DETERMINATION OF TRANSFORMATION MATRICES FOR KRON'S METHOD FOR CALCULATING NETWORKS AND FOR DIAKOPTICS

By

GY. TEVAN

Institute of Heavy Current Engineering,
Department of High Voltage Engineering
Technical University, Budapest.

Received April 9, 1979
Presented by Dr. P. O. GESZTI

1. Introduction

In computing large networks — in order to reduce preparatory work — it is advisable to avoid drawing the graph of the network and have instead a set of data, characteristic of the graph, as input. It can simply be carried out by using the traditional potential method. For the efficient use of Kron's general method for calculating networks and the diakoptics, we need an algorithm which determines the transformation matrices from a set of data characteristic of the graph.

2. Deriving the projector matrix assigned to the graph

To derive the actual algorithm, let us introduce certain projector matrices.

Let us consider, for example, the connected oriented graph shown in Fig. 1. An obvious base of the subspace M of irrotational (potential) voltage distributions, discussed by P. O. GESZTI and GY. TEVAN [1] in Chapter five, is obtained by taking the potential of one of the nodes as 1 and that of the others as zero.

If the base voltage vectors are characterized by the row-vectors, showing the voltage rise of the branches, then taking the potential of the sequence of nodes R, S, T, V to be 1, the so-called *incidence matrix* is obtained by writing the corresponding row vectors below each other.

$$[A] = \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & -1 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(The case where the potential of node U is 1 and that of the others are zero, it is equivalent to that where the potential of U is zero and that of the others are -1 and since this is the negative sum of the previous row vectors, it will not give a linearly independent distribution.) In case of v nodes (considering connected networks) matrix $[A]$ has $v-1$ linearly independent row. These rows result from that the branch arriving into the corresponding node gives $+1$, the branch leaving the node gives -1, and the branch having no connection with the node gives zero to the branch component of the row-vector. An *oriented graph can be uniquely characterized by the incidence matrix*. (The incidence matrix of a non connected network has $v-p$ independent rows where p denotes the number of connected subnetworks.)

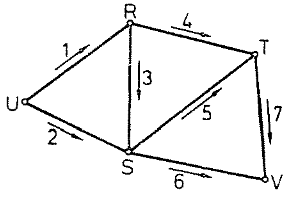

Fig. 1

Since the closed path current distributions falling into the divergenceless subspace G are orthogonal to the elements of subspace M [1], the column vectors representing the base vectors of subspace G are orthogonal to the row vectors of $[A]$. Moreover, it is also known that taking a (complete) tree in a connected graph, with one link branch (chord) and tree branches altogether $(e-v+1)$ linearly independent loop currents can be formed, which thus form the base of the $(e-v+1)$-dimensional subspace G.

Let us join the nodes of a graph, then let us separate them one by one and finally let us take a (complete) tree in all the graphs obtained. In Fig. 2 the tree branches and the loop branches are drawn in continuous and in dotted lines, respectively. The tree branch of a previous form must be considered as tree branch in the following as well. The incidence matrices are indicated beside all the graphs in the figure. The incidence matrix of the k-th form is seen to contain the first $(k-1)$ rows of the incidence matrix of the starting graph.

Let us assign quadratic matrices to all the graphs drawn in Fig. 2 in which the column vectors corresponding to the tree branch numbers are zero and the column vectors corresponding to the chord numbers represent the unit mesh current belonging to the chord.

Denote this matrix of the $(k+1)$-th graph $[P_k]$, so:

$$[P_0]=[E] \text{ (unit matrix; there is no tree branch)}$$

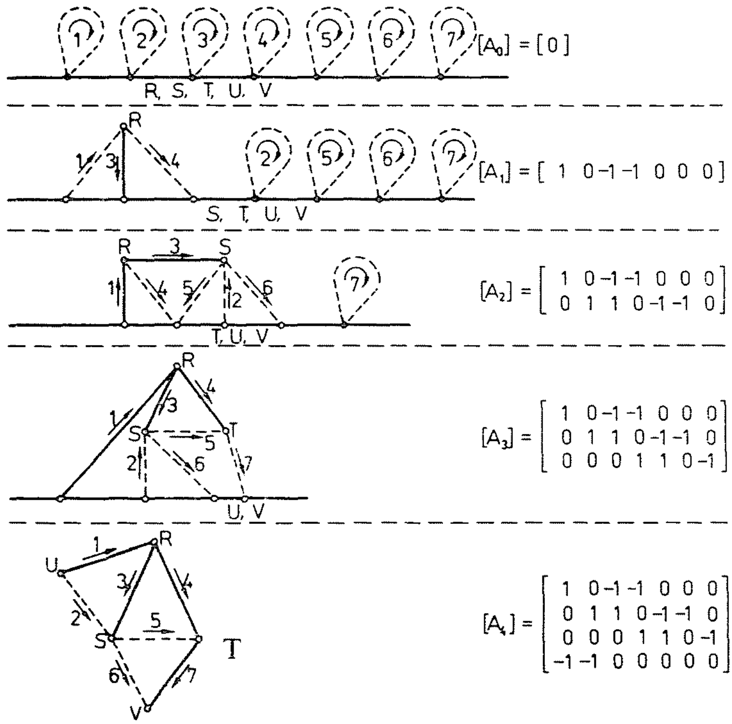

Fig. 2

$$[\mathrm{P}_1]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \quad [P_2]=\begin{bmatrix} 0 & -1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[P_3]=\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$[P_4]=\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

$f_1=3, f_2=1, f_3=4, f_4=7$ (tree branch subscripts)

In accordance with the equations $[P\,]^2=[P\,]$ valid to the projectors, it is simple to realize that these *matrices P_k are projector matrices;* in $[P_k]^2$ the zero column vectors are in the same place as in P_k, the column vectors corresponding to the loop currents will remain in $[P_k]^2$ too, because in the scalar multiplication by the rows the non-zero element corresponding to the tree branches are multiplied by zero and the multiplier 1 corresponding to the link will result in the original column vector.

The "left-hand" zero subspace of the matrix $[P_k]$ is generated by the row vectors $[a_1]_t$, $[a_2]_t \dots [a_k]_t$ of matrix $[A_k]$ because even concerning this $(k+1)$-th graph, the loop current falling into the sourceless subspace G_k are orthogonal to the voltage distributions, represented by the rows of $[A_k]$, of the irrotational (potential) subspace M_k [1]. The "right-hand" zero subspace of $[P_k]$ is generated by the f_1-st, f_2-nd, … f_k-th column vectors, $[e_{f1}]$, $[e_{f2}]$ … $[e_{fk}]$ respectively, of the unit matrix, where $f_1, f_2 \dots f_k$ are subscripts of the tree branches. So the nullicity of the projector $[P_k]$ is just equal to k, therefore its rank is $e-k$. The "right-hand" range of $[P_k]$ contains that of $[P_k+1]$ i.e.

$$R([P_{k+1}])\subset R([P_k]) \qquad (*)$$

because the columns of $[P_{k+1}]$ are orthogonal to the rows of $[A_{k+1}]$ and the columns of $[P_k]$ are orthogonal to the rows of $[A_k]$ included in $[A_{k+1}]$.

The matrix

$$[D_k]=[P_k]-[P_{k+1}]$$

can be stated that its columns of order $f_1, f_2, \dots f_k$ are zero vectors (similar to those $[P_k]$ and $[P_{k+1}]$) therefore they will annihilate the linearly independent column vectors $[e_{f1}]$, $[e_{f2}]$, … $[e_{fk}]$ of the unit matrix. Moreover, they will annihilate $e-(k+1)$ non-zero, linearly independent column vectors, corresponding to the mesh currents of $[P_{k+1}]$ because these are transformed unchanged — as vectors of the range — by $[P_{k+1}]$, and by $[P_k]$ in accordance with Eq. (*). So the nullicity of $[D_k]$ is $k+[e-(k+1)]=e-1$, *its rank is one*, so it can be written as a diad.

$$[P_k]-[P_{k+1}]=[d_{k+1}]\,[q_{k+1}]_t\,. \tag{1}$$

Since there are k zeros and $e-k$ 1 elements in the principal diagonal of $[P_k]$, therefore the trace of $[P_k]$ is

$$Tr([P_k])=e-k\;;$$

So taking the trace of Eq. (1), we may write;

$$(e-k)-[e-(k+1)]=[q_{k+1}]_t\,[d_{k+1}]\,,$$

i.e.

$$[q_{k+1}]_t\,[d_{k+1}]=1\,, \tag{2}$$

Let us multiply Eq. (1) first from the left-hand side by row vector $[a_{k+1}]_t$, being in the left-hand side zero subspace of $[P_{k+1}]$, and then from the right-hand side by column vector $[e_{f_{k+1}}]$, being in the right-hand side zero subspace of $[P_{k+1}]$:

$$[a_{k+1}]_t\,[P_k]=([a_{k+1}]_t\,[d_{k+1}])\,[q_{k+1}]_t\,,$$

$$[P_k]\,[e_{f_{k+1}}]=[d_{k+1}]\,([q_{k+1}]_t\,[e_{f_{k+1}}])\,.$$

From these and also using Eq. 2:

$$\frac{[P_k]\,[e_{f_{k+1}}]\,[a_{k+1}]_t\,[P_k]}{[a_{k+1}]_t\,[P_k]^2\,[e_{f_{k+1}}]}=[d_{k+1}]\,[q_{k+1}]_t\,,$$

the scalar multiplier is factored out to yield the following equations:

$$[d_{k+1}]=[P_k]\,[e_{f_{k+1}}] \tag{3a}$$

$$[q_{k+1}]_t=[a_{k+1}]_t\,[P_k]\cdot\frac{1}{[a_{k+1}]_t\,[P_k]\,[e_{f_{k+1}}]}\,. \tag{3b}$$

From the way of forming matrices $[P_k]$ in accordance with Fig. 2 it is realized that only one column becomes zero in $[P_{k+1}]$ relative to $[P_k]$ and there are no elements in the non-zero columns which would be changed from 1 to -1 or from -1 to $+1$ because the orientation of the branches is left unchanged. So the left-hand side of Eq. 1 (matrix $[D_k]$) contains only 0, 1 and -1 elements. In accordance with Eq. 3a $[d_{k+1}]$ has only 0, 1 and -1 elements as well because it is identical to the f_{k+1}-th columns of $[P_k]$. So then $[q_{k+1}]$ is also similar.

From Eq. (1) and (3) and from the interpretation of $[P_0]$, we can write:

$$[P_{k+1}]=[P_k]-[P_k]\,[e_{f_{k+1}}]\frac{[a_{k+1}]_t\,[P_k]}{[a_{k+1}]_t\,[P_k]\,[e_{f_{k+1}}]} \tag{4a}$$

$$[P_0]=[E] \tag{4b}$$

With the help of recurrent Eq. (4), the matrices $[P_k]$ end the vectors $[d_k]$, $[q_k]_t$ forming the diad can be determined consecutively without drawing the graph, since only the row vectors $[a_k]_t$ of the incidence matrix are needed for the calculations, the column vectors $[e_{f_{k+1}}]$ of the unit matrix can be chosen in an arbitrary sequence, only ensuring to have no zero in the denomination. This latter will enforce the selection of a tree of the network.

In our case,

$$[P_1]=[E]-\begin{bmatrix}0\\0\\1\\0\\0\\0\\0\end{bmatrix}\cdot[-1\quad 0\quad 1\quad 1\quad 0\quad 0\quad 0]\,;$$

$$[P_2]=[P_1]-\begin{bmatrix}1\\0\\1\\0\\0\\0\\0\end{bmatrix}[1\quad 1\quad 0\quad -1\quad -1\quad -1\quad 0]\,;$$

$$[P_3]=[P_2]-\begin{bmatrix}1\\0\\0\\1\\0\\0\\0\end{bmatrix}[0\quad 0\quad 0\quad 1\quad 1\quad 0\quad -1]\,;$$

$$[P_4]=[P_3]-\begin{bmatrix}1\\0\\0\\1\\0\\0\\1\end{bmatrix}[0\quad 0\quad 0\quad 0\quad 0\quad 1\quad 1]\,;$$

(The conformity of some of the row vectors with the row vectors of $[A]$ is only accidental).

In the non-zero columns of the last projector $[P_{v-1}]$, the mesh current vectors are found. The diad-disintegration of matrix $[P_{v-1}]$ is very simple.

In our example:

$$[P_4]=\begin{bmatrix}-1\\1\\-1\\0\\0\\0\\0\end{bmatrix}[0\quad 1\quad 0\quad 0\quad 0\quad 0\quad 0]+\begin{bmatrix}0\\0\\1\\-1\\1\\0\\0\end{bmatrix}[0\quad 0\quad 0\quad 0\quad 1\quad 0\quad 0]+$$

$$+\begin{bmatrix}0\\0\\1\\-1\\0\\1\\-1\end{bmatrix}[0\quad 0\quad 0\quad 0\quad 0\quad 1\quad 0]$$

These two disintegrations can be written in general as follows:

$$[\mathrm{E}]-[P_{v-1}]=\sum_{k=1}^{v-1}[d_k]\,[q_k]_t \tag{5}$$

and

$$[P_{v-1}]=\sum_{k=v}^{e}[d_k]\,[q_k]_t \tag{6}$$

Moreover the sum of these two is:

$$[E]=\sum_{k=1}^{e}[d_k]\,[q_k]_t \tag{7}$$

3. The transformation matrices

The disintegration of the projectors into a sum of as many diads as their rank number is known to results a biorthogonal system. (This is easy to be realized: Expanding $[P_{v-1}]\,[x]$ which can be derived from Eq. (6), it is clear that $[d_k]\in R\,([P_{v-1}])$; therefore $[P_{v-1}]\,[d_k]=[d_k]$, a possibility in accordance with Eq. (6) only if the systems of column and row vectors of the diads are biorthogonal.) So the right hand sides of Eqs (5), (6), and (7) are biorthogonal systems, what is more, the right hand side of Eq. (7) is also a complete one because $[E]$ is a projector ranging over the whole e-dimensional space; the projector-like characteristic of matrix $[E]-[P_{v-1}]$ can be proved by a simple squaring.

Therefore, a complete biorthogonal system has been derived from which the column vectors of $e-v+1$ pairs of vectors are unit loop currents, the row vectors of $(v-1)$ pairs of vectors are orthogonal to the loop current, so they are potential voltage distributions and as a consequence, the other column vectors have a meaning of opened path current distribution, and the other row vectors have a meaning of rotational voltage distribution. The matrices of the biorthogonal bases will serve as transformational matrices.

$$[T]=[\,[d_1],[d_2]\,\ldots\ldots\,[d_e]\,]; \qquad [T]^{-1}=\begin{bmatrix}[q_1]_t\\ [q_2]_t\\ \cdot\\ \cdot\\ \cdot\\ [q_e]_t\end{bmatrix}$$

In our example:

$$[T]=\left[\begin{array}{cccc|ccc} 0 & 1 & 1 & 1 & -1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 1 & 1 & 0 & 0 & -1 & 1 & 1\\ 0 & 0 & 1 & 1 & 0 & -1 & -1\\ 0 & 0 & 0 & 0 & 0 & -1 & -1\\ 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1 & 0 & 0 & -1\end{array}\right]$$

and

$$[T]^{-1}=\left[\begin{array}{rrrrrrr}-1 & 0 & 1 & 1 & 0 & 0 & 0\\ 1 & 1 & 0 & -1 & -1 & -1 & 0\\ 0 & 0 & 0 & 1 & 1 & 0 & -1\\ 0 & 0 & 0 & 0 & 0 & 1 & 1\\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0\end{array}\right].$$

Comparing our symbols to the symbols of new base vectors discussed in the third and sixth Chapter of [1], it may be stated that:

$\mathbf{h}_{k'}$ new base vector corresponding here to column matrix $[d_k]$ and

$\mathbf{h}^{k'}$ new base vector corresponding here to row matrix $[q_k]_t$

In case of a non connected network, the algorithm can be used in the sequence of connected parts.

Let us note that this method is an application of theorem 1a in [2] on the set of linear equations determined by the incidence matrix.

Finally, the handling of *nodal current generators* has to be dealt with often used in practical network calculations, naturally supposing that for all the nodes of a connected network, their algebraic sum is zero. So in $(v-1)$ nodes, their value may be arbitrary. These nodal current generators are satisfactorily be characterized only by opened path, active components since loop currents are not leaving the nodes. Let the current of the i-th nodal current generator be K^i where $1\leqq i\leqq v-1$ and let us consider it to be positive when it points towards the node. Let us look for the opened path components K^b; $1\leqq b\leqq v-1$ (compare it to Chapter 6 in [1].)

Considering an irrotational voltage distribution in which the value of potential in the i-th node is 1 and in all the others is zero, i.e. the distribution characterized by the i-the row of matrix $[A]$, it can be stated that the system of nodal current generators with this distribution will have a power of K^i value, because in the way of the opened path currents driven through by the generators supplying nodes other than the i-th, there is either no change of potential or the increase of potential is equal to its decrease. Writing down these powers in the new base derived with the help of the algorithm, results in the set of linear equations of quadratic matrix

$$F^i_{\cdot b}K^b\equiv(A^i_{\cdot k}\,t^k_{\cdot b})\,K^b=K^i \tag{8}$$

where $A^i_{\cdot k}$ is the k-th element of the i-th row of matrix $[A]$ as the k-th component of the mentioned i-th voltage distribution in the primitive base, $t^k_{\cdot b}$ are the first $(v-1)$ columns of the transformation matrix $[T]$, *i.e.* $F^i_{\cdot b}$ is the b-th

component of the i-th irrotational voltage distribution in the new base. From Eq. (8), components K^b can be determined.
In our example:

$$F^i_{\cdot b}=\begin{bmatrix} 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & -1 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}=$$

$$=\begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & -1 \end{bmatrix}$$

Matrix $F^i_{\cdot b}$ is a triangular one, therefore solving the set of Eqs (8) is no problem. This is not only by change, the *matrix $F^i_{\cdot b}$ is always a triangular one.*

Namely, it comes from the algorithm [4] that the row vectors of the diads subtracted one after the other—i.e. the upper rows of matrix $[T]^{-1}$—are linear combinations of the rows so far used, of matrix $[A]$ and it is also true inversely: The i-th row of matrix $[A]$ is a linear combination of $[q_1]_t$, $[q_2]_t$ $\ldots [q_i]_t$ and such that it is orthogonal to the $k>i$-th column $[d_k]$ of matrix $[T]$.

4. The transformation matrices of diakoptics

Using the method of diakoptics, the subnetworks are first to be transformed into loopless networks [1]. Let us determine the incidence matrix of this loopless network which has a number of nodes equal to that of the original subnetwork. In the i-th row of this incidence matrix, there is the row matrix which gives (in the base of the new branches) the irrotational voltage distribution at which the potential of the i-the node is 1 and that of the others are zero. The same row matrix characterizes the above-mentioned voltage distributions which can be interpreted in the original network too, in the current path base in such a way that this gives the irrotational components, while the rotational components are zero. This, however, can be obtained from the (original) primitive base components—from the i-th row of the original incidence matrix—by a transformation according to Eq. (37) of [1].

That is:

$$[\alpha^{i}_{\cdot 1}, \alpha^{i}_{\cdot 2}, \ldots, \alpha^{i}_{\cdot (v-1)}, \quad 0, 0, \ldots, 0] =$$

$$= [A^{i}_{\cdot 1}, A^{i}_{\cdot 2}, \ldots, A^{i}_{\cdot e}] \underbrace{[t^{k}_{\cdot a} \mid t^{k}_{\cdot r}]}_{[T]},$$

where $[\alpha]$ and $[A]$ are the incidence matrices of the substitutional and the original networks, respectively. With subscript notation, and comparing also to Eq. (8) it is:

$$\alpha^{i}_{\cdot a} = A^{i}_{\cdot k}\, t^{k}_{\cdot a} = F^{i}_{\cdot a} \tag{9}$$

It was already seen in Chapter 7 of [1] (dealing with diakoptics) that a suitable way of disintegration into subnetworks is done as follows: first determined nodes and their branch connections are set into subnetworks (first partition), then one more node is joined to the corresponding branches of all but one subnetworks. The branches definitely left out will form the so-called inter-subnetwork. According to the first partition the number of nodes of the subnetworks will be $v_1, v_2, \ldots v_i, v_m$, the subscript i refers to the subnetwork without branch increment. For disintegration into subnetworks (without drawing the graph of the network) let us start from the incidence matrix of the whole network but for this, it is convenient to number the branches of each subnetwork in a sequence, where the numbers denoting the branches of the intersubnetwork are the highest, moreover the denoting the nodes of a subnetwork are also numbered in sequence and the order of subnetworks should follow the numbering of the branches. The nodes will belong to the subnetworks according to the first partition but one node among those of the subnetwork without branch increment will be left out to ensure that matrix $[A]$ will have no linearly dependent rows.

Using a numbering system described above, this incidence matrix is built up according to Fig. 3.

There are zeros in the non-striated parts. In addition to the incidence matrix $[A_k]$, there is a truncated matrix row belonging to the k-th subnetwork $(k \neq i)$ because of the common nodes. (This is absent of course if the common node is left out.) The transformation matrices of the subnetworks have to be determined by the sub-incidence matrices $[A_1]$, $[A_2]$, $\ldots [A_m]$ and by the algorithm described above, then the parameters of the substituting subnetworks and the matrices $[F]$ Eq. (9) have to be calculated by means of these transformation matrices. The incidence matrix of the connected substituting network is shown in Fig. 4, where the submatrix corresponding to the inter-

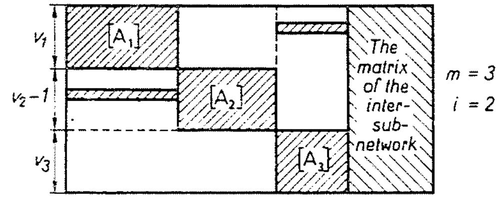

Fig. 3

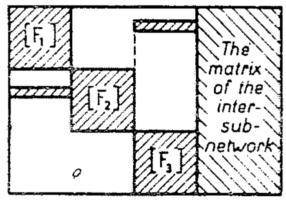

Fig. 4

subnetwork corresponds to the submatrix shown in Fig. 3, the quadratic matrices, aligned on the principal diagonal, are the new incidence matrices $[F]$, and the quadratic incidence matrix $[F_k](k \neq i)$ is completed into a singular matrix by the truncated matrix rows corresponding to the common nodes. On the basis of the big incidence matrix shown in Fig. 4, the passive currents and voltages can be determined, then coming back to the transformation matrices of the subnetworks, the currents of the original branches can be calculated.

As an example, let us consider the following incidence matrix of the graph, numbered in a way similar to the above:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

A non-abandoned common node is in the first row. From the incidence matrix belonging to the *first* subnetwork

$$[A_1] = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 \end{bmatrix}$$

the following projectors can be derived in accordance with algorithm (4):

$$[P_{11}]=[E]-\begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix}[1\quad 0\quad 0\quad -1\quad 0\quad -1]=\begin{bmatrix}0&0&0&1&0&1\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$$

$$f_1=1\ ;$$

$$[P_{12}]=[P_{11}]+\begin{bmatrix}0\\1\\0\\0\\0\\0\end{bmatrix}[0\quad -1\quad 0\quad 1\quad -1\quad 0]=\begin{bmatrix}0&0&0&1&0&1\\0&0&0&1&-1&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$$

$$f_2=2\ ;$$

$$[P_{13}]=[P_{12}]+\begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix}[0\quad 0\quad -1\quad 0\quad 1\quad 1]=\begin{bmatrix}0&0&0&1&0&1\\0&0&0&1&-1&0\\0&0&0&0&1&1\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$$

$$f_3=3\,.$$

So the transformation matrices are:

$$[T_1]=\begin{bmatrix}1&0&0&1&0&1\\0&-1&0&1&-1&0\\0&0&-1&0&1&1\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$$

$$[T_1]^{-1}=\begin{bmatrix}1&0&0&-1&0&-1\\0&-1&0&1&-1&0\\0&0&-1&0&1&1\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$$

The first subnetwork will be transformed into a loopless network by these matrices. The new sub-incidence matrix is, according to Eq. (9):

$$[F_1]=\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From the incidence matrix belonging to the *second* subnetwork:

$$[A_2]=\begin{bmatrix} 1 & -1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix}$$

The following projectors can be derived in accordance with algorithm (4):

$$[P_{21}]=[E]-\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}[1 \quad -1 \quad 0 \quad -1 \quad 0 \quad 0]=\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$f_1=1\ ;$$

$$[P_{22}]=[P_{21}]+\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}[0 \quad -1 \quad 1 \quad -1 \quad -1 \quad 0]=$$

$$=\begin{bmatrix} 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$f_2=2\ ;$$

$$[P_{23}]=[P_{22}]+\begin{bmatrix}0\\-1\\0\\1\\0\\0\end{bmatrix}[0\quad 0\quad 0\quad -1\quad -1\quad 1]=$$

$$=\begin{bmatrix}0&0&1&0&-1&0\\0&0&1&0&0&-1\\0&0&1&0&0&0\\0&0&0&0&-1&1\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$$

$$f_3=4\,.$$

So the transformation matrices are:

$$[T_2]=\begin{bmatrix}1&-1&0&1&-1&0\\0&-1&1&1&0&-1\\0&0&0&1&0&0\\0&0&-1&0&-1&1\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix},$$

$$[T_2]^{-1}=\begin{bmatrix}1&-1&0&-1&0&0\\0&-1&1&-1&-1&0\\0&0&0&-1&-1&1\\0&0&1&0&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}.$$

The second subnetwork will be transformed into a loopless network by these matrices. The new sub-incidence matrix is according to Eq. (9):

$$[F_2]=\begin{bmatrix}1&-1&0&-1&0&0\\-1&0&1&0&-1&0\\0&1&-1&0&0&1\end{bmatrix}\begin{bmatrix}1&-1&0\\0&-1&1\\0&0&0\\0&0&-1\\0&0&0\\0&0&0\end{bmatrix}=\begin{bmatrix}1&0&0\\-1&1&0\\0&-1&1\end{bmatrix}$$

So the incidence matrix of the whole substituting network becomes (the non-abandoned common node is in the first row):

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix}$$

From this, transformation matrices

$$[T]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$[T]^{-1}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

can be determined in accordance with algorithm (4). (The details of calculation are omitted.) The passive quantities of the substituting network will be calculated by the mesh method using these transformation matrices.

Summary

Taking the incidence matrix determining the graph as the starting point, an algoritm is described which calculates the base expanding the subspaces of the Kron-type opened and closed current paths. A method is developed for determining the transformation matrices for the practical application of diakoptics.

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Dr. György TEVAN, H-1521 Budapest