CALCULATION OF IMPEDANCES CHARACTERIZING THE 750 KV TRANSMISSION LINE

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Introduction

For the determination of the interdependence between voltages and currents of the 750 kV transmission line the whole system may be considered to consist of 16 sections, each homogeneous (Fig. 1). Each section is made up of

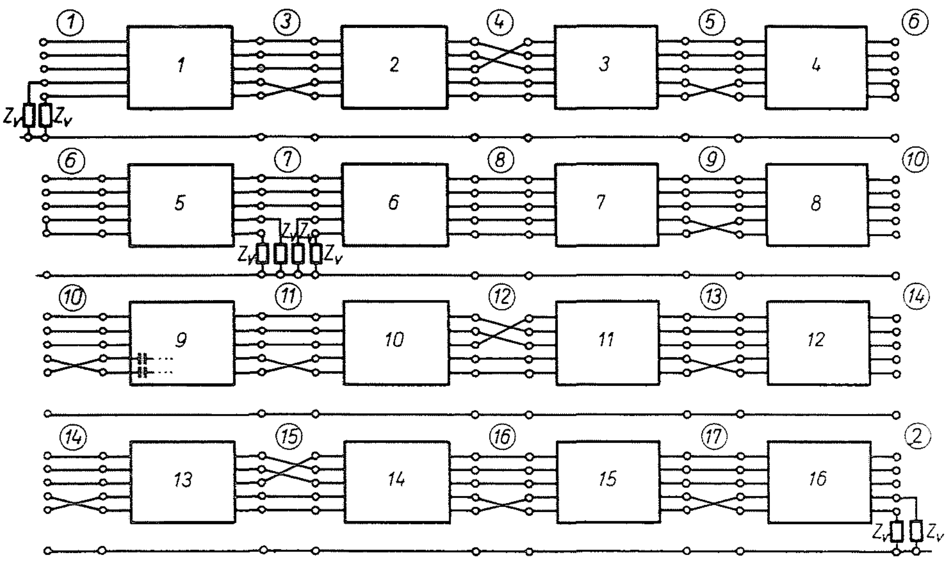

Fig. 1

5+1 conductors: 3 phase conductors, 2 ground wires and the earth. The sections form a chain-connection. The ends of the sections will be called connection points. For the calculations the connection points and the sections have been numbered. A method for the calculation of voltages and currents in networks consisting of transmission line sections and n-poles of concentrated parameters at the connection points is given in [1, 2, 3]. This procedure may not be directly applied to the system now considered. In the present case, apart from transposition of ground wires and phase conductors that can be taken into account in the method mentioned above, the ground wires are earthed through impedances at some connection points, at another point the ground wires of one section are connected to capacitors and at one connection point two ground wires are joined by a short-circuit. The calculation of such a system is possible by a modification of the method described in [3]. In the present paper this modified calculation is introduced for the determination of the interdependence between currents and voltages of phase conductors at the two ends of the transmission line, or more accurately: the determination of some matrices characterizing the network is dealt with. Let the column matrices of phase voltages and phase currents at connection point 1 be denoted by $\mathbf{U}_p$ and $\mathbf{I}_p$, and at connection point 2 by $\mathbf{U}_s$ and $\mathbf{I}_s$, respectively. For these the following may be written:

$$\begin{bmatrix} \mathbf{U}_p \\ \mathbf{U}_s \end{bmatrix} = Z \begin{bmatrix} \mathbf{I}_p \\ \mathbf{I}_s \end{bmatrix} . \tag{1}$$

Let us form the impedance matrix Z_s from the impedances at connection point 2, thus

$$\mathbf{U}_s = \boldsymbol{Z}_s \mathbf{I}_s . \tag{2}$$

The relationship between $\mathbf{U}_p$ and I_p is given by the input impedance matrix $\boldsymbol{Z}_{in}$, while the same between $\mathbf{U}_p$ and $\mathbf{I}_s$ is described by the transfer admittance matrix Y_t:

$$\mathbf{U}_p = \boldsymbol{Z}_{in} \mathbf{I}_p , \tag{3}$$

$$\mathbf{I}_s = Y_t \mathbf{U}_p . \tag{4}$$

From (2), using (4) we obtain:

$$\mathbf{U}_s = \boldsymbol{Z}_s \mathbf{I}_s = \boldsymbol{W} \mathbf{U}_p . \tag{5}$$

In the followings a method for the determination of matrices $\boldsymbol{Z}$, $\boldsymbol{Z}_{in}$, Y_t and $\boldsymbol{W}$ is presented.

Formulae for one transmission line section

The longitudinal layout of the z-directed section of length l_k is shown in Fig. 2. The column matrix of the currents of conductors is $\mathbf{i}_k(z)$, the column matrix of the voltages to earth is $\mathbf{u}_k(z)$. These may be written in the following form [1]:

$$\mathbf{u}_k(z)=e^{-\Gamma_k z}\,\mathbf{U}_k^{(+)}+e^{\Gamma_k z}\,\mathbf{U}_k^{(-)}, \tag{6}$$

$$\mathbf{i}_k(z)=Y_{0k}\,(e^{-\Gamma_k z}\,\mathbf{U}_k^{(+)}-e^{\Gamma_k z}\,\mathbf{U}_k^{(-)}), \tag{7}$$

where $\boldsymbol{\Gamma}_k$ is the propagation coefficient matrix and Y_{0k} is the wave admittance matrix of the section. These can be calculated in knowledge of geometrical

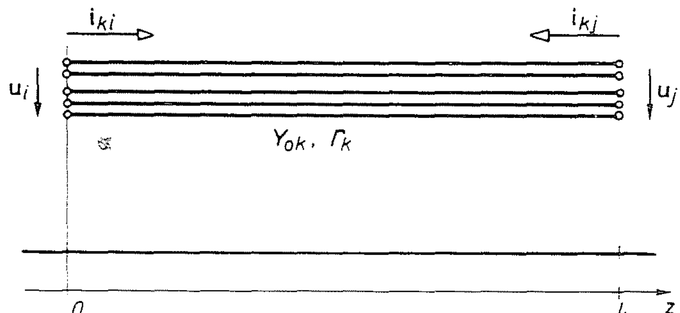

Fig. 2

dimensions and material constants. $\mathbf{U}_k^{(+)}$ and $\mathbf{U}_k^{(-)}$ are the column matrices of amplitudes at point $z=0$ of the voltage waves progressing in directions $+z$ and $-z$, respectively.

The k-th section of the transmission line, as it is seen from formulae (6) and (7), can be characterized by its wave admittance matrix Y_{0k}, propagation coefficient matrix $\boldsymbol{\Gamma}_k$ and length l_k. Let i denote the primary connection point of the section, j the secondary connection point, and the column matrices of currents and voltages at the same points be designated by $\mathbf{i}_{ki}$, $\mathbf{i}_{kj}$, $\mathbf{u}_i$, $\mathbf{u}_j$, respectively (Fig. 2). For these

$$\begin{bmatrix}\mathbf{i}_{ki}\\ \mathbf{i}_{kj}\end{bmatrix}=Y_k\begin{bmatrix}\mathbf{u}_i\\ \mathbf{u}_j\end{bmatrix}=\begin{bmatrix}p_k & r_k\\ r_k & p_k\end{bmatrix}\begin{bmatrix}\mathbf{u}_i\\ \mathbf{u}_j\end{bmatrix} \tag{8}$$

can be written, where

$$p_k=Y_{0k}\,\mathrm{ch}\,\Gamma_k l_k\,\mathrm{sh}^{-1}\,\Gamma_k l_k\,, \tag{9}$$

$$r_k=-\,Y_{0k}\,\mathrm{sh}^{-1}\,\Gamma_k l_k\,. \tag{10}$$

To each ground wire of the 9th section a capacitor is connected in series at the 10th connection point. This should be taken into account in determining the admittance matrix of the section. The inverse of the admittance matrix for the section without the capacitors is

$$Z_9'=\begin{bmatrix}\mathrm{ch}\,\Gamma_9 l_9 & 1\\ 1 & \mathrm{ch}\,\Gamma_9 l_9\end{bmatrix}\mathrm{sh}^{-1}\,\Gamma_9 l_9\,Y_{09}^{-1}\,, \tag{11}$$

where $\mathit{1}$ is the unit matrix of order 5. Due to the capacitors connected in series, this is modified as:

$$Z_9 = \begin{bmatrix} \operatorname{ch} \Gamma_9 l_9 + M & \mathit{1} \\ \mathit{1} & \operatorname{ch} \Gamma_9 l_9 \end{bmatrix} \operatorname{sh}^{-1} \Gamma_9 l_9 \, Y_{09}^{-1}, \tag{12}$$

where

$$M = \left[\begin{array}{c|cc} \mathit{0} & \multicolumn{2}{c}{\mathit{0}} \\ \hline & 1/j\omega C & 0 \\ \mathit{0} & & \\ & 0 & 1/j\omega C \end{array}\right] Y_{09} \operatorname{sh} \Gamma_9 l_9 . \tag{13}$$

Thus the admittance matrix of the section is

$$Y_9 = Z_9^{-1} = Y_{09} \operatorname{sh} \Gamma_9 l_9 \begin{bmatrix} p_9 & r_9 \\ r_9 & s_9 \end{bmatrix}, \tag{14}$$

where

$$p_9 = -\ (M + \operatorname{ch} \Gamma_9 l_9)\,(\mathit{1} - \operatorname{ch}^2 \Gamma_9 l_9 - M \operatorname{ch} \Gamma_9 l_9)^{-1} \tag{15}$$

$$r_9 = (\mathit{1} - \operatorname{ch}^2 \Gamma_9 l_9 - M \operatorname{ch} \Gamma_9 l_9)^{-1} \tag{16}$$

$$s_9 = -\operatorname{ch} \Gamma_9 l_9 \,(\mathit{1} - \operatorname{ch}^2 \Gamma_9 l_9 - M \operatorname{ch} \Gamma_9 l_9)^{-1} . \tag{17}$$

Let us form a diagonal hypermatrix from matrices p_k, r_k, s_k in the order of the numbers of the sections:

$$P = \langle p_1 \ p_2 \ldots p_{16} \rangle , \tag{18}$$

$$R = \langle r_1 \ r_2 \ldots r_{16} \rangle , \tag{19}$$

$$S = \langle s_1 \ s_2 \ldots s_{16} \rangle . \tag{20}$$

With the exception of the 9th section, $s_k = p_k$.

The graph of the network

A graph is correlated to the network in such a way that each transmission line section corresponds to a branch of the graph, and each connection point to a vertex of the graph (Fig. 3). The branches are given directions. It is appropriate to choose these to point from (1) to (2) at each branch, thus matching the direction of energy flow. Each node, to which the conductors of the sections are connected, as well as each conductor of the sections are given order numbers. The ordering at each section and connection point begins at 1.

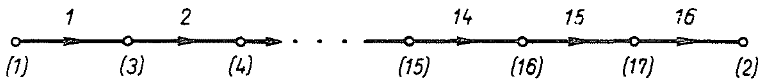

Fig. 3

The interconnection of the j-th connection point and the k-th section is characterized by a matrix $\mathbf{a}_{jk}$, in which the n-th element of the m-th row is 1, if at connection point j conductor n of the k-th section is connected to node m. Otherwise this element equals 0.

In the system under consideration (Fig. 1) the number of nodes at the majority of connection points is 5. At connection point 7, the number of nodes is 7: the phase conductors of the 5-th and 6-th sections connect to nodes 1, 2, 3; the ground wires of the 5-th section to nodes 4, 5, and the ground wires of the 6-th section to nodes 6, 7. At connection point 6 there are 4 nodes, the 4-th of which is connected to the ground wires of the 4-th and 5-th section.

The graph is characterized by non-reduced incidence matrices $\mathbf{A}$ and $\mathbf{A}_0$, which give information about the interconnections at each connection point. In hypermatrix $\mathbf{A}$ the rows of blocks correspond to connection points (vertices), the columns to sections (branches). The k-th block of the j-th row is $\mathbf{a}_{jk}$, if branch k is directed away from vertex j, $-\mathbf{a}_{jk}$ if branch k is directed towards vertex j and $\mathbf{0}$ if branch k and vertex j are not connected. $\mathbf{A}_0$ differs from $\mathbf{A}$ by ignoring the direction of branches, i.e. here $\mathbf{a}_{jk}$ stands for $-\mathbf{a}_{jk}$. For the considered network:

$$A=\begin{bmatrix} a_{11} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & -a_{216} \\ -a_{31} & a_{32} & 0 & 0 & \dots & 0 & 0 \\ 0 & -a_{42} & a_{43} & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & a_{1615} & 0 \\ 0 & 0 & 0 & 0 & \dots & -a_{1715} & a_{1716} \end{bmatrix}.$$

Equations for transmission line systems

The concentrated parameter circuits joining the connection points are taken into account by Norton-generators in star connection (Fig. 4). At connection point j column matrices $\mathbf{i}_{gj}$ and $\mathbf{i}_{cj}$ are formed from their source currents and branch currents, respectively, while diagonal matrix Y_{gj} is constructed from the admittances. With these:

$$\mathbf{i}_{cj}=\mathbf{i}_{gj}-Y_{gj}\mathbf{u}_j\,. \tag{21}$$

In the main diagonal of Y_{g2} the first three elements equal the load admittances of the transmission line.

$$Y_{g1}=\langle 0\ \ 0\ \ 0\ \ 1/Z_v\ \ 1/Z_v\rangle$$

$$Y_{g2}=\langle \mathbf{Z}_s^{-1}\ \ 1/Z_v\ \ 1/Z_v\rangle$$

$$Y_{g7}=\langle 0\ \ 0\ \ 0\ \ 1/Z_v\ \ 1/Z_v\ \ 1/Z_v\ \ 1/Z_v\rangle\,.$$

In the other cases $Y_{gj}=\mathbf{0}$. With the exception of connection points 1 and 2, $\mathbf{i}_{gj}=\mathbf{0}$.

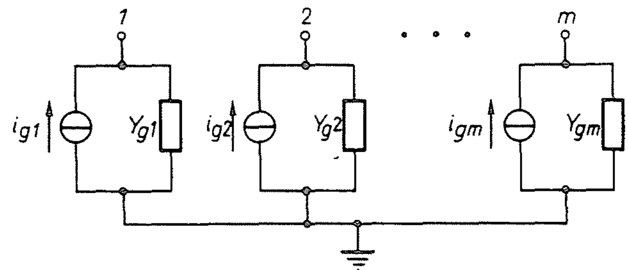

Fig. 4

The calculation is carried out with the aid of the node equations. To write these, currents are classified in three groups. At first those currents are written, which flow at the primary connection points of the sections, i.e. whose direction coincides with that of the corresponding branch in the graph. For section k, according to (8):

$$\mathbf{i}_{ki}=p_k\mathbf{u}_i+r_k\mathbf{u}_j\,. \tag{22}$$

Let $\mathbf{U}$ designate the hypermatrix constructed from matrices $\mathbf{u}_i$ $(i=1, 2,\ldots, 17)$, thus the hypermatrix of the above currents in the order of connection points is

$$\mathbf{I}_c'=\frac{1}{4}(A_0+A)\,[P\,(A_0+A)^+ + R\,(A_0-A)^+]\,\mathbf{U}\,, \tag{23}$$

where the transposition of matrices has been denoted by $^+$.

In the second group those currents are written, which flow at the secondary connection points of the sections, i.e. whose direction is opposite to that of the corresponding branch in the graph. For all of the sections in the order of connection points these are

$$\mathbf{I}_c'' = \frac{1}{4}(A_0 - A)\,[R\,(A_0 + A)^+ + S(A_0 - A)^+]\,\mathbf{U}\,. \tag{24}$$

The currents of the third group are the branch currents of the concentrated parameter circuits. These may be summarized in accordance with (21) in the following form:

$$\mathbf{I}_c = \mathbf{I}_g - Y_g \mathbf{U}\,, \tag{25}$$

where $\mathbf{I}_c$, $\mathbf{I}_g$ are the hypermatrices formed from matrices $\mathbf{i}_{cj}$, $\mathbf{i}_{gj}$ $(j = 1, 2, \ldots, 17)$, respectively, and

$$Y_g = \langle Y_{g1}\, Y_{g2}\, 0\, 0 \ldots 0\, Y_{g13}\, 0\, 0 \ldots 0 \rangle \tag{26}$$

is a diagonal matrix. The node equations of Kirchhoff for the complete network are

$$\mathbf{I}_c' + \mathbf{I}_c'' - \mathbf{I}_c = \mathbf{0}\,. \tag{27}$$

Substituting formulae (23), (24) and (25) we get:

$$(Y_c + Y_g)\mathbf{U} = \mathbf{I}_g\,, \tag{28}$$

where

$$\begin{aligned} Y_c = \frac{1}{4}\,[A_0\,(P + 2R + S)\,A_0^+ + A\,(P - 2R + S)\,A^+ + \\ + A_0 (P - S)\,A^+ + A\,(P - S) A_0^+]\,. \end{aligned} \tag{29}$$

$\mathbf{U}$ may be derived from (28):

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \vdots \\ \mathbf{u}_{17} \end{bmatrix} = (Y_c + Y_g)^{-1} I_g = Z_m \begin{bmatrix} \mathbf{i}_{g1} \\ \mathbf{i}_{g2} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}, \tag{30}$$

Let us give two different numberings to the nodes of the network. First the nodes are numbered in the order of the connection points. At the second numbering those nodes of connection points 1 and 2 are given the order numbers 1, 2, . . ., 6 which are connected to phase conductors, further nodes are assigned the order numbers 7, 8, . . ., . The relation between the two numberings is characterized by matrix $\boldsymbol{H}$, in which the j-th element of the i-th row: $h_{ij}=1$ if the order number of the i-th node according to the first numbering is j at the second numbering, otherwise $h_{ij}=0$. Let us form the transformed matrices

$$\mathbf{U}_h=\boldsymbol{H}\mathbf{U}=\begin{bmatrix}\mathbf{U}_p\\ \mathbf{U}_s\\ \mathbf{U}_t\end{bmatrix}, \quad \mathbf{I}_h=\boldsymbol{H}\mathbf{I}_g=\begin{bmatrix}\mathbf{I}_p\\ \mathbf{I}_s\\ \mathbf{0}\end{bmatrix}, \tag{31}$$

$$\boldsymbol{Z}_h=\boldsymbol{H}\boldsymbol{Z}_m\boldsymbol{H}^+ . \tag{32}$$

For the calculation of $\boldsymbol{Z}$ the Norton-generators at connection point 2 are chosen ideal, i.e. $\boldsymbol{Z}_s^{-1}=\boldsymbol{0}$ and thus $\mathbf{i}_{g2}=\mathbf{I}_s$.

After transformation, Eq. (30) yields:

$$\begin{bmatrix}\mathbf{U}_p\\ \mathbf{U}_s\\ \mathbf{U}_t\end{bmatrix}=\begin{bmatrix}\boldsymbol{Z}_a & \boldsymbol{Z}_b\\ \\ \boldsymbol{Z}_b & \boldsymbol{Z}_c\end{bmatrix}\begin{bmatrix}\mathbf{I}_p\\ \mathbf{I}_s\\ \mathbf{0}\end{bmatrix}, \tag{33}$$

where $\boldsymbol{Z}_a$ is a quadratic block of matrix $\boldsymbol{Z}_h$ of order six. Comparing with (1), it is evident that $\boldsymbol{Z}_a=\boldsymbol{Z}$.

For the following calculation let us presume, that the source currents of the Norton-generators at connection point 2 equal zero, and their inner impedances equal the load impedances. Thus transforming (30):

$$\begin{bmatrix}\mathbf{U}_p\\ \mathbf{U}_s\\ \mathbf{U}_t\end{bmatrix}=\begin{bmatrix}\boldsymbol{Z}_{11} & \boldsymbol{Z}_{12} & \boldsymbol{Z}_{13}\\ \boldsymbol{Z}_{21} & \boldsymbol{Z}_{22} & \boldsymbol{Z}_{23}\\ \boldsymbol{Z}_{31} & \boldsymbol{Z}_{32} & \boldsymbol{Z}_{33}\end{bmatrix}\begin{bmatrix}\mathbf{I}_p\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}, \tag{34}$$

where $\boldsymbol{Z}_{11}$ and $\boldsymbol{Z}_{22}$ are quadratic blocks of order three. From this:

$$\mathbf{U}_p=\boldsymbol{Z}_{11}\mathbf{I}_p=\boldsymbol{Z}_{in}\mathbf{I}_p, \tag{35}$$

i.e. the input impedance matrix has been determined.

From (34):

$$\mathbf{U}_s=\boldsymbol{Z}_{21}\,\mathbf{I}_p \tag{36}$$

Expressing $\mathbf{I}_p$ from (35) and substituting into (36):

$$\mathbf{U}_s=\boldsymbol{Z}_{21}\boldsymbol{Z}_{11}^{-1}\mathbf{U}_p \tag{37}$$

i. e. according to (5)

$$\boldsymbol{W}=\boldsymbol{Z}_{21}\boldsymbol{Z}_{11}^{-1} \tag{38}$$

is the voltage transfer matrix of the network.

Summary

The paper gives a method for the calculation of the impedance, input and transfer impedance, as well as the voltage transfer matrices for the 750 kV transmission line, in which wave phenomena, transposition of conductors, and impedances connected to certain sections are all taken into account.

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