}\\T\sqrt{x_1[n]-x_2[n]}\\T\end{bmatrix},\qquad \boldsymbol{\varphi}_2=\begin{bmatrix}\varphi_{21}\\\varphi_{22}\\\varphi_{23}\end{bmatrix}=\begin{bmatrix}T\sqrt{x_1[n]-x_2[n]}\\T\sqrt{x_2[n]-x_3[n]}\\T\sqrt{x_2[n]-x_4[n]}\end{bmatrix};$$

$$\boldsymbol{\varphi}_3=\begin{bmatrix}\varphi_{31}\\\varphi_{32}\end{bmatrix}=\begin{bmatrix}T\sqrt{x_2[n]-x_3[n]}\\T\end{bmatrix},\qquad \boldsymbol{\varphi}_4=\begin{bmatrix}\varphi_{41}\\\varphi_{42}\end{bmatrix}=\begin{bmatrix}T\sqrt{x_2[n]-x_4[n]}\\T\end{bmatrix}.$$

In (8), the model is assumed to evaluate the output signals $\hat{x}_i[n+1]$ point by point, (the upper mark refers to the output), based on the measured x_i i.e. pressure values.

Let the dynamic identification aim at determining parameter vectors **a**, **b**, **c** and **d** so that the mean-square errors between the measured (in the real distributing system) and model output pressures expected to be minimum:

$$
\begin{aligned}
I_1(\mathbf{a})&=M\{(x_1-\hat{x}_1)^2\}=M\{(\Delta x_1-\mathbf{a}^T\boldsymbol{\varphi}_1(\mathbf{x}))^2\}\to\min_{\mathbf{a}},\\
I_2(\mathbf{b})&=M\{(x_2-\hat{x}_2)^2\}=M\{(\Delta x_2-\mathbf{b}^T\boldsymbol{\varphi}_2(\mathbf{x}))^2\}\to\min_{\mathbf{b}},\\
I_3(\mathbf{c})&=M\{(x_3-\hat{x}_3)^2\}=M\{(\Delta x_3-\mathbf{c}^T\boldsymbol{\varphi}_3(\mathbf{x}))^2\}\to\min_{\mathbf{c}},\\
I_4(\mathbf{d})&=M\{(x_4-\hat{x}_4)^2\}=M\{(\Delta x_4-\mathbf{d}^T\boldsymbol{\varphi}_4(\mathbf{x}))^2\}\to\min_{\mathbf{d}},
\end{aligned}
\tag{9}
$$

where

$$\Delta x_i[n]=x_i[n+1]-x_i[n]. \tag{10}$$

At the extreme value point the corresponding gradients are zero.

$$
\begin{aligned}
\frac{dI_1(\mathbf{a})}{d\mathbf{a}}&=-2M\{(\Delta x_1-\mathbf{a}^T\boldsymbol{\varphi}_1(\mathbf{x}))\boldsymbol{\varphi}_1\}=\mathbf{0},\\
\frac{dI_2(\mathbf{b})}{d\mathbf{b}}&=-2M\{(\Delta x_2-\mathbf{b}^T\boldsymbol{\varphi}_2(\mathbf{x}))\boldsymbol{\varphi}_2\}=\mathbf{0}.
\end{aligned}
\tag{11}
$$

$$\frac{dI_3(\mathbf{c})}{d\mathbf{c}} = -2M\{(\Delta x_3 - \mathbf{c}^T\varphi_3(\mathbf{x}))\varphi_3\} = \mathbf{0},$$

$$\frac{dI_4(\mathbf{d})}{d\mathbf{d}} = -2M\{(\Delta x_4 - \mathbf{d}^T\varphi_4(\mathbf{x}))\varphi_4\} = \mathbf{0}.$$

Regression equation of this type can be solved by the apparatus of quasi-optimal learning algorithms [8], [9], [16]

$$\begin{aligned}
\mathbf{a}[n+1] &= \mathbf{a}[n] + \Gamma_1[n](\Delta x_1[n] - \mathbf{a}^T[n]\varphi_1(\mathbf{x}[n]))\varphi_1(\mathbf{x}[n]),\\
\mathbf{b}[n+1] &= \mathbf{b}[n] + \Gamma_2[n](\Delta x_2[n] - \mathbf{b}^T[n]\varphi_2(\mathbf{x}[n]))\varphi_2(\mathbf{x}[n]),\\
\mathbf{c}[n+1] &= \mathbf{c}[n] + \Gamma_3[n](\Delta x_3[n] - \mathbf{c}^T[n]\varphi_3(\mathbf{x}[n]))\varphi_3(\mathbf{x}[n]),\\
\mathbf{d}[n+1] &= \mathbf{d}[n] + \Gamma_4[n](\Delta x_4[n] - \mathbf{d}^T[n]\varphi_4(\mathbf{x}[n]))\varphi_4(\mathbf{x}[n])
\end{aligned} \tag{12}$$

where, in the case of vectors **a** and **b**

$$\Gamma_i[n] = \begin{bmatrix} \gamma_1[n] & 0 & 0 \\ 0 & \gamma_2[n] & 0 \\ 0 & 0 & \gamma_3[n] \end{bmatrix},$$

$$\gamma_m[n] = l_{mm}[n] / \sum_{i=1}^{3} l_{mi}^2[n]\,; \qquad (m = 1, 2, 3)\,; \tag{13}$$

and the case of vectors **c** and **d**

$$\Gamma_i[n] = \begin{bmatrix} \gamma_1[n] & 0 \\ 0 & \gamma_2[n] \end{bmatrix},$$

$$\gamma_m[n] = l_{mm}[n] / \sum_{i=1}^{2} l_{mi}^2[n]\,, \qquad (m = 1, 2)\,; \tag{14}$$

where $l_{mi}[n]$ is an element of the matrix $L[n]$.

$$L[n] = \sum_{m=1}^{n} \varphi[m]\varphi^T[m].$$

The computer outputs proving the usefulness of the above dynamic algorithms are presented in the Appendix.

Although the basic concepts of dynamic identification of intermediate pressure gas distributing systems by learning algorithms have been presented here on a simple subsystem, for the sake of illustration, the results can be generalized for systems of any topology, without theoretical difficulties.

The efficiency of the presented dynamic model has its limitations. Partly, for a network greater from the aspect of computer realization, the dimensionality of the model becomes prohibitive, and partly, the real gas flow processes are only correctly described by the above model with the restrictions under 2.c.

Conclusions

Dynamic identification algorithms describing the dynamic behaviour if intermediate pressure gas distributing systems, based on the analysis of published relationships are suggested. The algorithms are based on the theory of quasi-optimal learning algorithms valid under indeterminate conditions, assuming no independent samples.

Formulation of the problem of dynamic identification aimed at establishing a model simulating the closest possible the dynamic behaviour of system if simultaneous measurement are taken at different points of the network with equal time intervals. The solution involved apparatus of quasi-optimal learning algorithms developed by Tsypkin [8].

As an illustration, computer outputs of the dynamic identification are presented in the Appendix. For cases involving the assumptions of a partially compressible model medium no-loss flow while the system with distributed parameters is approximated by a system with concentrated parameters.

The results of the dynamic analysis show the actual one-hour samplings to be inadequate for improving the approximate pattern of the system obtained from the static model by taking the results of dynamic analysis into consideration, namely the transient processes take a few minutes. The computer outputs presented in the Appendix illustrate the efficiency of the suggested algorithms.

Appendix

The computer outputs of the dynamic identification.

To demonstrate the efficiency of the algorithms under 3, for the dynamic identification of gas distributing systems, the experimental computer program below has been developed.

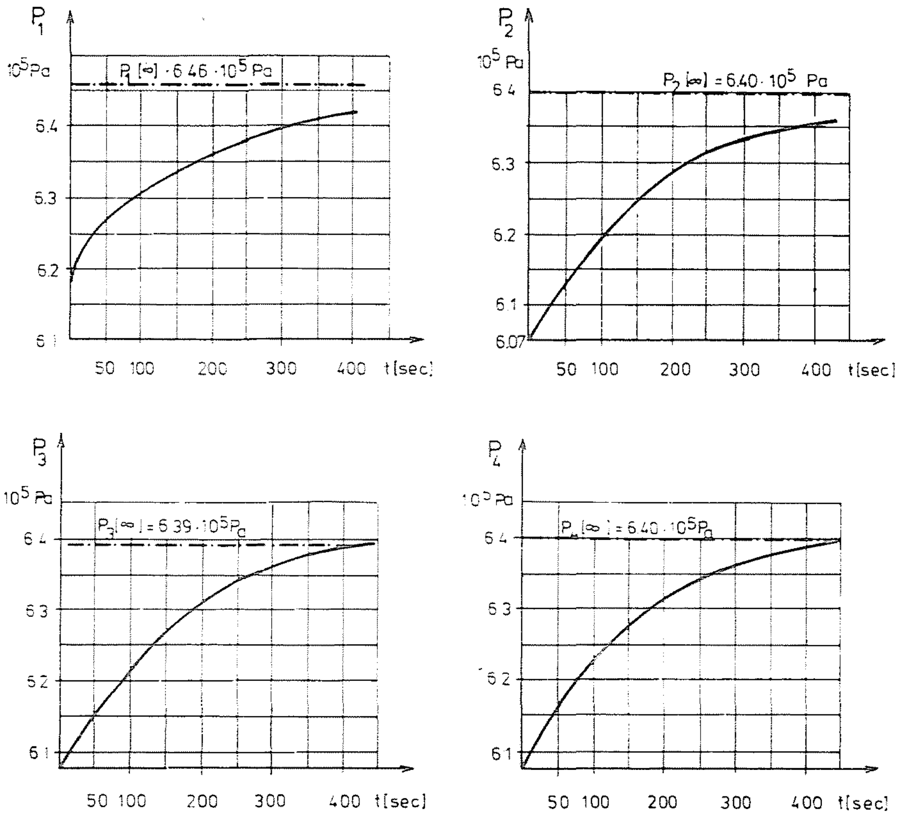
The experimental program operates on the following principle:

The first part of the program simulates the dynamic behaviour of the medium-pressure gas distributing system in the general case, according to (3) and (4), and specifically for the subsystem in Fig. 2, according to the system of differential equations (5), using Euler's approximative formula [15]. The characteristic pressure and flow data of the process, and the connected pairs of

$x_i[n]$ and $\varphi_i(\mathbf{x}[n])$, were provided by this part of the program for the identification subprogram Eq. (12).

The second part computes the unknow parameters **a**, **b**, **c** and **d** of dynamic identification, according to Eqs (12) and (13). Running started checking the function of the simulation program part with outputs for the transient processes generated by two different switch-off effects in *Fig. 3*. The transient processes are seen to take but a few minutes.

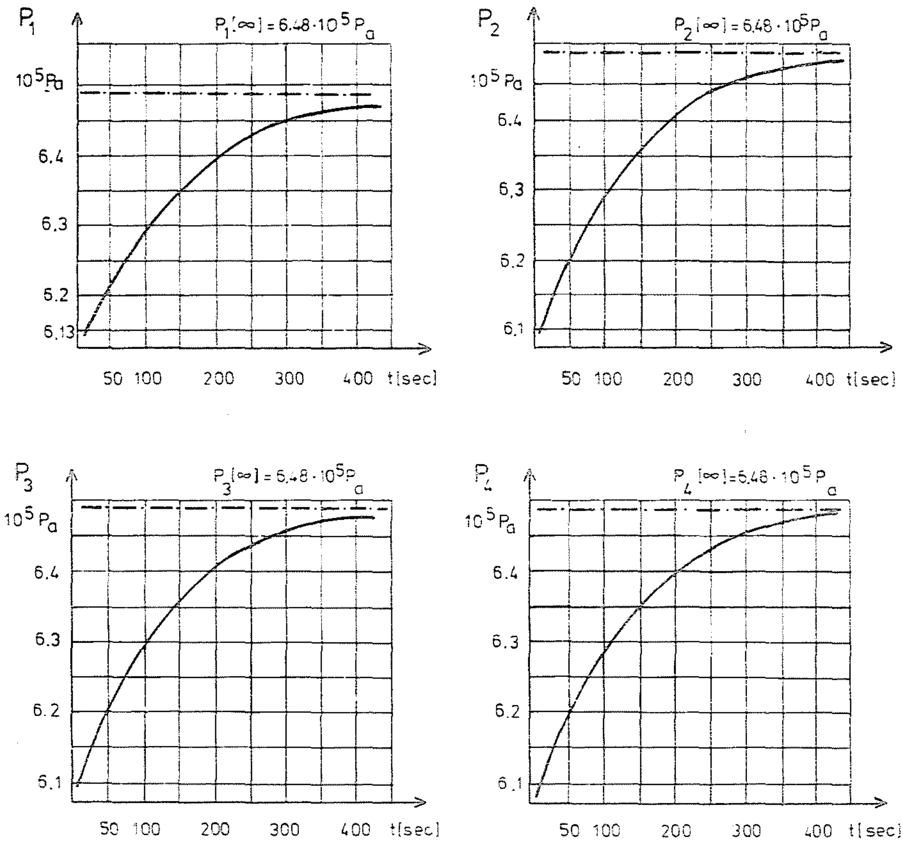
Operation of the dynamic identification was examined by the following theoretical experiment. The input (flow) and the output (pressure) data, provided by the simulation program, were considered as measurement data.



$\Phi_{12} = 0.981 \to 0$ kgsec^{-1}

$\Phi_{33} = 1.067$ kgsec^{-1}

$\Phi_{44} = 0.052$ kgsec^{-1}



$\Phi_{12} = 0.981 \quad kgsec^{-1}$

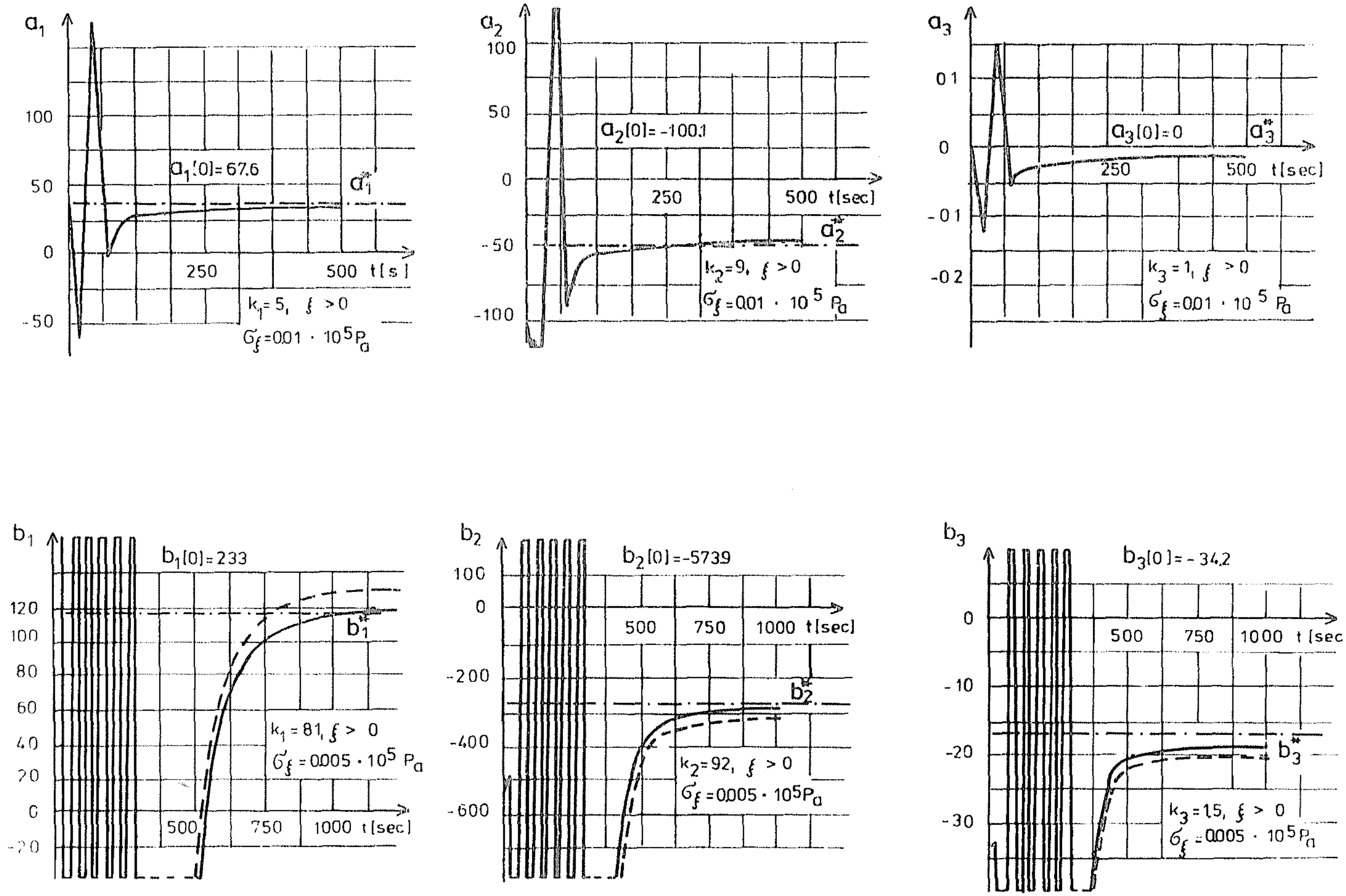
$\Phi_{33} = 1.067 \rightarrow 0 \quad kgsec^{-1}$

$\Phi_{44} = 0.052 \quad kgsec^{-1}$

Fig. 3. Transient responses of the model

Gaussian additive noise with zero mean and given variance was superimposed to the output data, and the identification procedure was started with given initial parameter vector values. The results are presented in *Fig. 4*. Asterisks refer to optima of the different parameters. To accelerate the convergence, the convergence factors calculated from (13), were multiplied by different constant k_i, also indicated in Fig. 4.

Quasi-optimal learning algorithms are seen to suit dynamic identification of the approximate dynamic behaviour of medium pressure gas distributing systems, according to the principles in chapter 3.


a1
a1[0]=67.6
k1=5, ξ>0
σξ=0.01 · 10^5 Pa
a2
a2[0]=-100.1
k2=9, ξ>0
σξ=0.01 · 10^5 Pa
a3
a3[0]=0
k3=1, ξ>0
σξ=0.01 · 10^5 Pa
b1
b1[0]=233
k1=81, ξ>0
σξ=0.005 · 10^5 Pa
b2
b2[0]=-5739
k2=92, ξ>0
σξ=0.005 · 10^5 Pa
b3
b3[0]=-34.2
k3=1.5, ξ>0
σξ=0.005 · 10^5 Pa
t[sec]

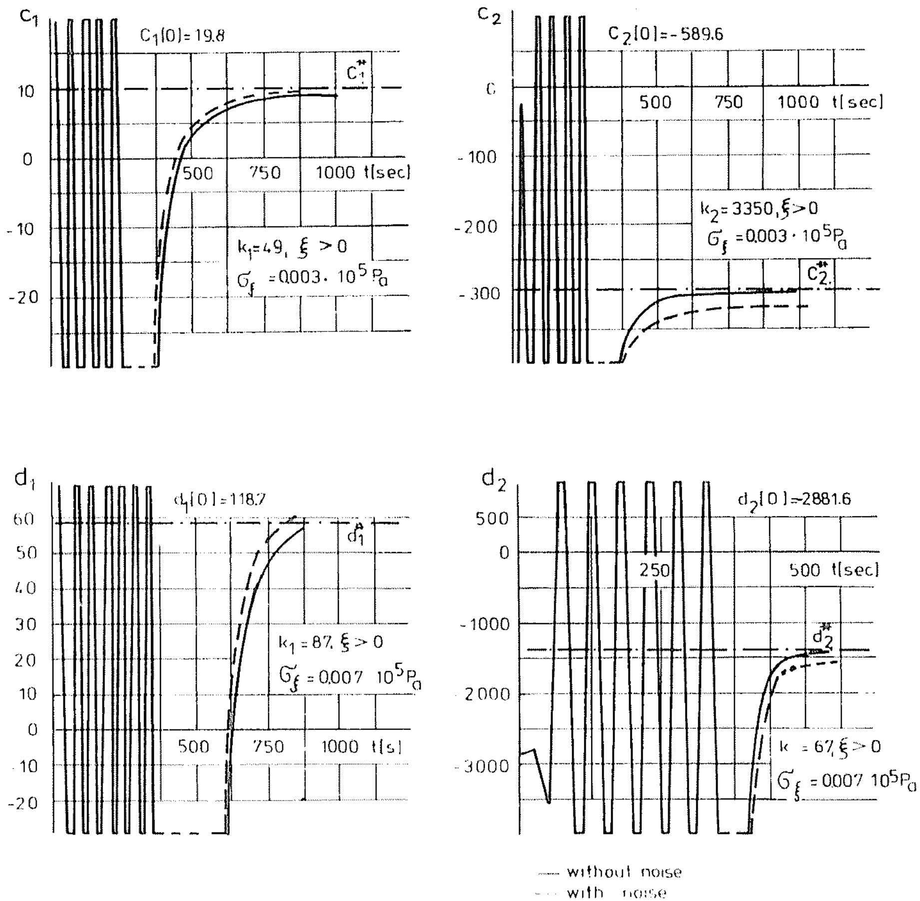


Fig. 4. Transient processes of dynamic identification for $\Phi_{12} = 0.981\ \mathrm{kg\,sec^{-1}} \rightarrow 0\ \mathrm{kg\,sec^{-1}}$

Summary

The principles of the approximate mathematical description of the controlled processes of intermediate pressure gas distributing networks are presented. The algorithms of the dynamic identification for gas distributing systems are examined, based on the theory of quasi-optimal learning algorithms.

The suggested, modern theoretical relationships are illustrated on the sampled data of a section of the Budapest intermediate pressure gas distributing system.

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Éva LEBOVITS-KÁLMÁN
Prof. Dr. Imre SZABÓ
Dr. János KOCSIS
Prof. Dr. Frigyes CSÁKI

H-1521 Budapest

[1] in Hungarian