THE MINIMUM LOSS OF IMPEDANCE MATCHING TWO-PORTS

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1. Characterization of impedance matching two-ports

Figure 1 shows a matching two-port terminated by generator impedance Z_S and load resistance R_L. Generally, in the design of communication systems the task is to transfer power from a given source to a given load (for example U_S, Z_S and $Z_L = R_L$, or in the opposite direction). This problem often involves the design of a lossless matching two-port to transform the load impedance into the complex conjugate of the source impedance. This question was first considered by H. W. Bode [1] for a restricted class of impedances. R. M. Fano [2] extended Bode's result to the case of an arbitrary passive impedance. D. C. Youla [3] developed an alternate theory which relied upon the normalized scattering parameters and bypasses some difficulties encountered in Fano's work. Youla's theory can be generalized to active impedances, too [4].

The problem mentioned before is the so-called broadband matching problem and it is practically solved. Sometimes a similar task arises, called the broadband impedance matching problem. In the ideal case of impedance matching $Z_{1in} = Z_S = R_S + jX_S$ and $Z_{2in} = Z_L = R_L$ (*Fig. 1*). Such questions arise in wirebound telecommunication systems using transmission lines in frequency ranges where the characteristic impedance is frequency-dependent and undesirable reflections are to be avoided. Now, let us investigate some properties of impedance matching networks.

Suppose the matching two-port is a lossless one, therefore the normalized scattering matrix is uniter [5] which implies

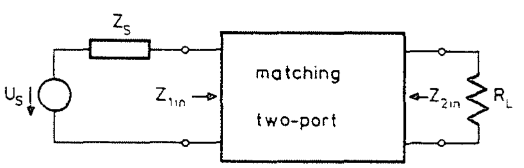

Fig. 1. Matching two-port

$$|S_{11}|=|S_{22}|, \tag{1.a}$$

$$|S_{12}|=|S_{21}|, \tag{1.b}$$

$$|S_{11}|^2+|S_{21}|^2=1. \tag{1.c}$$

(S_{11} and S_{22} are complex reflection coefficients, S_{12} and S_{21} are transmission coefficients). According to Fig. 1, for the ideal case we get:

$$S_{11}=\frac{Z_{1in}-Z_S^*}{Z_{1in}+Z_S}=j\frac{X_S}{Z_S}, \tag{2.a}$$

$$|S_{11}|^2=\left|\frac{X_S}{Z_S}\right|^2=\sin^2\varphi_S \tag{2.b}$$

where * denotes the complex conjugate and $\varphi_S=\mathrm{arc}\, Z_S$. For example at low frequencies the phase of the characteristic impedance of a transmission line is $\varphi_t\approx-\frac{\pi}{4}$, i.e. $|S_{11}|^2\approx\frac{1}{2}$. If the transmission line is connected through a lossless matching two-port to an equipment with input impedance R_L, the problem arises at the second port, because $|S_{22}|=|S_{11}|$ (see Eq. 1.a), and at this port the normalized reflection coefficient is equivalent to the impedance reflexion coefficient ($R_2^*=R_2$)

$$S_{22}=\frac{Z_{2in}-Z_L^*}{Z_{2in}+Z_L}=\frac{Z_{2in}-R_2}{Z_{2in}+R_2}=r_{22} \tag{3}$$

For a rigorous specification a reflection attenuation of $20\lg\frac{1}{|S_{22}|}=3$ dB is too low, therefore a lossy matching two-port is needed. One possibility for the circuit is given in *Fig. 2* which is suitable for matching between transmission lines and equipments [6].

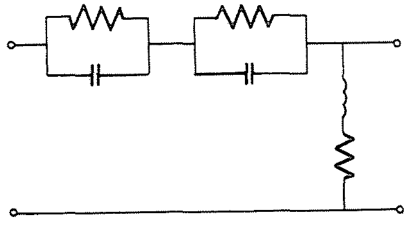

Fig. 2. Circuit for a possible impedance matching two-port

2. Calculation of the minium loss

In the previous section, the impedance matching two-port was seen to be lossy having greater attenuation than a lossless circuit if it is well matched. This is the lower limit of the attenuation. Taking this into account, according to Eq. 1.c:

$$|S_{12}|^2 = 1 - |S_{11}|^2 = \cos^2\varphi_S. \tag{4}$$

In the case of $\varphi_t \approx -\pi/4$, we have $|S_{12}|^2 = \frac{P_2}{P_0} = \frac{1}{2}$ namely, the half of the available power is reflected. Of course, in using lossy elements, a part of the power will be dissipated, too, so the attenuation will be higher.

Theorem: Let us have a generator of input impedance Z_1 and a load resistance R_2. If a lossless impedance matching two-port is designed, which gives a perfect impedance matching at the generator side, then the power attenuation is

$$\left(\frac{P_0}{P_2}\right)^{dB} = 20 \lg \frac{1}{\cos(\text{arc } Z_1)} \tag{5}$$

In practical cases the power attenuation is greater, because the matching two-port is lossy.

3. Example

Let the characteristic impedance of a transmission line be given in Table I. Designing the impedance matching two-port [6] results in the circuit in *Fig. 3* for an equipment impedance $R_2 = 123$ ohm. Measured data have been compiled in Table II, where

Table I

Characteristic impedance of a transmission line

f(kHz)	6	12	24	36	60	108
$ReZ_c(\Omega)$	164	144	130	123	123	123
$-ImZ_c(\Omega)$	96	61.5	33.3	24.1	13.6	7.7

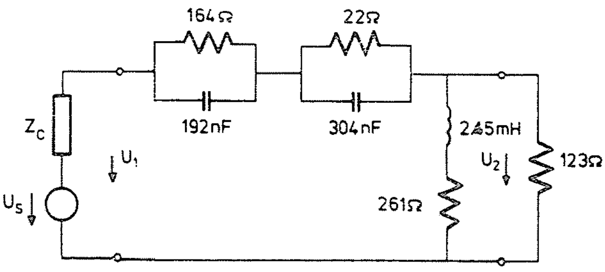

Fig. 3. Impedance matching two-port designed for the example

Table II

Reflection and attenuation data versus frequency

f(kHz)	6	12	24	36	60	108
a_{r1}(dB)	25.2	26.9	26.1	26.9	32.6	36.5
a_{r2}(dB)	15.6	16.1	17.4	19.1	22.6	27.8
a_u(dB)	7.04	4.00	1.56	0.78	0.30	0.13

$$a_{r1}=20\ \lg\left|\frac{Z_{1in}-Z_c}{Z_{1in}+Z_c}\right| \qquad a_{r2}=20\ \lg\left|\frac{Z_{2in}-R_2}{Z_{2in}+R_2}\right|$$

are the reflection attenuation at ports 1 and 2, resp., $a_u=20\ \lg\left|\frac{U_S}{2U_2}\right|$ is the voltage attenuation. From the view-point of power attenuation the worst case is at 6 kHz. From Eq. 4. at this frequency the lower limit of power attenuation is

$$\left(\frac{P_0}{P_2}\right)^{dB}=20\ \lg\frac{1}{\cos\left(\text{arc tg}\frac{-96}{164}\right)}=1.28\ \text{dB}.$$

Actually the power attenuation is 5.79 dB, which can be checked from the given voltage attenuation $a_u=7.04$ dB.

4. Acknowledgement

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Summary

In the usual broad-band matching theory the attenuation in the pass-band approximates zero. In the wirebound telecommunication the transmission lines have a frequency-dependent characteristic impedance and if the reflection coefficient is prescribed both for the transmission line side and for the equipment side, a matching two-port is needed, which is lossy and causes power-loss. Based on the scattering matrix, the minimum loss can be calculated.

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