PRACTICAL APPLICATION OF THE PROBABILITY THEORY OF LIGHTNING STROKES

By

T. HORVÁTH

Institute of Heavy Current Engineering,
Department of High Voltage Engineering
Technical University, Budapest

Received November 21, 1979

One of the usual methods of determining the efficiency of lightning protection systems is based on observing the places of the strokes of lightning. Such observations, however, involve cases very different from each other and are not frequent enought to allow general conclusions. The other method utilises the similarity between lightning and the long spark produced in laboratory and applies impulse voltage model experiments to determine the point of stroke and the shielded zone protected from strokes. However, there is a considerable difference between the physical natures of the two kinds of discharge, therefore it is a general opinion that a discharge produced in a laboratory cannot substitute for the lightning. The concept of shielded zone itself is contradictory, as strokes have been observed which demonstrate that no zone completely protected from lightnings exists at all.

Bases of the probability theory

A result of the investigation of lightning protection at the Dept. of High-Voltage Engineering of the Institute for Heavy-Current Engineering is the probability theory based on the process of the emergence of lightning strokes. The theory starts from the observation that the leader of a lightning approaches the earth independently of the effect of earthy objects on a randomly formed path and aims at the point of stroke only after reaching the so-called orientation point. The orientation point is determined by the fact that, upon the effect of the field intensity produced near the earth surface by the charges of the leader channel, an upward leader starts from the earthy objects and leads the further path of the lightning to its own starting point. As there is a direct connection between the charge of the leader channel and the peak value of the current wave of the lightning, the orientation distance is dependent also on the lightning current and depends in addition, mainly on the height of the earthy object. The frequency distribution of the lightning current is known from numerous measurements of which also the frequency distribution of the orientation distances can be deduced. Its function of probability density can be written as

$$\frac{\mathrm{d}w}{\mathrm{d}r} = \frac{k}{\sqrt{2\pi}}\,\frac{1}{r}\,e^{-\frac{k^2}{2}\left(\ln\frac{r}{r_m}\right)^2} \tag{1}$$

where k is a constant characteristic of the relationship between the observed frequency of lightning currents and the orientation distance, r is the orientation distance and r_m its median value dependent mainly on the height of the earthy object. As can be seen from Fig. 1, the distribution of orientation distances is asymmetrical, both the low and the high values occur with small frequency and larger orientation distances belong to higher objects (M).

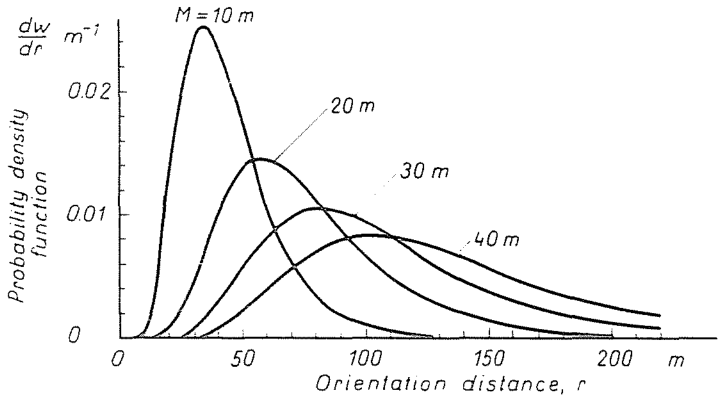

Fig. 1. The probability density function of the orientation distance in the case of several height of structures

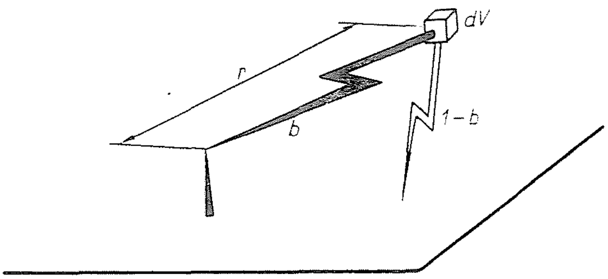

Fig. 2. The striking ratio b and the orientation distance r at a volume element dV

The section of the lightning path from the orientation point to the point of stroke is more markedly defined. This section can be mapped also in a laboratory by impulse voltage flashes in which two leader channels grow toward each other in a manner similar to the downward and upward leaders. This is not any more characteristic of a longer section of the lightning path. From the same orientation point the lightning may strike several points if upward leaders can start from several points, therefore the striking ratio b has to be calculated which will determine the proportion of the strokes from a given orientation point striking the object under consideration or its part to be protected (Fig. 2).

If the probability density function of the orientation distance and striking ratio b in the points of a space over an earthy object are known, the frequency of strokes can be expressed in the general form

$$v = B \int_V \frac{\mathrm{d}w}{\mathrm{d}r} b \cdot \mathrm{d}V \qquad \frac{\text{strokes}}{\text{year}} \tag{2}$$

where B is the specific lightning density characteristic of area, expressed as lightnings/km^2 per year.

As a first approximation, the striking ratio b can be defined also in the way that $b = 1$ in all points lying nearer to the structure considered than to anything else. This is modified by the polarity of the lightning and, accordingly, the attractive zone of the structure is somewhat smaller for positive lightning and larger for negative lightning. Outside the attractive zone the value $b = 0$ is assumed. This method, too, can be applied to towers and horizontal wires, and as there are sufficient observation data at disposal, the theoretical results can be compared with them. The comparisons performed from several points of view gave fair agreement, supporting the validity and applicability of the theory.

Determining the stroke frequency by a model experiment

As it was demonstrated in the foregoing, the section of the lightning path between the orientation point and the striking point can be replaced by laboratory discharges, which offer the possibility of determining the striking ratio b by a model experiment in any point of the space over the structure to be protected. In the experiment, an impulse voltage sufficient to flash over the distance measured from the grounded model is applied to a rod-shaped electrode pointing downward. The rod represents the downward leader and its lower end is the orientation point. From a higher number (50...100) of discharges it is determined in which proportion the strokes hit the object parts to be protected. In the nodes of a regular (e.g. rectangular or axially symmetrical) coordinate system one can determine the value of b and scan the whole attractive zone of the structure to be protected. Of course, the reductions following from the symmetry can be utilized.

In correspondence with the coordinate system, to each point examined belongs a volume element in which the value of b can be regarded as approximately constant. Similarly, to each point a density function according to formula (1) can be assigned with the use of which the striking probability pertaining to the structure to be protected can be approximated, e.g. by orthogonal coordinates

$$v = B\sum_x \sum_y \sum_z \frac{\mathrm{d}w}{\mathrm{d}r} b \Delta x \Delta y \Delta z \qquad \frac{\text{strokes}}{\text{year}} . \tag{3}$$

This calculation is carried out practically on a computer with the data of the model experiment used as input data.

The method based on model experiments is suitable for examining the lightning protection of any structure, but, considering the time and cost requirements, it is worth being used only in cases of special or very valuable objects. In the past years, the Department has examined by this method the lightning protection of large hydrocarbon gas holders, the Theater in Győr (Hungary), the Sports Hall under construction in Budapest, as well as the standardized 400 kV substation and the 750 kV substation of Albertirsa in Hungary.

The results are better demonstrated if the reciprocal value of the frequency of strokes calculated with formula (3), i.e., the average time elapsing between the strokes is considered. The average period of time free of strokes was found to be 300 years for the 400 kV substation and about 1300 years for the 750 kV substation with reducing the number of the lightning rods originally planned to 60 per cent. Thus, the examination yielded also a direct economic result. Based on the stroke frequency, also the expected lightning damage could be estimated, whereby a possibility was offered to find the economic optimum in the design of lightning installations.

Designing the arrangement of air termination

The methods discussed in the foregoing permit only to determine the stroke frequency in the case of a given system. In the design work, on the other hand, the adequately arranged air termination system has to be constructed for assumed safety. With the use of the model experiment, e.g., this problem could be solved only by tiresome iteration. Therefore, a simplified procedure using slight neglections has been elaborated for constructing the arrangement of the air termination system.

In compliance with the calculations, when the rods and wires of the protective system are arranged in the way that, approaching from above, a sphere with radius R just touches the object to be protected when the sphere lies on the air terminals according to Fig. 3, the stroke frequency decreases as a function of radius R, height M of the structure and height m of the air terminals, as compared with the stroke frequency without lightning protection. If this striking ratio δ is assumed to be given in correspondence with the safety requirements, the radius of the sphere required to the construction can be calculated with the following formulas:

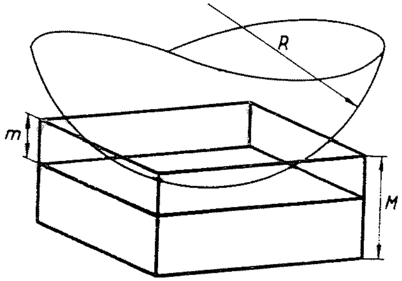

Fig. 3. Construction of the arrangement of air terminals with a sphere attaching the structure to be protected and bearing on the air terminals

$$R=\frac{R_0}{1+\mu-\frac{1}{2}\sqrt{\mu(\mu+1)}} \tag{4}$$

$$R_0=(12.8\delta+1.15)M+81.1\delta+7.33m \tag{5}$$

$$\mu=\frac{m}{R_0}. \tag{6}$$

This construction method has the advantage that it can be used for any object and any kind of air terminal (e.g. rods and wires together).

The probability theory offers also the possibility of estimating the frequency of strokes without lightning protection, with the use of following formulas:

$$v_0=B(A+C_1\cdot p\cdot M+C_2\cdot M^2)\cdot 10^{-6} \quad \frac{\text{strokes}}{\text{year}} \tag{7}$$

$$C_1=1.71+2.15\left(\frac{M}{10}\right)^{-0.24} \tag{8}$$

$$C_2=42\left(\frac{M}{10}\right)^{-0.2}+8.6\left(\frac{M}{10}\right)^{-0.7} \tag{9}$$

where M is the height of the structure to be protected, A is its area and p its perimeter. With this value the permissible striking ratio will be found from the formula:

$$\delta = \frac{v}{v_0} \tag{10}$$

where v is the permissible stroke frequency in spite of the lightning protection, which can be expressed by the reciprocal of a prescribed average period of time without strokes in the parts to be protected. Substituting this into formula (5), one can calculate that radius R of a sphere with which the air termination system can be constructed as based on the safety dependent on the sensitivity and the importance of the structure.

As the neglections required for the elaboration of the method may involve considerable errore, the results of the construction were compared with observations made on overhead lines, according to which the conductors of about 20 m high overhead lines with shielding angle of about 30° are struck once about every 10 years, related to a line length of 100 km. The Table indicates the shielding angles calculated with the use of the sphere construction, as functions of the line height and the desired period free of strokes. The ratio of the height of the conductor to that of the ground wire was in each case 3 to 4.

Table

Height of the ground wire	Shielding angle for stroke-free periods of		
	5 years	10 years	20 years
10 m	38.5°	36.6°	35.5°
15 m	33.2°	31.6°	30.8°
20 m	29.9°	28.6°	27.9°
25 m	27.7°	26.5°	25.9°
30 m	26.1°	25.0°	24.5°
35 m	24.8°	23.9°	23.4°
40 m	23.9°	23.0°	22.6°

The Table reflects the practical experience that the required shielding angle decreases with the increase of height and, instead of the shielding angle of about 30° suitable for 20. . . 25 m height, a shielding angle of about 20° must be employed for higher overhead lines. In an indirect way, this examination gives the basis of using the sphere method for constructing the air terminals of geometrically complicated arrangement in the case of buildings and outdoor substations. The Hungarian Standard of Lightning Protection introduced the spheric construction for explosive objects as early as 1962, and its general introduction is going on at present. Of course, the Standard states simpler rules of determining the radius of the sphere. At international conferences and in the literature it was first stated in 1977 that this method is the most adaptable one and in best agreement with our knowledge about the mechanism of lightning stroke. However, nowhere has it come to determining the radius of the sphere and to taking account the required safety.

Summary

The frequency of lightning strokes in a structure can be estimated on the basis of a theory of the striking process as well as the ratio between the number of strokes with lightning protection to them without it. The air terminals have to be arranged in such a way that the surface to be protected must not be attached by a shpere approaching downwards without bearing on the air terminals. The radius of the sphere to be applied has been determined as a function of the height of air terminals, the dimensions of the structure, the lightning earth-flash density and an assumed risk, which means the average frequency. of strokes (years/stroke) in the structure in spite of lightning protection. The results have been verified by their comparison with the experience got at high-voltage transmission lines or at such typical structures on which there are enough observations for evaluation.

References

1. Horváth T.: Eine neue Methode zur Ermittlung der Wahrscheinlichkeit von Blitzeinschlägen. Elektrie Bd. 17 (1963) pp. 216—220.
2. Horváth T.: Wirtschaftlicher Blitzschutz von Bauwerken auf Grund der Wahrscheinlichkeitstheorie. Periodica Polytechnica Electr. Eng. Vol. 11 (1967) pp. 253—259.
3. Horváth T.: Gleichwertige Fläche und relative Einschlaggefahr als charakteristische Ausdrücke des Schutzeffektes von Blitzableitern. 11. Intern. Blitzschutz Konf. München, 1971. Themengruppe 2.
4. Horváth I.—Berta I.: Nagyfeszültségü alállomások villámvédelme. Electrotechnika, Vol. 70 (1977) pp. 437—440.
5. Wiesinger, J.—Hasse, P.: Handbuch für Blitzschutz und Erdung. VDE Verlag, Berlin, 1977.
6. Horváth T.: Hinweise zur praktischen Schutzwirkung von Fangvorrichtungen. ETZ-A Vol. 99 (1978) pp. 661—663.
7. Golde, R. H.: Lightning protection. Edward Arnold, London, 1973.

Prof. Dr. Tibor Horváth, H-1521 Budapest