DIGITAL SIMULATION OF THE OPERATION OF THERMOMECHANICAL THERMOSWITCHES

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In engineering practice a great variety of thermomechanical actuating devices are applied. Among them, the thermoswitches used for the thermal protection of electrical machines play an important role.

In designing thermomechanical systems, it is necessary to determine in advance the expected working conditions. In the following a model concept and a physical-mathematical model suitable for this purpose will be presented, the describing equation systems and the methods of solution will be discussed.

Construction of the thermoswitch

The schematic arrangement of the thermoswitch is shown in Fig. 1. Its sensing elements are the phase bimetal strips ((B_R, B_S and B_T) surrounded by heating shields. The heating shields are heated proportionally by the current flowing across the phase coils of the motor to be protected. The phase bimetal

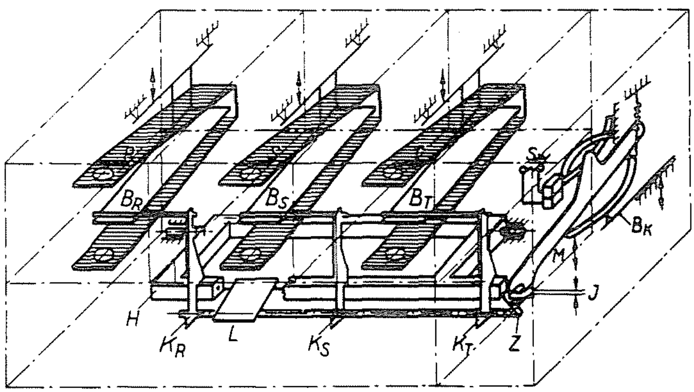

Fig. 1. Schematic arrangement of the thermoswitch

strips warming in correspondence with the thermal coupling bend upwards as shown in the figure. The ends of the strips are hinged to stirrups K_R, K_S, K_T and are supported both by the bridge-like frame H and the axle-like arm. Axle Z has a bearing formed in frame H by means of plate L. The end of axle Z is attached to the forked end of single-arme lever M and, lifting it, actuated switch S_w. The rotation centre of lever M is varied by the position of compensating bimetal B_K to an extent depending of the ambient temperature.

Operation of the thermoswitch

The thermoswitch has two fundamental functions of protection

1) in case of an overload against overheating,

2) in case of a phase asymmetry.

In case of an overload the bimetal strips bending to a greater extent lift the stirrups and axle Z, the deflection of lever M increases and switch S_w gets switched out.

To examine the protective action against phase asymmetry suppose that phases S and T drop out completely and heat will develop only in heating shield R. In this case stirrup K_R will move upward, taking with itself axle Z up to the point where the initial play J cesses. After this, axle Z will rotate around the axle of plate L supported in H, and also frame H will move upward and lever M will be lifted until S_w is switched off.

Model conception

In the thermal switch the temperature field and the position field of the structural elements are in interaction. Therefore, the thermoswitch may be considered as a thermomechanical system. For modelling, a simultaneous model built from thermal and mechanical part models will be constructed (Fig. 2). A heat flow network model will be applied as thermal part model, and a model based on the displacement method as mechanical part model. Coupling model conditions will serve for their coupling [5].

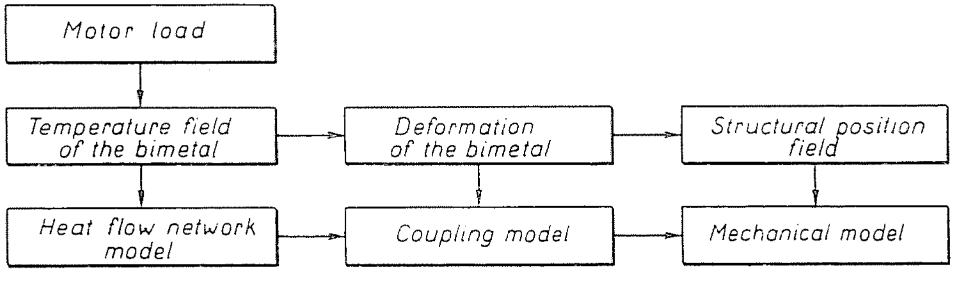

Fig. 2. Construction of the simultaneous model

The heat flow network part model

The thermoswitch was divided into discrete parts according to the geometrical functions and material properties of the structural elements. The division spaces on the elements of interest were decreased. The effects of the particular discrete parts were replaced functionally by concentrated heat capacitances and heat flow sources and coupled by resistences of heat conduction, convection and radiation.

The equation system describing the thermal part model

The heat flow network model of the thermoswitch has 104 degrees of freedom, and it is a nonlinear heat flow network whose transient equation system can be written as

$$\mathbb{C}\dot{\mathbb{T}} + \mathbb{K}(T_i, T_j)\mathbb{T} = \mathbf{\Phi}(T_i, T_a). \tag{1}$$

Initial conditions are: $\tau = 0$, and $\mathbb{T} = \mathbb{T}_a$,
where $i = 1, \ldots, n$
$j = 1, \ldots, n$
$i \neq j$,
the nodes of the network,
$\mathbb{C}$ heat capacitance matrix,
$\mathbb{K}$ conductivity matrix,
$\mathbb{T}$ temperature column vector,
$\mathbf{\Phi}$ column vector of heat sources,
$\mathbb{T}_a$ ambient (reference) temperature.

The heat transfer between the nodes consists of parts corresponding to the basic forms: conduction (v), convection (k) and radiation (s). As an example: the total heat conductivity between the i-th and j-th nodes 5 is

$$K_{ij} = K_{ij}^{(v)} + b_{ij}^{(k)}(T_i - T_j)^p + b_{ij}^{(s)}(T_i^2 + T_j^2)(T_i + T_j). \tag{2}$$

For the total heat transfer between the i-th node and the external ambiency, relationship (2) is valid in the case of $j = a$.

The transient solution of the equation system of the nonlinear heat flow network was performed with the use of the following three-level difference scheme [1, 2, 3]:

$$\mathbb{C}\frac{\mathbb{T}_2 - \mathbb{T}_0}{2\Delta\tau} + \mathbb{K}(\mathbb{T})\frac{\mathbb{T}_2 + \mathbb{T}_1 + \mathbb{T}_0}{3} = \mathbf{\Phi}, \tag{3}$$

where the solutions pertinent to different time intervals are

$$\mathbf{T}_0 = \mathbf{T}(\tau - \Delta\tau), \tag{4a}$$

$$\mathbf{T}_1 = \mathbf{T}(\tau), \tag{4b}$$

$$\mathbf{T}_2 = \mathbf{T}(\tau + \Delta\tau), \tag{4c}$$

and $\Delta\tau$ is the time step.

Introducing the time average of the heat conductivity matrix:

$$\hat{\mathbf{K}} = \frac{1}{2\Delta\tau} \int_{\tau-\Delta\tau}^{\tau+\Delta\tau} \mathbf{K}[\mathbf{T}(\tau)] \, \mathrm{d}\tau . \tag{5}$$

The linearized form of equation (3) is

$$\mathbf{C} \frac{\mathbf{T}_2 - \mathbf{T}_0}{2\Delta\tau} + \hat{\mathbf{K}} \frac{\mathbf{T}_2 + \mathbf{T}_1 + \mathbf{T}_0}{3} = \mathbf{\Phi} . \tag{6}$$

Introduce, within the time step $\Delta\tau$ the variable $\xi = \frac{\tau}{\Delta\tau}$, and assume that, with the chosen time step, the temperature changes linearly beyond the time step:

$$\frac{\mathbf{T}_2 - \mathbf{T}_1}{\Delta\tau} = \frac{\mathbf{T}_1 - \mathbf{T}_0}{\Delta\tau} . \tag{7}$$

The temperature of the i-th node within the time interval is

$$T_i = T_{1i} + \xi (T_{1i} - T_{0i}) . \tag{8}$$

After integration, the following expression will be obtained from equation (5), with taking (2) into account:

$$\hat{K}_{i,j} = \frac{1}{2\Delta\tau} \int_{\tau-\Delta\tau}^{\tau+\Delta\tau} K_{i,j} \mathrm{d}\tau \cong K_{i,j}^{(v)} + b_{ij}^{(k)} (T_{i1} - T_{j1})^{1/4} +$$

$$+ b_{i,j}^{(s)} \left[(T_{i1} + T_{j1})(T_{i1}^2 + T_{j1}^2 + \frac{\Delta T_i^2 + \Delta T_j^2}{3} + \right.$$

$$\left. + \frac{2}{3} (T_{ij} \Delta T_i + T_{j1} \Delta T_j)(\Delta T_i + \Delta T_j) \right] . \tag{9}$$

After substitution equation system 6 was solved with the application of varying time steps.

The mechanical part model

The mechanical part model was constructed with the assumption of the model conditions based on the structural properties (Fig. 3):

1. considering the slow movement of the elements, their kinetic energy is neglected,
2. based on small displacements of the elements, the elastic deformations are described by the first-order theory with sufficient accuracy,
3. the elements are unloaded in the horizontal plane, therefore it is sufficient to consider only the deformations of vertical direction,
4. the calculated spring stiffness taken into account includes also the effect of possible friction,
5. lever M, frame H and stirrups K are modelled as rigid bodies,
6. the point contacts of the latter are modelled by means of "substitute springs" loadable only for pressure.

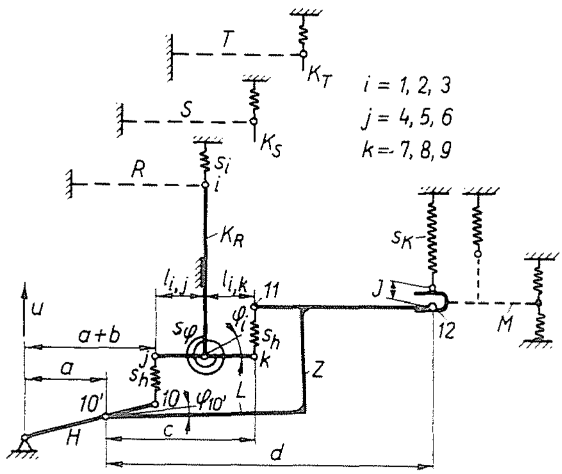

Fig. 3. Scheme of the mechanical part model

The mechanical part model (Fig. 3) built from rigid bodies, linear and nonlinear springs are described with use of the displacement method. When choosing the stiffness s_h of the substituting spring, it is desirable to make the equilibrium equation system well conditioned on the one hand and to use the most realistic description on the other.

The statical equilibrium equation system of the mechanical part model

I. *Transformational equation system* of the displacement u of rigid bodies—with the use of the notations of Fig. 3:

$$\begin{bmatrix} u_k \\ u_j \end{bmatrix} = \begin{bmatrix} 1 & l_{i,k} \\ 1 & -l_{i,j} \end{bmatrix} \begin{bmatrix} u_i \\ \varphi_i \end{bmatrix}, \tag{10}$$

$$u_{10} = \frac{a+b}{a} u_{10'}, \tag{11}$$

$$\begin{bmatrix} u_{10'} \\ u_{11} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & c \end{bmatrix} \begin{bmatrix} u_{10'} \\ \varphi_{10'} \end{bmatrix}. \tag{12}$$

II. *Equilibrium equation system* for the substituting springs and for lever M:

$$s_i u_i = F_i, \tag{13a}$$

$$s_h \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_{10} \end{bmatrix} = \begin{bmatrix} F_j \\ F_{10} \end{bmatrix}, \tag{13b}$$

$$s_h \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_k \\ u_{11} \end{bmatrix} = \begin{bmatrix} F_k \\ F_{11} \end{bmatrix}, \tag{13c}$$

$$\begin{bmatrix} s_k & s_k d \\ s_k d & s_k d^2 \end{bmatrix} \begin{bmatrix} u_{10'} \\ \varphi_{10'} \end{bmatrix} = \begin{bmatrix} F_{10'} \\ M_{10'} \end{bmatrix}, \tag{14}$$

if $J=0$.

III. *Incremental form of the equilibrium equation*:

$$\mathbf{S}(\mathbf{u})\Delta\mathbf{u} = \Delta\mathbf{F}, \tag{15}$$

where: $\mathbf{S}$ is the so-called tangential stiffness matrix with the use of equations (10)—(14),

$\mathbf{u}$ is the column matrix of the position field and

$\Delta\mathbf{u}$ is the change of the position field taking place upon the effect of the load change

$$\Delta\mathbf{F} = \mathbf{F}(\tau+\Delta\tau) - \mathbf{F}(\tau), \tag{16a}$$

$$\Delta\mathbf{u} = \mathbf{u}(\tau+\Delta\tau) - \mathbf{u}(\tau), \tag{16b}$$

i.e.,

$$\mathbf{u} = [\Delta u_i\ \Delta u_j\ \Delta u_k\ \Delta\varphi_i\ \Delta\varphi_{10'}\ \Delta\dot{u}_{10'}]^*. \tag{17}$$

Initial condition:

$$\mathbf{u}(\tau=0)=\mathbf{u}_0=\mathbf{0}\,. \tag{18}$$

The spring stiffness of the bimetal strips is a function of temperature. Because of this, equation system (15) is not linear, and therefore it is written and solved for each time step.

IV. *Equation system describing the changes of connection*

Let the position difference u_1-u_2 of ends 1 and 2 of some substituting spring be interpreted and marked u_{12}:

$$u_{12}=u_1-u_2 \tag{19}$$

If u_{12} changes to positive (i.e., the previously pressured spring becomes pulled), then the spring will be eliminated.

A connection change takes place when

$$sgn\,[u_{12}(\tau)]+sgn\,[u_{12}(\tau+\Delta\tau)]=0\,. \tag{20}$$

A connection arises when $u_{12}(\tau)<0$, and cesses when $u_{12}(\tau)>0$ was before the change.

The equation determining the position field that belongs to time τ^* of the first connection change within the time step will be written in the form

$$\mathbf{u}(\tau^*)=\mathbf{u}(\tau)+\lambda^*\Delta\mathbf{u},\;\;(\tau\leqq\tau^*\leqq\tau+\Delta\tau)\,; \tag{21}$$

where

$$\lambda^*=\min\left\{abs\left[\frac{u_{12}(\tau)}{u_{12}(\tau)-u_{12}(\tau+\Delta\tau)}\right]\right\}. \tag{22}$$

The next connection change within the time step, and all the later changes with taking the state in the actual time τ^* for "initial" state are calculated in the above discussed way.

The state at time τ^* means the position field, the unbalanced external load distribution ($\Delta\mathbf{F}$) and the model corresponding to the change of relation:

$$\Delta\mathbf{F}(\tau^*)=\Delta\mathbf{F}(\tau)\,(1-\lambda^*)\,. \tag{23}$$

Coupling the thermal and mechanical part models

The part models are coupled by the dual function of the bimetal strips: the warming as signal-shaping and the deformation as control action. It is assumed that the movement of the mechanical model modifies the thermal model to a slight extent only. In modelling the operation of the bimetal strips we have determined those values of the external concentrated forces acting on the ends of the bimetal strips that produce free deformations (x) exactly belonging to the temperature change of the bimetals [4, 5]. The general form of this coupling equation is (with $i=1, 2, 3$)

$$F_i=s_i[T(\tau, x)]\cdot u_i[T(\tau, x)]\,. \tag{24}$$

Solution of the equation system of the simultaneous model

Steps of the solution are as follows:

1. selection of the time step,
2. generation of the heat flow network equation,
3. a) solution of the heat flow network equation system,
   b) calculation of the next time step with the known $\mathbf{T}_2$,
4. solution of coupling equation (24) on the basis of $\mathbf{T}_2$,
5. a) generation and solution of the equation system (15) describing the mechanical part model,
   b) if equation (20) is fulfilled, equations (21)...(23) have to be calculated, the model has to be modified and the calculation continued with a).

After this the computation procedure will be continued from step 2.

Results, evaluation

The complete examination was carried out by simulating various operational conditions.

Some model elements have been corrected by parameter identification based on particular measurements.

Fig. 4 and Fig. 5 illustrate the thermal and the free deformation characteristics resp. of the "S" phase bimetall root determined by calculation and by measurement at $I_r=36$ A.

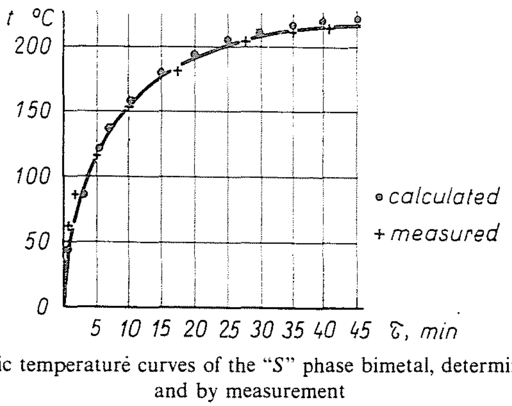

Fig. 4. Characteristic temperature curves of the "S" phase bimetal, determined by calculation and by measurement

Figure 6 shows the warming curve for phase bimetal S in the case of $1.05I_r$. When the steady state is approximated, an overload of $1.2I_r$ and $1.5I_r$ arises. The switching-off calculated took place in 3 and 1 minutes, respectively.

In Fig. 7 the curve of the displacement of lever M is seen.

Based on the results obtained, certain structural elements were modified to achieve their more favourable behaviour.

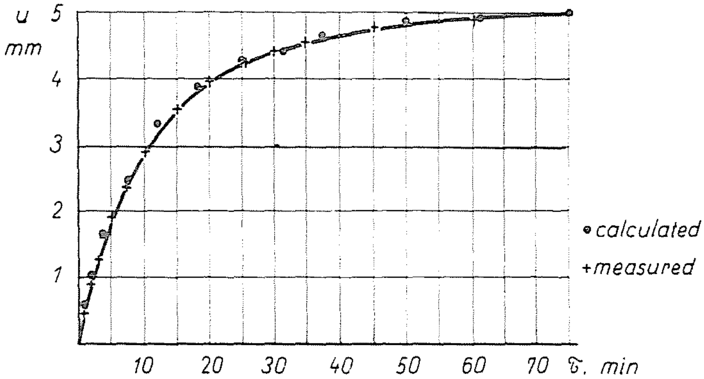

Fig. 5. Curve of the free deformation of the "S" phase bimetal

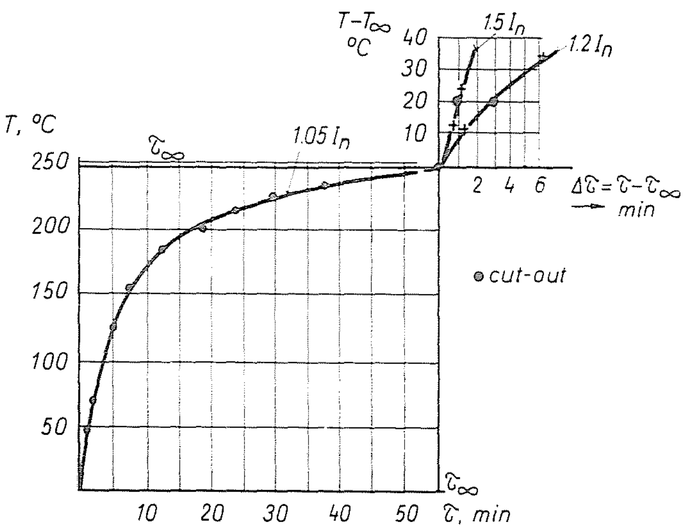

Fig. 6. Warming curve of the root of "S" phase bimetal in the case of $1.05I_r$

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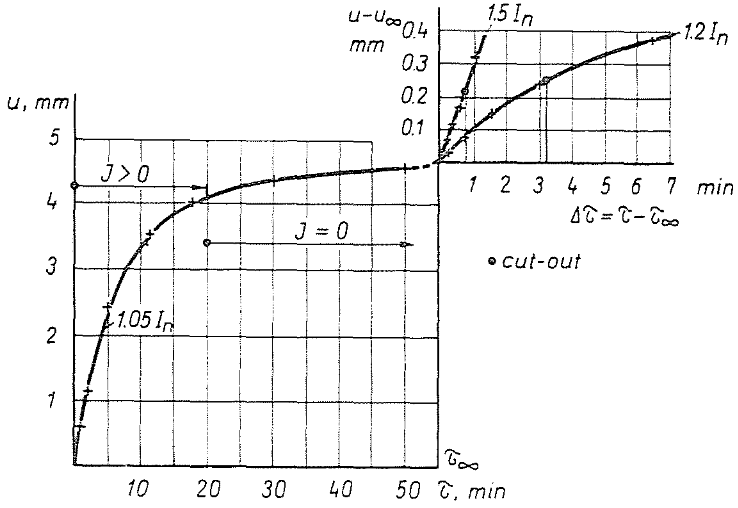

Fig. 7. Displacement curve of the disconnecting lever "M"

Summary

The electrical engineering practice applies a great variety of thermomechanical actuating devices. Among them, the bimetal thermoswitches are widely used for the thermal protection of electrical rotating machines. It is necessary in the stage of designing and developing work to know the expectable behaviour of thermoswitches under different operational conditions.

In a thermoswitch the temperature field and the position field of the structural elements are in interaction with each other. The physical-mathematical model of a thermoswitch consists of simultaneous thermal and mechanical part models. A nonlinear heat flow network model can be applied as thermal part model, and a sectionally linear structural model as mechanical part model.

The operation can be simulated for any combination of the operational parameters by solving the equation system of the complete simultaneous model numerically, applying varying time steps.

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