

## THE ROLE OF THE CHARACTERISTIC PARAMETERS IN THE WARMING OF OIL-TRANSFORMERS

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The conditions of warming are decisive for the service life of oil-transformers and for their unit power.

The calculation of warming requires the knowledge of different (heat transfer, mass flow and other) parameters and data. So, e.g., important parameters are inside the winding: the heat transfer coefficient and the flow resistances; for the heat exchanger: the overall heat transmission coefficient; the characteristic curve of the pump. Important data are: the temperature of the environment, heat transfer surface of the heat exchanger, material characteristics, etc.

The accuracy of the warming calculation is greatly affected, in addition to the errors of modelling and numerical operations, by the reliability degree of the functions of the above parameters and factors.

Researchers usually make efforts to determine the transfer coefficients experimentally. The complicated and irregular geometry, the instability effects arising in the flow of the oil and in some cases also the interpretation of the individual transport coefficients raise a number of difficulties in the concepts and in the measurement technique.

Difficulties and uncertainties are involved also in the validation of the measurement results of the expensive and longlasting experiments to other cases.

Thus, it may be of great practical advantage to examine the question of what degree of accuracy is required in respect of the parameters and how carefully the dimensions and operational data have to be selected.

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### Sensitivity matrix

The greater the influence of some parameter on the conditions of warming, the higher are the accuracy requirements.

The oil-transformer is a multiparameter, nonlinear electro-thermo-hydrodynamical system. In the course of calculating the warming, the transformer is replaced by its simultaneous heat and mass flow network model [1...7].

In the examination of the transformer model, all the components for which the accuracy requirements are examined will be treated arbitrarily as "input" parameters, while all those directly influencing or determining the degree of warming will be regarded as "output" parameters.

The system reacts to the different input parameters ( $B$ ) to differing extent. There are more or less complicated action paths between the input and output ( $K$ ) parameters. The stronger the connection on an action path, the greater is the change caused by the same input parameter in the value of the output parameter belonging to the given action path. The strength of the action paths can be characterized by the quotients of absolute and relative changes, and these ratios are termed sensitivity factors (or transfer coefficients [2, 3]). The absolute sensitivity factor is interpreted for the changes taking place in the vicinity of working point  $M$ :

$$A_{ij} = \left( \frac{\partial K_j}{\partial B_i} \right)_M, \quad (1)$$

$$\begin{aligned} i &= 1, 2, \dots, x && \text{input parameters,} \\ j &= 1, 2, \dots, y && \text{output variables.} \end{aligned}$$

The relative sensitivity factor is

$$R_{ij} = \left( \frac{\partial K_j}{\partial B_i} \right)_M \left( \frac{B_i}{K_j} \right)_M. \quad (2)$$

The sensitivity factors interpretable in the system are contained in sensitivity matrices  $\mathbf{A}$  and  $\mathbf{R}$ :

$$\begin{bmatrix} A_{11} & \dots & A_{1j} & \dots & A_{1y} \\ \vdots & & \vdots & & \vdots \\ A_{i1} & \dots & A_{ij} & \dots & A_{iy} \\ \vdots & & \vdots & & \vdots \\ A_{x1} & \dots & A_{xj} & \dots & A_{xy} \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} R_{11} & \dots & R_{1j} & \dots & R_{1y} \\ \vdots & & \vdots & & \vdots \\ R_{i1} & \dots & R_{ij} & \dots & R_{iy} \\ \vdots & & \vdots & & \vdots \\ R_{x1} & \dots & R_{xj} & \dots & R_{xy} \end{bmatrix} \quad (4)$$

### Determination of the sensitivity matrix

The number of the rows of the sensitivity matrix is identical with the number of the input parameters, and the number of its columns with that of the output parameters examined. The elements of the matrix can be determined in the following way:

1. The physical-mathematical model of the warming of the transformer is constructed;

2. on the basis of a given parameter system the describing equation system of the model is solved and the working point state space determined;

3. in relationships (1) and (2) the differential quotients are approximated by difference quotients:

$$A_{ij} \cong \left( \frac{\Delta K_j}{\Delta B_i} \right)_M, \quad (5)$$

$$R_{ij} \cong \left( \frac{\Delta K_j}{\Delta B_i} \right)_M \cdot \frac{B_i}{K_j}; \quad (6)$$

4. to produce the elements of each row of the matrix, the input parameters will be modified to an adequately small extent ( $\Delta B_i$ ), then the modified states determined by repeated solution of the equation system. After this, using the data of the original (index "e") and of the modified (index "m") state spaces, the values  $\Delta K_j$  of (5) and (6) will be determined:

$$\Delta K_j = K_{j(e)} - K_{j(m)}. \quad (7)$$

Thus, to obtain all the elements of the matrix, the describing equation system has to be solved  $(x + 1)$  times.

### Describing equation system of the transformer

The describing equation system will be discussed in its general form (for more detail see [6...8]).

*Mass flow network*

*Nodal equation system*

$$\left( \sum_{b=1}^n \Phi_{mb} \right)_c = 0, \quad (8)$$

$b = 1, \dots, n$  number of the branches,  
 $c = 1, \dots, k$  number of the nodes.

*Loop equation system*

$$\left( \sum_d \sum_{z_d} (\Delta p_{z_d} + \Delta p_{R_{z_d}}) \right)_l = 0, \quad (9)$$

$l = 1, 2, \dots, n-1$  number of the loops,  
 $d =$  number of the branches in the loop,  
 $z_d =$  number of the mass flow network elements in the branch  $d$ .

*Branch equations*

$$\Delta p_{bR} = R_b(\Phi_m, T)\Phi_m. \quad (10)$$

*Heat flow network equation system*

$$\begin{bmatrix} \mathbf{K}_{1x} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{xy} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{yz} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{T}_{1x} \\ \mathbf{T}_{xy} \\ \mathbf{T}_{yz} \end{bmatrix} = \begin{bmatrix} \Phi_{1x} \\ \Phi_{xy} \\ \Phi_{yz} \end{bmatrix} + \begin{bmatrix} \mathbf{V}_{1x} \\ \mathbf{V}_{xy} \\ \mathbf{V}_{yz} \end{bmatrix} \cdot \mathbf{T}_W, \quad (11)$$

where  $\mathbf{K}_{1x}$ ,  $\mathbf{K}_{xy}$  and  $\mathbf{K}_{yz}$  are corrected heat conductivity matrices (for parts of divided winding discs) which are quadratic matrices of  $x$ -th,  $(y-x)$ -th and  $(z-y)$ -th order:  $\mathbf{T}_{1x}$ ,  $\mathbf{T}_{xy}$ ,  $\mathbf{T}_{yz}$  are column matrices of temperature:  $\Phi_{1x}$ ,  $\Phi_{xy}$ ,  $\Phi_{yz}$  are column matrices of the source heat-flow and  $\mathbf{V}_{1x}$ ,  $\mathbf{V}_{xy}$ ,  $\mathbf{V}_{yz}$  are column matrices of the coupling conductivity.

### Equation system of the dual-function elements

$$\Delta p_{bz} = F_p(L, \rho, \Delta T)_{bz}, \quad (12)$$

$$\Delta T_{wbz} = F_T(\Phi_h, \Phi_m, c)_{bz}, \quad (13)$$

where  $\rho$ : mass density,  $c$ : specific heat;  $L$ : characteristic length.

### Equations of the variators

State equation of the *mixer*:

$$\left( \sum_b \Phi_{mb} c_b T_b \right)_K - \Phi_{hW} = 0, \quad (14)$$

where  $\Phi_{hW}$  is the loss heat flow of the mixer toward the environment.

State equation of the heat exchanger:

$$\Phi_{h_z(b=1)} = k(\Phi_{m1}, T_{z1})A\Delta T_{z1}(L), \quad (15)$$

where  $k$ : coefficient of the overall heat transmission,  
 $A$ : heat transferring surface.

### Results and discussion

The solution of the simultaneous equation system was evolved in detail in another papers [6. . . 8]. In possession of the repeated solutions according to equation (5) . . . (7) the sensitivity coefficients and the elements of the sensitivity matrices for a 75 MVA ONAN transformer were generated.

Considering that the system is not linear, the sensitivity factors can be regarded as valid only in the vicinity of the working point belonging to the input data system. With a differing input data system, the working point may deviate considerably.

Table 1 presents the relative sensitivity matrix, in a tabulated form. The headings of the rows and columns indicate the symbols of the parameters examined and their values in the operational working point ( $M$ ).

An evaluation of the results furnishes some suprising information.

In winding 1 (of low voltage) the influence  $\alpha_1$  of the heat transfer coefficient developing on the surfaces of the divided winding disc weak, as can be seen:  $R_{(Thsp1-\alpha_1)} = -11.7\%$ . For a change of 100% of  $\alpha_1$  the average temperature of winding 1 and the hot-spot temperature are modified only about by 12%. Thus an error made in the value of  $\alpha_1$  has no significant effect. Whether the value of  $\alpha_1$  is 60 or 80 W/m<sup>2</sup> °C, means a deviation of about 2 °C in the temperature of the hot-spot.

The overall heat transmission coefficient  $k_{rad}$  the radiator has a greater role. Its effect on the hot-spot temperature manifest itself through the action path: thermosyphon actuating pressure—oil flow—oil warming. Its influence is great ( $(R_{Thsp1-k_{rad}}) = -38.9\%$ ), thus, it is an important parameter and an error in its value has significant consequences. If, e.g., 7.5 W/m<sup>2</sup> °C is assumed instead of  $k = 5$  W/m<sup>2</sup> °C, the error in the hot-spot temperature will amount to 10 °C.

The influence of the ambient temperature ( $t_w$ ) is other than supposed practically. According to practical supposition a rise of  $t_w$  by about 1 °C raises all the temperatures by about 1 °C.

As can be seen from the Table 1, this is not so. It has varying effects on the various temperatures. For the hot-spot temperature, e.g., it is 35 to 38 per cent: ( $R_{Thp1-t_w} = 37.73\%$  and  $R_{Thsp2-t_w} = 35.74\%$ ). For with the rise of  $t_w$  the overall

Table 1

Relative sensitivity matrix of 75 MVA ONAN cooled transformer ( $R_{ij}$  values in ‰)

$K_j$		M: 21.673 kg/s	9.549 kg/s	6.746 kg/s	5.378 kg/s	57.40 °C	48.68 °C	8.72 °C	62.43 °C	68.38 °C	63.90 °C	71.06 °C
		$\Sigma\Phi_m$	$\Phi_{mr1}$	$\Phi_{mr2}$	$\Phi_{mr}$	$T_{0, in}$	$T_{0, out}$	$T_0$	$T_{t1}$	$T_{hsp1}$	$T_{t2}$	$T_{hsp2}$
60	$\alpha_1, \text{W/m}^2 \text{ } ^\circ\text{C}$	-0.55	-1.05	-0.30	0.00	-0.35	-0.41	0.00	-10.89	-11.70	-0.31	0.00
5	$k_{rad}, \text{W/m}^2 \text{ } ^\circ\text{C}$	-31.65	-31.84	-32.61	-30.12	-44.60	-58.34	32.11	-44.53	-38.90	-43.19	-36.59
25	$t_w, \text{ } ^\circ\text{C}$	31.65	32.05	32.61	29.75	43.21	56.70	32.11	43.25	37.73	41.94	35.74
2.7	$C_{1r}$	-11.63	-38.54	10.97	7.81	1.05	-0.82	11.47	1.92	4.09	-1.25	-1.97
$5.9 \cdot 10^9$	$C_{rad}$	-6.74	-7.54	-6.52	-4.46	0.70	-0.41	5.96	0.00	0.58	+0.31	+0.56
1.08	$Z_r$	42.17	62.83	40.91	6.69	5.92	14.28	-41.28	7.69	3.51	8.42	5.07
2.44	$L_R$	8.03	28.07	5.93	-24.54	-38.33	-43.55	-9.17	-36.84	-35.10	-34.43	31.52
96	$R'$	-1.20	16.76	-3.85	-31.61	-37.28	-44.37	2.29	-36.52	-34.22	-34.43	-30.68
25	$g_R$	4.34	23.67	2.08	-27.15	-37.98	-43.96	-4.59	-36.84	-34.81	-34.43	-31.24
0.144	$A_{ctr}, \text{m}^2$	24.64	68.70	0.59	-22.69	7.67	13.56	-25.23	6.41	2.05	10.64	9.57
$4.12 \cdot 10^{-3}$	$r'_{t1}, \text{m}$	36.64	109.12	-12.75	-31.24	6.27	13.97	-36.70	4.81	-1.75	12.21	11.82
1.62	$H_{iv}, \text{m}$	-15.96	1.68	0.00	-66.20	11.50	10.68	16.06	8.33	7.60	8.45	7.60

L. JIMRE et al.

 $T = t - t_w$  : over-temperature $C_{1r}, C_{rad}$  : flow resistance coefficients $Z_r$  : coil-radiator level difference $L_R$  : radiator length $l_r$  : number of radiator elements in a group $g_r$  : number of radiator groups $A_{ctr}$  : coil channel cross section $r'_{t1}$  : hydraulic radius $H_{iv}$  : core length

temperature level rises, the viscosity of the oil decreases, and thus the oil flows increase. As a consequence, the oil in the windings warms up to a lesser extent. Therefore, the hot-spot temperature rises to a lesser extent than the  $t_w$ .

It can be observed that the action path of the hydraulic radius of the oil duct pertaining to its own oil flow undergoes a strengthening: ( $R_{\varphi_{t1}-r_{t1}} = 109.12\%$ ), and consequently the  $T_{hotp}$  slightly decreases ( $-1.75\%$ ). The overall oil mass flow rate  $\varphi_m$  increases, owing to the decrease of the flow resistance in the whole winding system ( $36.64\%$ ). The whole surplus of the oil, however, flows through duct 1, and even also some of the oil flows of winding 2 and of the iron core ( $R_{\varphi_{t2}-r_{t1}} = -12.75\%$  and  $R_{\varphi_{v-r_{t1}}-r_{t1}} = -31.24\%$ ).

Owing to this circumstance, the overall and hot-spot temperature of winding 2 increase ( $12.21\%$ ;  $11.82\%$ ). It is interesting at the same time that, since the efficiency of the heat exchanger decreases, ( $R_{T_{ki}-r_{t1}} = 13.97\%$ ) the average temperature of winding 1 slightly increases: ( $R_{T_{t1}-r_{t1}} = 4.8\%$ ), so, the temperature  $T_{hs p1}$  decreases all the same to a small extent: ( $R_{T_{hp1}-r_{t1}} = -1.75\%$ ).

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### Summary

The temperature conditions of the windings of oil transformers are influenced by a number of heat transfer and flow parameters. The endeavours to determine these parameters put the research and development instituts to considerable expenses all over the world. A generalization of the experimental results meets with difficulties because of the complicated phenomena and geometry. If the influence of each parameter on warming is known, it becomes possible to judge which factors require maximum accuracy and which are the ones whose approximate values can be satisfactory. To judge the influence of the parameters in the vicinity of the working point the sensitivity matrix is applied. The elements of the matrix can be determined numerically with the use of a simultaneous heat and mass flow network model.

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