CALCULATION OF ELECTROMAGNETIC FIELD IN THE AIR GAP OF ASYNCHRONOUS MACHINES BY VARIATIONAL METHOD

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1. Introduction

In the air gap and rotor slots of asynchronous machines quasi-stationary electromagnetic field is generated by the excitation of travelling waves. In this paper a variational method for the calculation of quasi-stationary electromagnetic field is briefly described. This method is suitable for the numerical, approximate determination of the field in the air gap and rotor slots. The procedure replaces the solution of the differential equation relating to the vector potential with the solution of a linear algebraic system of equations, by the application of Ritz's method [3, 4, 6, 7] and the conditional extremum calculus of Lagrange [1, 2, 4]. The unknowns of the system of equations are coefficients, with the aid of which the vector potential, and thus magnetic induction and electric field intensity, can be expressed as linear combinations of coordinate functions suitably chosen in advance.

The complex formalism generally used for description of phenomena with sinusoidal time dependence is employed. Both the unknown coefficients and the coordinate functions are considered to be complex for the solution. Thus field quantities are easily obtained in travelling wave form.

The results are applied to calculate the electromagnetic field in the air gap and rotor conductors of a squirrel-cage asynchronous machine. In the model considered the permeability of the iron is taken to be infinite and the machine is extremely long, thus a two-dimensional, linear problem of field determination is derived. The given stator excitation is taken into account by surface currents calculated by Fourier expansion.

Numerical calculations have been carried out for the determination of the field in a 6/24 pole pole-changing asynchronous motor at different values of slip. It has been established that in some cases, the approximation usual in literature [5, 8], that recognizes the effect of the basic harmonic only in stator excitation, causes considerable error.

2. Calculation of quasi-stationary electromagnetic field by variational method

For the calculation of quasi-stationary electromagnetic field in steady state with sinusoidal excitation the partial differential equation

$$\mathrm{rot}\,\mathrm{rot}\,\mathbb{A} + j\omega\mu\sigma\mathbb{A} = \mathbf{0} \tag{1}$$

has to be solved for the vector potential at suitable boundary conditions, where

$$\mathbb{B} = \mathrm{rot}\,\mathbb{A}\,, \tag{2}$$

and the usual complex formalism has been employed.

According to variational calculus the solution of differential equation (1) is equivalent to finding the formal extremal function of the following complex functional:

$$I = \int_V (\mathrm{rot}\,\mathbb{A}\,\mathrm{rot}\,\mathbb{A}^* + j\omega\mu\sigma\mathbb{A}\,\mathbb{A}^*)\,\mathrm{d}V + \int_S \mu\mathbb{K}\mathbb{A}^*\,\mathrm{d}S\,, \tag{3}$$

where V is the region considered, S is the surface bounding the region, $\mathbb{K}$ is surface current, and sign "*" denotes conjugation. For the determination of an approximate formal extremal function satisfying boundary conditions Ritz's method and the conditional extremum calculus is applied (1, 2, 3, 4). Let the vector potential be approximated by a linear combination of the elements of an entire function set:

$$\mathbb{A} \approx f^+ a = \sum_{k=1}^{n} a_k f_k\,, \tag{4}$$

where column matrix f consists of the elements f_k of the entire function set, which may also be complex valued, while a is the column matrix of the complex coefficients a_k to be determined. It is presumed that the satisfaction of boundary conditions may be ensured with the fulfillment of linear equations by the unknowns a_k:

$$\mathbb{F}a = d\,. \tag{5}$$

The natural boundary condition of functional (3) is that the tangential component of magnetic field intensity equals the negative value of surface current on the boundary. Where surface current is zero, magnetic field is perpendicular to the boundary.

The Lagrangian for the determination of conditional formal extremum is

$$L(\boldsymbol{a}, \lambda) = \int_V (\mathrm{rot}\, \boldsymbol{f}^{+} \boldsymbol{a}\, \mathrm{rot}\, \boldsymbol{f}^{+*} \boldsymbol{a}^{*} + j\omega\mu\sigma \boldsymbol{f}^{+} \boldsymbol{a} \boldsymbol{f}^{+*} \boldsymbol{a}^{*})\, \mathrm{d}V + \\ + \int_S \mu\, \mathrm{f}^{+*} \boldsymbol{a}^{*} \mathbf{K}\, \mathrm{d}S + \lambda^{+} (\mathbf{F}^{*} \boldsymbol{a}^{*} - \boldsymbol{d}^{*}), \tag{6}$$

where λ is the column matrix of Lagrange multiplicators. The necessary condition of formal extremum is

$$\frac{\partial L}{\partial \boldsymbol{a}^{*}} = \mathbf{0}\,; \quad \frac{\partial L}{\partial \lambda} = \mathbf{0}\,. \tag{7}$$

These yield the following for the coefficients to be determined:

$$\mathbf{A}\boldsymbol{a} + \mathbf{F}^{+*}\lambda = \boldsymbol{e}\,, \tag{8}$$

$$\mathbf{F}\boldsymbol{a} = \boldsymbol{d}\,, \tag{9}$$

where

$$\mathbf{A} = \int_V (\mathrm{rot}\, \boldsymbol{f}^{*}\, \mathrm{rot}\, \boldsymbol{f}^{+} + j\omega\mu\sigma \boldsymbol{f}^{*} \boldsymbol{f}^{+})\, \mathrm{d}V, \tag{10}$$

$$\boldsymbol{e} = \int_S \boldsymbol{f}^{*} \mathbf{K}\, \mathrm{d}S\,. \tag{11}$$

From the set of linear Eqs (8), (9) the unknown column matrix $\boldsymbol{a}$, and thus according to (4) the function approximating the vector potential may be determined.

3.1. The model of asynchronous machine

The method described above is applied to determine the electromagnetic field in the air gap and rotor conductors of an induction motor.

The machine may be considered to be of infinite length, thus the problem is two-dimensional in the plane perpendicular to the axis. The permeability of iron has been presumed to be infinite, and so magnetic field intensity has normal component only on the surface of iron. The stator excitation has been assumed to be given. Since the size of slot mouth on the stator is extremely small, the tangential (azimuthal) component of magnetic field intensity can be taken to be constant along the slot mouth, and its value is readily calculated as the quotient of slot current and width of slot mouth. On the surface of teeth the tangential component of field intensity equals zero. Thus the azimuthal component of magnetic field intensity with sinusoidal time variation is known along the stator surface considered to be smooth. Expanding it into Fourier series stator excitation may be taken into account by the sum of travelling wave surface currents of different orders.

$$\mathbf{K} = \sum_{\nu} K_{\nu m} \cos(\omega_0 t - p\nu\varphi)\mathbf{e}_z, \tag{12}$$

where ω_0 is network angular frequency, p is the number of pole pairs, ν is the order of different harmonics, $K_{\nu m}$ is the amplitude of the ν-th harmonic of surface current. The solution is sought in the coordinate system rotating with the rotor. On transformation to this system the surface current is:

$$\mathbf{K}' = \sum_{\nu} K_{\nu m} \cos(s_\nu \omega_0 t - p\nu\varphi)\mathbf{e}_z, \tag{13}$$

where

$$s_\nu = \frac{\omega_0/(p\nu) - \omega}{\omega_0/(p\nu)}, \tag{14}$$

is the slip for the ν-th harmonic, and ω is the mechanical angular velocity of the rotor.

In comparison with other usual models [5, 8], this model is more accurate insomuch, that it takes all harmonics of excitation into account.

The form of rotor slot has been simplified. The walls of the slot has been taken to be of radial direction, the bottom of the slot is a circular arc. This is acceptable for great number of slots. The slot is filled with conductor of given conductivity. The simplified model of the electric machine is shown in Fig. 1.

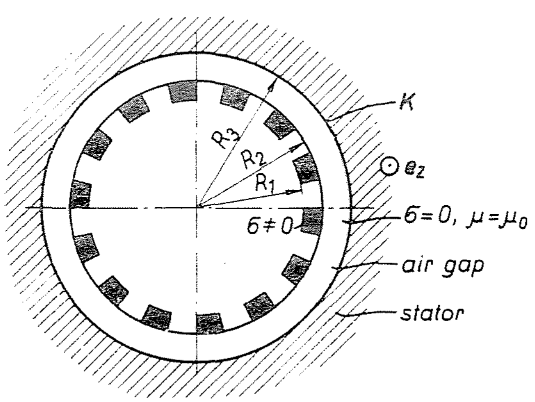

Fig. 1

3.2. Application of variational method

In the forthcomings the effect of surface current of order ν is dealt with only, since due to the linearity of our model the field may be calculated as the sum of fields generated by different harmonics. The excitation being a travelling wave, the solution has to be such, that after the excitation having turned one slot pitch, the initial condition is to recur at a distance of one slot pitch from every angle φ. This condition is fulfilled if the vector potential is chosen as follows:

$$\mathbf{A}_\nu = \mathbf{e}_z \sum_i \sum_k a_{ik} \left(\frac{r}{R_3}\right)^{p\nu + iz_2} e^{-j(p\nu + kz_2)\varphi}$$

$$i = 0, \pm 1, \pm 2, \ldots; \qquad k = 0, \pm 1, \pm 2 \ldots, \tag{15}$$

where z_2 is the number of rotor slots, a_{ik} are the coefficients to be determined. It can be shown that for such a choice it suffices to consider a region of one slot pitch. Fig. 2. shows the conditions of the region to be examined.

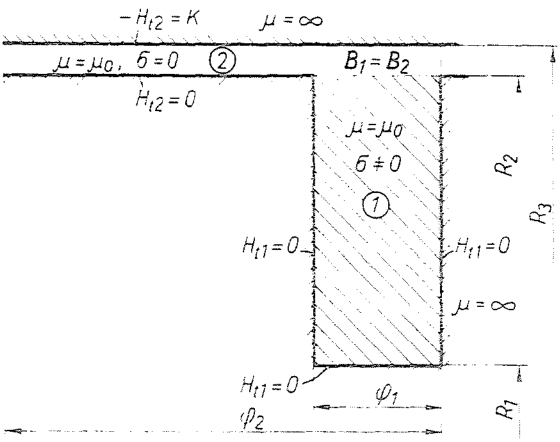

Fig. 2

The region has been divided into two subregions. Subregion 1 is the slot, while subregion 2 is the air gap. The form of the vector potential in subregions 1 and 2 is:

$$\mathbf{A}_{\nu 1} = \mathbf{e}_z \sum_i \sum_k a_{ik} \left(\frac{r}{R_3}\right)^{p\nu + iz_2} e^{-j(p\nu + kz_2)\varphi} \tag{16}$$

$$\mathbf{A}_{\nu 2} = \mathbf{e}_z \sum_i \sum_k b_{ik} \left(\frac{r}{R_3}\right)^{p\nu + iz_2} e^{-j(p\nu + kz_2)\varphi} \tag{17}$$

The boundary conditions are:

$$H_{\varphi 1}(r=R_1) = H_{r1}(\varphi=0)=H_{r1}(\varphi=\varphi_1)=H_{\varphi 2}(r=R_2)=0\,, \tag{18}$$

$$-H_{\varphi 2}(r=R_3)=K_{\nu m}e^{-jp\nu\varphi}\,. \tag{19}$$

On the curve separating the two subregions the following continuity conditions are to be fulfilled:

$$H_{\varphi 1}(r=R_2)=H_{\varphi 2}(r=R_2)\,, \tag{20}$$

$$B_{r1}(r=R_2)=B_{r2}(r=R_2)\,. \tag{21}$$

Conditions (18) are natural boundary conditions of functional (3), thus these are not prescribed. Conditions (19)—(21) may be written in matrix form:

$$\mathbf{F}_2\boldsymbol{b}=\boldsymbol{d}_2\,, \tag{22}$$

$$\mathbf{F}_{12}\boldsymbol{a}+\mathbf{F}_{21}\boldsymbol{b}=\mathbf{0}\,, \tag{23}$$

where column matrices $\boldsymbol{a}$ and $\boldsymbol{b}$ consist of the coefficients to be determined in subregions 1 and 2,respectively. The construction of matrices $\mathbf{F}_2$, $\mathbf{F}_{12}$, $\mathbf{F}_{21}$ and $\boldsymbol{d}_2$ is not dealt with here.

Consequently partial differential equations

$$\text{rot rot}\,\mathbf{A}_{\nu 1}+js_\nu\omega_0\mu\sigma\mathbf{A}_{\nu 1}=\mathbf{0}\,, \tag{24}$$

$$\text{rot rot}\,\mathbf{A}_{\nu 2}=\mathbf{0} \tag{25}$$

have to be solved in the region of Fig. 2, with boundary conditions (22)—(23). (It should be noted that s_ν appears in equation (24) due to the coordinate transformation.) Applying the procedure described in 2, the following set of equations is derived:

$$\mathbf{A}\boldsymbol{a}+\mathbf{F}_{12}^{+*}\lambda_{12}=\mathbf{0}\,, \tag{26}$$

$$\mathbf{B}\boldsymbol{b}+\mathbf{F}_2^{+*}\lambda_2+\mathbf{F}_{21}^{+*}\lambda_{12}=\boldsymbol{e}_2\,, \tag{27}$$

$$\mathbf{F}_2\boldsymbol{b}=\boldsymbol{d}_2\,, \tag{28}$$

$$\mathbf{F}_{12}\boldsymbol{a}+\mathbf{F}_{21}\boldsymbol{b}=\mathbf{0}\,, \tag{29}$$

where matrices $\mathbf{A}$, $\mathbf{B}$ and $\boldsymbol{e}_2$ can be written in accordance with (10)—(11).

In knowledge of the complex coefficients $\boldsymbol{a}$ and $\boldsymbol{b}$ obtained as the solution of equations (26)—(29), the components of electromagnetic field in the two subregions are

$$\mathbf{B}_1 = \mathrm{rot} f^+ \boldsymbol{a}, \tag{30}$$

$$\mathbf{E}_{z1} = -js_\nu \omega_0 f^+ \boldsymbol{a}, \tag{31}$$

$$\mathbf{B}_2 = \mathrm{rot} f^+ \boldsymbol{b}, \tag{32}$$

$$\mathbf{E}_{z2} = -js_\nu \omega_0 f^+ \boldsymbol{b}. \tag{33}$$

4. Example

Calculations have been carried out for the determination of the electromagnetic field in a given 6/24 pole pole-changing induction motor. In the 6 pole case rated angular speed has been examined. In the 24 pole case the field harmonics have been determined for some angular speeds occuring during loss of speed. Orders 1, 5, 7 and 11 have been taken into account from the components of excitation.

In the 6 pole case the effect of upper harmonics was not considerable as compared to that of the basic harmonic, in the 24 pole case, however, to neglect them would cause substantial error.

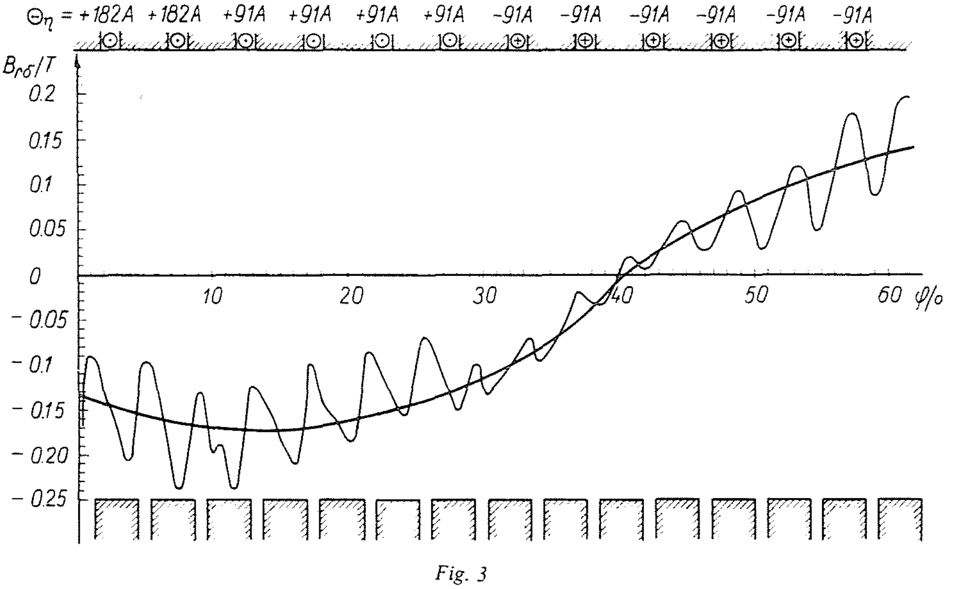

Fig. 3

The calculation has been executed on the computer R32. The field harmonics have been approximated in both the air gap and the rotor slots by 45 terms. To determine the effect of one excitation harmonic at one angular speed took about 4 minutes.

Instead of presenting numerical results the radial component of magnetic induction in the air gap has been plotted in Fig. 3 in a given moment at the rated angular speed of the 6 pole case $s_n = 0.1$; $I_n = 14$ A. The effect of slots is easily recognizable on the plot, it causes the curve of induction to become "rippled".

Summary

A variational method suitable for numerical determination of quasi-stationary electromagnetic field in the air gap and rotor slots of induction motors is presented. In the model end effects of the motor and nonlinearity is neglected, stator excitation is presumed to be given, and modelled by surface currents. The method is suitable for the examination of the effect of the harmonics in the excitation. Results are obtained in travelling wave form, readily utilizable in practice.

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