OPTIMIZING ANALYSES OF A DOUBLE-SIDED LINEAR INDUCTION MOTOR

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The computing method described in "Computation of a Double-Sided Linear Induction Motor by Undamped Traveling Waves" [1] with the help of some further simplifications permits to draw general conclusions for optimizing the performance of linear motors. Since this work is closely connected to the computing method, the numbering of equations in this second paper is a continuation of that of the previous one. Reference to Eqs (1) to (40) means the equations of the previous paper without further notations.

1. Neglecting skin effect; analysis of an "infinite" linear motor

For deducing general conclusions, relationships will be simplified by approximations irrelevant to the quality of decisive connections.

First, the skin effect will be omitted; it is possible since the secondary part is usually thin compared to the pole pitch, so that the skin effect affects the values of impedance and force to a degree irrelevant to the dependence on other parameters. Also harmonics of orders $\nu=-5$ and $\nu=7$ will be neglected; namely the $b_\nu \xi_\nu$ value in impedance and force expressions (38) and (40) is low compared to $b_1 \xi_1$ in (9) and (2) for the same harmonics. In the following, linear induction motors with only three traveling fields, of orders:

$$\nu=1, \qquad \nu=1-\frac{1}{2p_1} \quad \text{and} \quad \nu=1+\frac{1}{2p_1}$$

will be discussed. In case of a secondary part thin compared to the pole pitch, functions tg and tanh in (22) and (20) will be approximated by their respective arguments:

$$\mathbf{k}_\nu^\infty \cong \frac{j\tau_p}{\pi\nu d(1+jm_\nu)} \tag{41}$$

$$\mathbf{K}_\nu^\infty \cong \frac{\mathbf{k}_\nu^\infty + j\frac{\pi\nu}{\tau\rho}(\delta - d)}{1 - j\mathbf{k}_\nu^\infty \frac{\pi\nu}{\tau_p}(\delta - d)}$$

substituting the latter formula into the former one, transforming and neglecting the "two-fold small" value $\frac{d(\delta-d)}{\tau_p^2}$ yields:

$$\mathbf{K}_\nu^\infty = \frac{j\tau_p}{\pi\nu(\delta + jm_\nu d)} \tag{42}$$

After these preliminaries, let us examine tractive force for one slot of "infinite" linear induction motors (hence of "infinite length" and "infinite width"). Because of "infinite length", only the fundamental traveling wave has to be considered with a relative amplitude according to

$$\xi^2 = \lim_{p\to\infty} (b_1\xi_1 + b_{1+\frac{1}{2p_1}}\xi_{1+\frac{1}{2p_1}} + b_{1-\frac{1}{2p_1}}\xi_{1-\frac{1}{2p_1}}) \cong 0{,}905\,\xi_1^2$$

obtained from (4), (5), (2*) and (9), since now also the harmonics of order $1 \pm \frac{1}{2p_1}$ are converted to fundamental harmonics. (40) yields the force for a slot along a section of width l of the motor of infinite width:

$$F^\infty = 3q\frac{l}{\tau_p}\mu_0\Theta^2\xi^2\, Re\,(\mathbf{K}_1^\infty)$$

hence, according to (42):

$$F^\infty = \frac{3}{\pi}ql\mu_0\xi^2\Theta^2\,\frac{m_1 d}{\delta^2 + m_1^2 d^2} \tag{43}$$

where, according to (17) and (14):

$$m_1 = \frac{\gamma\mu_0\tau_p^2\omega_0}{\pi^2}s \tag{44}$$

a value proportional to the slip. Accordingly, the force-slip characteristic under constant excitation (current) of an "infinite" machine has a maximum (*Fig. 1*).

The maximum is at $m_1 = \frac{\delta}{d}$; it is therefore advisable to choose the nominal slip of the motor accordingly, hence, from (44):

$$s_n = \frac{\pi^2 \delta}{d\gamma\mu_0\tau_p^2\omega_0} = \frac{1}{G} \tag{45}$$

where

$$G = \frac{2\tau_p^2\mu_0 f}{\frac{\pi}{d\gamma}\delta} \tag{46}$$

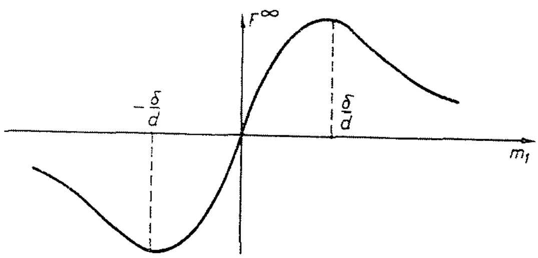

Fig. 1

the Laithwait goodness factor. Remark that the characteristic of constant voltage has a much higher slip for the max. force. Θ_n is a nominal excitation, product of the useful slot cross section by the thermally permissible current density, determining, with an impedance Z for s_n from (39), the terminal voltage for a slot, half of the voltage for a turn; by adequately choosing the number of turns connected in series for each phase, the linear motor can be adapted to the network voltage. With the nominal slip in (45), the nomial force for each slot [substituting $m_1 = \frac{\delta}{d}$ in (43)]:

$$F_n^\infty = \frac{3}{\pi} q\mu_0\xi^2 \frac{l}{2\delta} \Theta_n^2;$$

the slot pitch being:

$$\tau_s = \frac{\tau_p}{3q}$$

we obtain:

$$F_n^\infty = \frac{\mu_0}{\pi} \xi^2 \frac{l}{2\delta} \frac{\tau_p}{\tau_s} \Theta_n^2 \tag{47}$$

Construction reasons do not admit a too narrow slot or a too narrow tooth from saturation aspects: τ_s has a lower bound. The same is true for 2δ, since the real air gap $2(\delta - d)$ cannot be smaller than a given value from construction aspects, and the thickness $2d$ of the secondary part from strength aspects. Thus, for the same useful slot cross section hence the same copper consumption, the pole pitch has to be the highest possible to further increase F_n^∞.

The nominal efficiency of the secondary part, according to the relationship

$$\eta_{2,n} = 1 - s_n = 1 - \frac{1}{G}$$

obtained from (45) is the higher, the higher the goodness factor G, becoming, by comparing (11) and (46):

$$G = \frac{\mu_0 d \gamma v_0 \tau_p}{\pi \delta} \equiv \frac{\mu_0 d \gamma}{\pi \delta} \frac{v_0^2}{2f};$$

thus, for a specified synchronous velocity v_0, the pole pitch is made as large as possible, also from secondary efficiency aspects.

This is otherwise equivalent to reducing the frequency. From the latter relationships:

$$f = \frac{\mu_0 \gamma v_0^2 d}{2\pi\delta} (1 - \eta_{2,n})$$

(For instance, in case of $\frac{\delta}{d} = 2$, a secondary efficiency $\eta_{2,n} \geqq 0.9$ is possible in aluminium at a frequency $f \leqq 0.3 v_0^2$.)

2. Analysis of the influence of the edge-effect and of the end-effect; optimum performance of linear induction motors

In case of a linear motor of finite width, in the force term $\mathbf{K}_v^\infty$ has to be replaced by $\mathbf{K}_v$ from (37). $\mathbf{k}_v$ will be replaced by

$$\mathbf{k}_v = \mathbf{k}_v^\infty - \mathbf{k}_v' = \mathbf{k}_v^\infty \left(1 - \frac{\mathbf{k}_v'}{\mathbf{k}_v^\infty}\right)$$

according to (34), and the real air gap will be multiplied by a factor $\beta > 1$.

The relative correction value of $\mathbf{k}_v$ from Eqs (32) and (41):

$$\frac{\mathbf{k}_v'}{\mathbf{k}_v^\infty} = \frac{\frac{2\tau_p}{\pi v l}}{\kappa(1 + jm_v) + \sqrt{1 + jm_v}\,\coth \mathbf{a}}$$

where, from (30):

$$\mathbf{a} = \frac{v\pi l}{2\tau_p}\sqrt{1 + jm_v}$$

and using notations in (24):

$$\kappa = \tanh\left(\frac{\pi v l_s}{2\tau_p}\right).$$

For high τ_p/l values $\operatorname{cth} \mathbf{a} = \frac{1}{\mathbf{a}}$ hence:

$$\frac{\mathbf{k}_v'}{\mathbf{k}_v^\infty} \cong \frac{1}{1 + \kappa \frac{\pi v l}{2\tau_p}(1 + jm_v)}$$

For $\tau_p/l \to \infty$, $\mathbf{k}_v'/\mathbf{k}_v^\infty \to 1$, and $\mathbf{k}_v \to \mathbf{0}$. Upon increasing κ, this condition will be approximated for higher τ_p/l values, therefore it is advisable to chose the overhang l_s of the secondary part to have $\kappa \approx 1$. The factor β expressing the virtual air gap increase also grows by τ_p/l according to (35). Thus the more the transversal end effect reduces the force related to both the secondary part and the air gap, the higher the ratio τ_p/l; it becomes important where the pole pitch approximates the motor width. The pole pitch of an infinite linear motor was seen above to be advisably taken as large as possible. Now it appears that the finite width confines the advisable pole pitch, that is,—for a specified synchronous velocity—the frequency decrease.

The longitudinal end effect of the linear induction motor with a finite primary part will be considered for the simpler case with no transversal end effect; thus, the motor will be considered as of infinite width, having a thrust, from Eqs (40) and (42):

$$F=\frac{3}{\pi}ql\mu_0\Theta^2\sum_\nu\frac{b_\nu\xi_\nu}{\nu}\frac{m_\nu d}{\delta^2+m_\nu^2d^2},$$

summation affecting harmonics of orders $\nu=1$, $\nu=1-\frac{1}{2p_1}$ and $\nu=1+\frac{1}{2p_1}$.

Substituting (17), (14) and (46):

$$F=\frac{3}{\pi}q\mu_0\frac{l}{\delta}\Theta^2\sum_\nu\frac{b_\nu\xi_\nu}{\nu}\frac{\frac{Gs_\nu}{\nu^2}}{1+\left(\frac{Gs_\nu}{\nu^2}\right)^2} \tag{48}$$

where, from (13)

$$s_\nu=1-\nu(1-s); \qquad s=1-\frac{1}{\nu}+\frac{s_\nu}{\nu}$$

Thus, the force-slip characteristic referring to the constant excitation (current) in (48) may be considered as resultant of three part characteristics, having zeros at $s_\nu=0$ hence, in terms of fundamental harmonic slip s at

$$s_0^{(\nu)}=1-\frac{1}{\nu} \tag{49}$$

Extremal value places $s_{\nu 1,2}=\pm\frac{\nu^2}{G}$ are, in terms of s:

$$s_{1,2}^{(\nu)}=1-\frac{1}{\nu}\pm\frac{\nu}{G}. \tag{50}$$

Extremal values [substituting identity $3q=\frac{\tau_p}{\tau_s}$ and using (46) and (11)]:

$$F_{\nu 1,2}=\pm\frac{l}{2\tau_s}\frac{G}{v_0 d\gamma}\Theta^2\frac{b_\nu\xi_\nu}{\nu}. \tag{51}$$

For instance, for a linear motor that is a fully wound four-pole machine, hence $p_1 = p = 2$ cf. (4) and $G = 5$, the three harmonics are of orders 1, $\frac{5}{4}$ and $\frac{3}{4}$, part characteristic zeros being, according to (49), at:

$$s_0^{(1)} = 0, \qquad s_0^{(\frac{5}{4})} = \frac{1}{5}, \qquad s_0^{(\frac{3}{4})} = -\frac{1}{3}$$

and extremal values, according to (50) at:

$$s_{1,2}^{(1)} = \pm\frac{1}{5}, \quad s_{1,2}^{(\frac{5}{4})} = \frac{1}{5} \pm \frac{1}{4}, \quad s_{1,2}^{(\frac{3}{4})} = -\frac{1}{3} \pm \frac{3}{20},$$

the $\frac{b_\nu \xi_\nu}{\nu}$ values in (51) from (9) and (5) being:

$$0.5\xi_1^2, \qquad 0.25\xi_1\xi_{5/4}, \qquad 0.415\xi_1\xi_{3/4}$$

in this order.

The corresponding part characteristics are seen in Fig. 2 (slip is indicated as decreasing from 1). Thrust of a machine without longitudinal end effect is seen to arise as an (approximate) resultant around $s = 1$, while about the

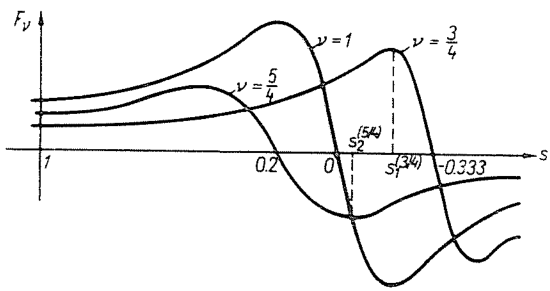

Fig. 2

maximum of the fundamental harmonic, a much lower resultant force is obtained than for the motor of infinite length, hence part characteristics are summed with a shift. The most of deviations result at about the generator maximum of the harmonic characteristic of order $1 + \frac{1}{2p_1}$ due to the negative partial force. The higher the reducing effect, the higher (the positiver) the slip value of the generator extremal value place of the characteristic of order $1+\frac{1}{2p_1}$ and the lower (the negativer) the slip value of the motor extremal value place of the characteristic of order $1-\frac{1}{2p_1}$. Thus, from the aspect of the degree of the longitudinal end effect, slip values

$$s_2^{\left(1+\frac{1}{2p_1}\right)} = \frac{1}{2p_1+1} - \frac{2p_1+1}{2p_1} \cdot \frac{1}{G}$$

and

$$s_1^{\left(1-\frac{1}{2p_1}\right)} = -\frac{1}{2p_1-1} + \frac{2p_1-1}{2p_1} \cdot \frac{1}{G}$$

from (50) are advisably plotted and analyzed as a function of $1/G$ (Fig. 3).

From a comparison between Figs 2 and 3 it is obvious that the influences of the longitudinal end effect to reduce the force grows with decreasing $1/G$, hence with increasing G.

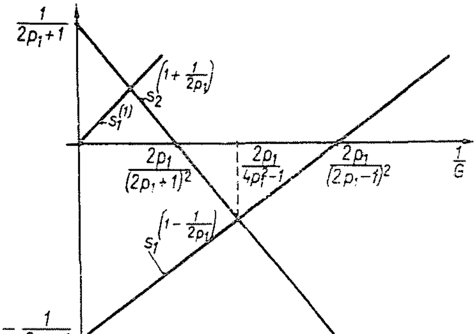

Fig. 3

According to (51), the maximum force increase (for each harmonic) by G/v_0, hence by the pole pitch τ_p according to (46); in conformity with the above, however, increasing G reduces the force.

Thereby also the finite length limits the pole pitch increase, hence—for a specified synchronous velocity—the frequency reduction. Increase of the goodness factor G beyond all measures distorts the resultant characteristic to a degree to have several zeros, namely then $s_2^{\left(1+\frac{1}{2p_1}\right)}$ outgrows $s_1^{(1)}$ (Line at 45° in Fig. 3).

In case of the specified synchronous velocity, the frequency optimum for the force or the efficiency may be numerically determined by means of the algorithm in [1]. Neglecting the transversal end effect permits, however, to construct a formula for assessing the optimum frequency. The above permit to state that the optimum G value is about where the longitudinal end effect is medium. Looking at Fig. 3, this is seen to be about the abscissa of the intersection of lines $s_2^{\left(1+\frac{1}{2p_1}\right)}$ and $s_1^{\left(1-\frac{1}{2p_1}\right)}$ thus:

$$\frac{1}{G_{opt}} \cong \frac{2p_1}{4p_1^2-1} \cong \frac{1}{2p_1} \cong \frac{1}{2p} \tag{51}$$

Then, $s_2^{\left(1+\frac{1}{2p_1}\right)}$ and $s_1^{\left(1-\frac{1}{2p_1}\right)}$ are negative and coincident, in the range of low positive slips the resultant of the characteristics of fraction order is about horizontal, thus, the resultant force of all three harmonics has a maximum at about that of the fundamental harmonic force. Selecting the nominal slip at this place according to (45):

$$s_{n,\,\mathrm{opt}} \cong \frac{1}{G_{\mathrm{opt}}} \approx \frac{1}{2p} \tag{52}$$

Primary part of the linear motor being of length:

$$L = 2p\tau_p$$

(51) can also be written as:

$$\tau_p G_{\mathrm{opt}} = L$$

yielding the assessed value for the optimum frequency, using Eqs (11) and (46):

$$f_{\mathrm{opt}} \approx \sqrt{\frac{\mu_0 \gamma d v_0^3}{4\pi L\delta}} \tag{53}$$

and the optimum nominal slip:

$$s_{n,\,\mathrm{opt}} \approx \frac{\tau_p}{L} = \frac{v_0}{2f_{\mathrm{opt}}L} \tag{54}$$

Specifying the condition

$$s_{n,\,\mathrm{opt}} < 0.1$$

in order to avoid poor secondary efficiency and comparing (53) and (54) yields the inequality:

$$Lv_0 > \frac{100\pi}{\mu_0\gamma}\frac{\delta}{d} \tag{55}$$

the primary part of low-speed linear induction motors has to be made long (multi-poled).

Finally, let us consider conditions optimum from force aspects in applications where slip of the linear motor little differs from $s=1$ (e.g. door opening).

In this case the longitudinal end effect was seen to little affect the force. From (43), substituting $3q=\frac{\tau_p}{\tau_s}$:

$$(F^{\infty})_{s=1}=\frac{\mu_0}{\pi}\frac{l}{\tau_s}\xi^2\Theta^2\frac{\tau_p(m_1)_{s=1}d}{\delta^2+[(m_1)_{s=1}d]^2}$$

where, according to (44) and (46):

$$(m_1)_{s=1}=\frac{2\gamma\mu_0\tau_p^2 f}{\pi}=\frac{\delta}{d}G$$

expressing τ_p from the latter, the force for a slip $s=1$:

$$(F^{\infty})_{s=1}=\mu_0\frac{l}{\tau_s}\xi^2\Theta^2\frac{1}{\sqrt{\mu_0\gamma\pi f\delta d}}\frac{G\sqrt{G}}{1+G^2}.$$

Optimum force of the goodness factor is:

$$G_{\text{opt}}=\sqrt{3}$$

yielding the pole pitch using (46).

The optimum force is higher for lower frequencies.

Summary

General conclusions are drawn in this paper concerning linear induction motors based on the computing method described in the author's previous paper. First the performance of the "infinite" linear motor is investigated neglecting the skin effect, then, as an approximation, the edge-effect and the end-effect are taken into consideration one-by-one for investigating the optimum performance of linear motors.

Reference

1. TEVAN, GY.: Computation of a Double-Sided Linear Induction Motor by Undamped Traveling Waves. Periodica Polytechnica, El. Eng. Budapest 23, No. 2. (1979).

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