SYNTHESIS BY BLOCK CHARACTERIZATION OF OP-AMP ACTIVE FILTERS

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1. Modelling with blocks

It is well-known that conjugated pairs of poles in the transfer function can be generated by applying the sum or difference decomposition method. The inverting and non-inverting inputs of the op-amp offer a simple way to realize the substraction. There are two extreme cases of second-order blocks with only one op-amp: the output signal is fed back frequency-dependent to only the inverting input or the output signal is totally fed back to the inverting input (voltage follower) and at the same time a feedback with frequency dependence is applied at the non-inverting input. These two extremes of the so-called infinite gain and controlled source realizations and the other non-extreme cases with Q-enhancement seem to be mutually related through the rules of complementary transformation [1].

The two input terminals of an op-amp are ideally suited to compose the difference of two components. This fact is easy to be modelled by blocks in a diagram and utilised as a useful tool for the synthesis. The partitioned blocks are generated consequently for the signal of the generator applied as well as for the components fed back from the output [2]. A recent publication [3] restricts this method to the feedback components.

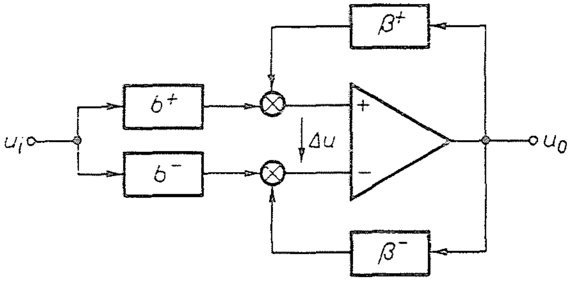

Fig. 1. Feedforward and feedback model

As a matter of fact the suggested synthesis is not a direct one, the term used indicates only the (partly heuristic) usefulness of the results derived from the analysis. For example the method is readily applicable to investigate the inherent possibilities and limitations of complementary systems.

Fig. 1 shows the block structure, which is quite suitable for analysis purposes of single op-amp filter sections. If the common mode gain cannot be neglected, the models in [1], [3], [4] and [8] are improper and the derived results should be revised. In the following sections the common mode parameters are supposed to be ideal.

In accordance with the block scheme shown in Fig. 1

$$\Delta u = \frac{u_0}{A} = bu_i - \beta u_0,$$

where A is the open loop differential gain,

$$b = b^+ - b^- = \left.\frac{\Delta u}{u_i}\right|_{u_0=0},$$

and

$$\beta = \beta^- - \beta^+ = -\left.\frac{\Delta u}{u_0}\right|_{u_i=0}.$$

The symbol b will denote below the feedforward transfer ratio and β the feedback transfer ratio. Rearranging the terms results:

$$A_f = \frac{u_0}{u_i} = \frac{b}{\beta}\frac{A}{1+A} = \frac{b}{\beta}\frac{L}{1+L} = \frac{b}{\beta}e.$$

A_f is the resulting filter voltage transfer function, $L = A\beta$ is the loop gain and e the error function. Assuming $|L| \gg 1$ gives:

$$A_f \approx \frac{b}{\beta}.$$

2. Single op-amp second order blocks as complementary circuits

The block scheme of the most widely used basic configurations are shown in Figs 2 and 3. In the case of Fig. 2 $b = -b^-$ and $\beta = \beta^-$, while in Fig. 3 $b = b^+$ and $\beta = 1 - \beta^+$. To keep the number of passive components as low as possible the passive network is generally a three-port structure, in which each component plays an important role in both of the above-mentioned transfer functions.

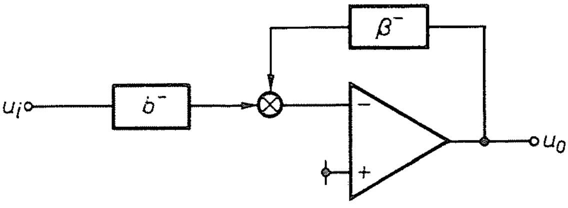

Fig. 2. Infinite gain configuration

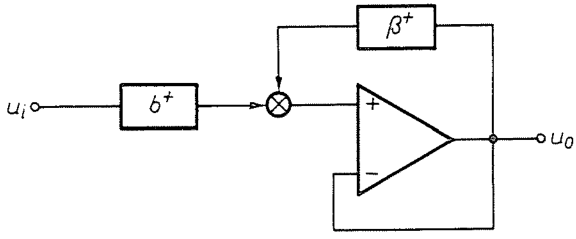

Fig. 3. Controlled source (follower) configuration

Supposing that b^- in Fig. 2 is the same as b^+ in Fig. 3 and at the same time β^- in Fig. 2 equiles the $1-\beta^+$ difference of the structure in Fig. 3 all of the parameters of the related filters—irrespective of the 180° phase difference—would be the very same. The generation of these pairs of networks of the same component values, sensitivities and other parameters seems to be possible by applying the procedure in [1].

The second order section according to Fig. 2 is to be realized by the multiloop infinite gain feedback structure in Fig. 4. The voltage transfer ratios are the following:

$$b=-b^-=\frac{-Y_1Y_3}{(Y_3+Y_5)(Y_1+Y_2+Y_4)+Y_3Y_5}$$

and

$$\beta=\beta^-=\frac{Y_5(Y_1+Y_2+Y_3+Y_4)+Y_3Y_4}{(Y_3+Y_5)(Y_1+Y_2+Y_4)+Y_3Y_5}.$$

The block structure in Fig. 3 can be realized by applying a so-called finite gain voltage controlled voltage source, in our extreme case a voltage follower.

The corresponding parameters (Fig. 5) are the following:

$$b = b^{+} = \frac{Y_1 Y_3}{(Y_3 + Y_5)(Y_1 + Y_2 + Y_4) + Y_3 Y_5}$$

and

$$\beta = 1 - \beta^{+} = \frac{Y_5(Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4 + Y_1 Y_3}{(Y_3 + Y_5)(Y_1 + Y_2 + Y_4) + Y_3 Y_5}.$$

It should be noted that for a bandpass filter according to Fig. 4 Y_2 is not necessary and a lowpass or highpass filter according to Fig. 5 can be realized even without the component Y_4.

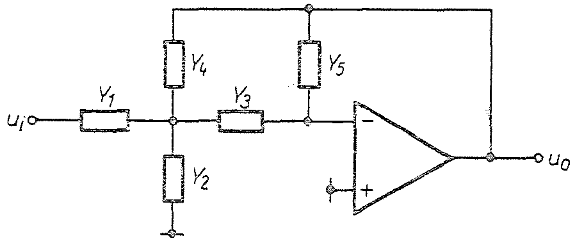

Fig. 4. Infinite gain multiloop filter section

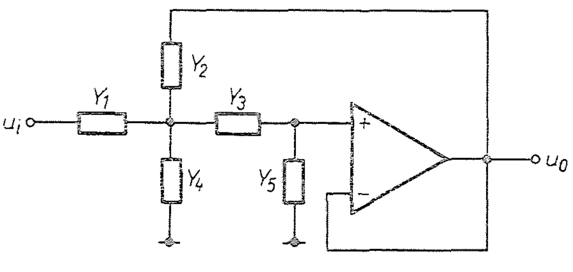

Fig. 5. Controlled source (follower) filter section

The numerator of the β-function in the above-mentioned cases differs in the term $Y_1 Y_3$, consequently there is no possibility to find complementary networks having the same b- and β-functions neither with the same nor with different passive components. The networks in Figs 4 and 5 are b-invariant networks undefined up to now.

According to the theory of complementary transformation the β-function remains unchanged if the grounded terminals of passive components are connected to the output of the op-amp, while the terminals connected to the output are grounded, the connections to the inverting and non-inverting inputs are interchanged and finally instead of grounding the non-inverting input a follower feedback is applied or vice versa. Based on these and the results obtained in Section 3 the requirements can be generalized to the Q-enhancement cases. The b-function remaines unchanged (irrespective of the sign) if and only if the generator is connected to the same passive component terminal in both cases. As the generator terminal is grounded for the calculation of the β-function, the requirements for equal b and β values lead to contradiction and cannot be fulfilled simultaneously.

Taking the above-mentioned facts into account the necessary and sufficient condition of the realizability of networks with similar transfer functions and β-complementary structures can be stated. The original network has to have a topology in which the generator and the ground are connected to a common node *via* impedances differing only by a constant factor and/or similar condition should be fulfilled referring to the generator and output terminals. Naturally in such cases the component values for a given b/β function and the b- and β-functions are different.

A well-known bandpass multi-loop feedback infinite gain filter realization is shown in Fig. 6. The network in Fig. 7 is less generally used. Its topology was constructed using the above-derived results. The component values as well as the network parameters are obviously quite different.

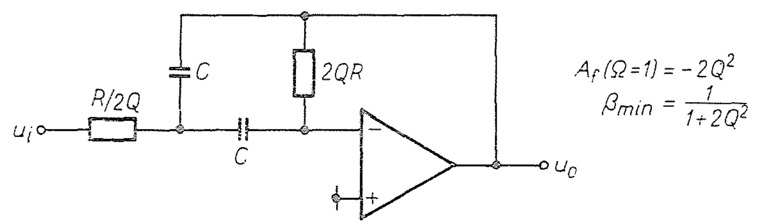

Fig. 6. Example for multiloop network

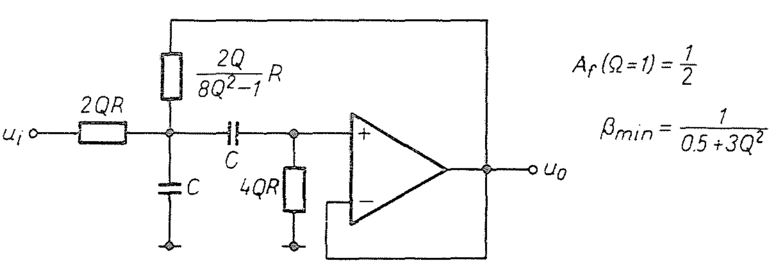

Fig. 7. Filter of complementary structure to Fig. 6

For the sake of simplicity the normalized complex frequency variable $S=sRC$ will be used (R and C are the unit values). The transfer subfunctions b and β of second order sections have the following form:

$$b=\frac{\mp N(S)}{S^2+\frac{1}{q}S+1}$$

$$\beta=\frac{S^2+\frac{1}{Q}S+1}{S^2+\frac{1}{q}S+1}.$$

The condition that the S^2 and constant terms in the denominator polynominal have unity coefficients is advantageous and is here supposed to have been fulfilled. If the components in structures of Figs 4 and 5 are either purely resistive of purely capacitive in character, $N(S)$ is constant, a pure linear or a pure second-order function of S, that is, only lowpass, bandpass or highpass filters can directly result. Fundamental inequalities are: $q \leqq Q$ and $q \leqq 0.5$; in practical cases q is of the $1/2Q$ order.

3. Additional frequency-independent feedforward and feedback

Three conductances are added to the network in Fig. 4 (Fig. 8). As a result the b- and β-functions are changed. The deviations are:

$$\Delta b=b^+=\frac{G_6}{G_6+G_7+G_8}$$

and

$$\Delta\beta=-\Delta\beta^+=\frac{-G_7}{G_6+G_7+G_8}$$

giving

$$b=\frac{-N(S)+b^+\left(S^2+\frac{1}{q}S+1\right)}{S^2+\frac{1}{q}S+1}$$

and

$$\beta = \frac{S^2 + \dfrac{\dfrac{1}{Q} - \dfrac{\beta^+}{q}}{1-\beta^+} S + 1}{S^2 + \dfrac{1}{q} S + 1} (1 - \Delta\beta^+).$$

Likewise from Figs 9 and 5:

$$\Delta b = -\Delta b^- = \frac{-G_7}{G_6 + G_7 + G_8}$$

and

$$\Delta\beta = \Delta\beta^- = -1 + \frac{G_6}{G_6 + G_7 + G_8} = \frac{-(G_7 + G_8)}{G_6 + G_7 + G_8}.$$

The character of the changes is similar in both cases.

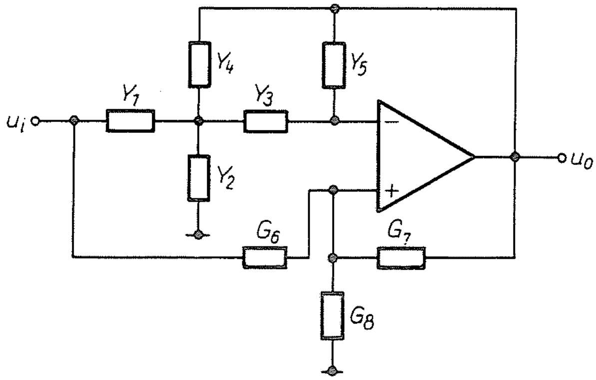

Fig. 8. Additional feedforward and feedback at an infinite gain filter

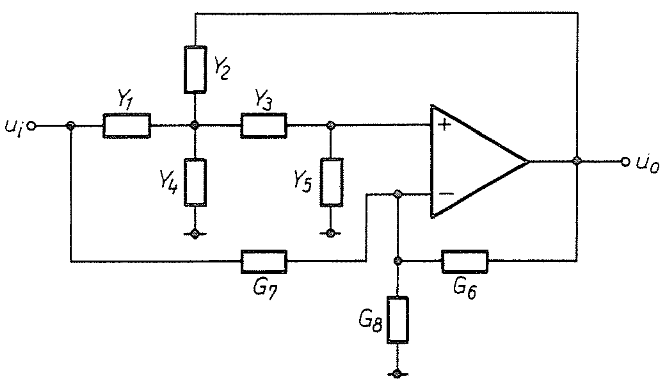

Fig. 9. Additional feedforward and feedback at a controlled source filter

Additional feedforward results in a change in the numerator of the filter transfer function, additional feedback has a consequence of Q-enhancement. Taking the difference of two independent expressions results in higher relative sensitivities, but the application of Q-enhancement increases the loop gain minimum related to the extreme structures, as in [5]. The main conclusions are the following:

3.1. With the help of additional feedforward the numerator is changed. The realization of the mathematical possibilities is limited in some cases by resulting high sensitivity values.

3.2. Additional feedback means Q-enhancement. Supposing the same β-functions and choosing the values of G_8 to be zero and the ratio G_7/G_6 to be equal in both structures (Figs 8 and 9) the new β-functions are also the same (Rauch and Deliyannis filters).

3.3. In [4] analytical expressions are derived for the nonlinear properties of feedback systems with energy storage elements using the Volterra series techniques. One of the conclusions should be modified, though. The "positive feedback" and "negative feedback" structures containing the same active devices and linear passive elements resulting in the same b- and β-functions must show the very same nonlinear character.

4. Noise considerations

A lot of publications deal with the noise contribution of passive filter components, such as [2], [6], [7]. The analysis is much simplified in [8]. The op-amp noise contribution can be calculated according to [2] in the framework of the block representation. The only further parameter is the impedance function Z defined as an input impedance of the passive network between the inverting and non-inverting input of the op-amp supposing the generator and output terminals to be grounded (Fig. 10). The complementary structures in Figs 4 and 5 have the same expression for Z:

$$Z=\frac{Y_1+Y_2+Y_3+Y_4}{(Y_3+Y_5)(Y_1+Y_2+Y_4)+Y_3Y_5}.$$

In the case of Q-enhancement a resulting equivalent resistive term has to be added.

The voltage transfer ratio for the equivalent reduced input noise voltage source (u_n in Fig. 10):

$$A_t=\frac{1}{\beta}\frac{L}{1+L}.$$

Supposing a function $\beta(S)$, the same as in Section 2, and $e\approx 1$ substituting $S=j\Omega$ results in:

$$A_{t\,\max}=\frac{1}{\beta_{\min}}=|A_t(\Omega=1)|=\frac{Q}{q}\,;$$

$$\lim_{\Omega\ll 1}|A_t|=1\,;$$

$$\lim_{\Omega\gg 1}|A_t|=1\,.$$

The shape of the curves without Q-enhancement are shown in Fig. 11.

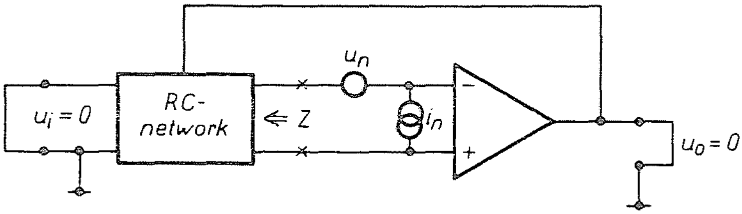

Fig. 10. Noise equivalent circuit

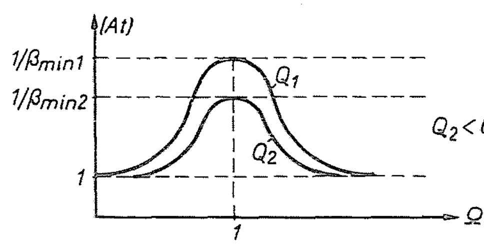

Fig. 11. Noise voltage gain *vs.* frequency

In case of Q-enhancement the value of $\beta_{\min}$ for a given Q-value becomes greater, if the extent of Q-enhancement is increased. This results in lower noise contribution levels. The transfer impendance for the equivalent reduced input noise current source (i_n in Fig. 10) is:

$$Z_t=\frac{1}{\beta}\,\frac{L}{1+L}\,Z\,.$$

For instance circuit in Fig. 6 (structure in Fig. 4) with $e\approx 1$:

$$Z_t=\frac{Y_1+Y_3+Y_4}{(Y_3+Y_5)\,(Y_1+Y_4)+Y_3\,Y_5}=\frac{(2Q+2S)R}{S^2+\dfrac{1}{Q}\,S+1}\,.$$

The results:

$$|Z_t(\Omega=1)|=2QR\ ;$$

$$\lim_{\Omega\ll 1}|Z_t|=2QR\ ;$$

$$\lim_{\Omega\gg 1}|Z_t|=0$$

5. Generation of new elliptic filters

The results easily lead to the patented structure of [9]; the author in the proposed elliptic sections gets use of a b-modifying correction, too.

In the well-known structure of Fig. 12, $R_1=R$ results in a band reject filter while $R_1=R/2$ in an allpass filter.

The heuristic power of the described synthesis concept is illustrated by two new elliptic filter sections applying two op-amps each. The zero frequencies can be tuned by one component without effecting the other filter parameters.

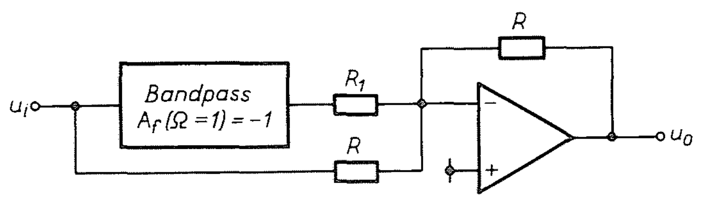

Fig. 12. Feedforward by summer

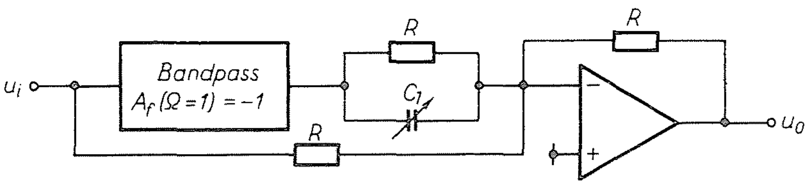

Fig. 13. New lowpass elliptic filter configuration

The configuration suggested in Fig. 13 with its frequency-dependent weighting of the output signal of the first filter block and the additional frequency-independent feedforward signal path has the following transfer function:

$$\frac{U_0(S)}{U_i(S)}=\frac{S^2\left(1-\frac{1}{Q}\frac{C_1}{C}\right)+1}{S^2+\frac{1}{Q}S+1}.$$

The zero frequency of such a lowpass elliptic section is determined by the value of C_1.

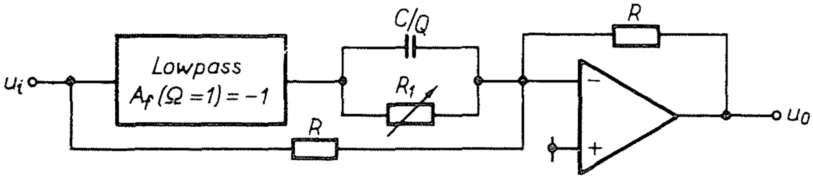

Fig. 14. New highpass elliptic filter configuration

The other structure is very similar to the first one and is shown in Fig. 14. Here:

$$\frac{U_0(S)}{U_i(S)}=\frac{S^2\dfrac{R_1}{R_1-R}+1}{S^2+\dfrac{1}{Q}S+1}.$$

The zero frequency of the higphass elliptic section can be tuned by varying the value of R_1.

Summary

Single op-amp active *RC* filter sections are investigated on the basis of feedforward and feedback subfunctions referring to the inverting and non-inverting input. Information concerning the possibility of two basic configurations with equal parameters, including the *Q*-enhancement cases, are gained. The conditions and possibilities of a unified complementary transformation, the effect of applying additional feedforward or feedback and the equivalence of nonlinear parameters of different filters as well as noise parameters are investigated and new elliptic filter configurations are proposed.

References

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