MODIFIED SYMMETRICAL COMPONENT THEORY AND ITS APPLICATION IN THE THEORY OF ASYMMETRICAL INDUCTION MOTORS

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1. Introduction

Several attempts have been made to develop a generalized theory of asymmetrical induction machines where also the winding axes have been assumed to be asymmetrically displaced [1, 2]. BROWN and JHA [1] have shown that the behaviour of a machine with asymmetrically displaced stator windings cannot be analysed by the conventional symmetrical component theory, except where the winding displacement angle is a submultiple of 2π electrical radians. They suggested a general rotating field theory. It can be shown, however, that by the application of a new general modified symmetrical component theory the behaviour of m/n-phase induction motors can be discussed even for a winding displacement angle other than $2\pi/m$ (m and n being the phase numbers of stator and rotor windings, respectively) or not a submultiple of 2π electrical radians. In case of two-phase induction machines VASKE [3] and VAS [4] used two-phase symmetrical components for the analysis of two-phase winding displaced by angles other then $\pi/2$ radians. However, the transformation introduced — but not derived mathematically or physically — by VASKE does not lead exactly to the well-known right angle two-phase symmetrical components. In this paper an a-priori mathematical deduction will be presented for modified n-phase symmetrical component transformation, also physical derivation will be shown.

It must be pointed out that the general voltage equations derived by using the general rotating field theory are analogue to those derived by the new modified symmetrical component theory, however, the forward field operators [1, 2] are applied on the phase quantities and the resulting symmetrical components will be the new generalized symmetrical components.

In the followings, derivation of the new, modified m-phase symmetrical component transformation will be presented.

2. Derivation of modified m-phase symmetrical component transformation

The analysis of the m-phase unbalanced system is based on the fact that a single angle asymmetrical system of m-phase vector quantities is equivalent to m-separate angle-asymmetrical systems of order $k = 1, 2, \ldots, m$. The effect of the asymmetrical system is the synthesis of the separate effects of the m-(modified) systems. Be the phase currents of the angle asymmetrical m-phase system $I_a, I_b, \ldots, I_m$. Resolution of these to m generalized symmetrical components leads to

$$\begin{aligned} I_a &= I_{a1} + I_{a2} + \ldots I_{am} \\ I_b &= I_{b1} + I_{b2} + \ldots I_{bm} \\ &\vdots \qquad\qquad\qquad\quad \vdots \\ I_m &= I_{m1} + I_{m2} + \ldots I_{mm} \end{aligned} \tag{1}$$

where I_{jk} is the k^{th} modified symmetrical component of phase j. Figure 1 shows the m-phase system, where the displacement angle between phase i and phase a is α_{ai} and the angle between phases i and $i + 1$ is $\gamma_{i(i+1)}$.

From Fig. 1 it follows that $\gamma_{i(i+1)} = \alpha_{a(i+1)} - \alpha_{a(i)}$, (2).

Figure 2 shows the k^{th} symmetrical component currents of phases i, $i + 1$ and $i + 2$ $(I_{i(k)}, I_{i+1(k)}, I_{i+2(k)})$.

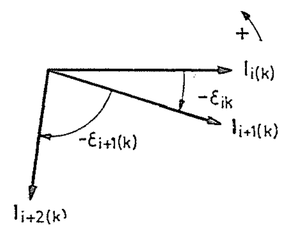

Fig. 1. Angle asymmetrical m-phase system

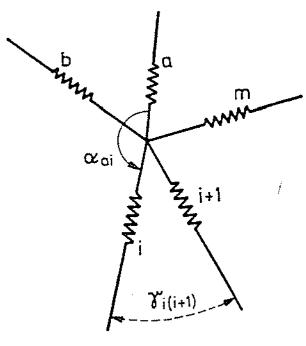

Fig. 2. The k^{th} symmetrical components of i, $i + 1$ and $i + 2$ phases, respectively

The k^{th} (modified) symmetrical component currents are in a time delay by $\varepsilon_{i(k)}$ to the k^{th} component current. It follows that the k^{th} component current of phase (i + 1) in Eq. (1) expressed in terms of the k^{th} component current of phase i will be:

$$I_{i+1(k)} + I_{i(k)} \exp\left(-j\varepsilon_{i(k)}\right) \ . \tag{3}$$

Due to angle asymmetry, values of $\varepsilon_{i(k)}$ differ from each other for a fixed k. If no winding displacement exists, in Eq. (3) exp $[-jk2\pi/m]$ stands, as the vectors of the k^{th} system are shifted by an angle $-k2\pi/m$ from each other in a direction opposite to the revolving of the symmetrical system. Negative direction was assumed, as in the positive sequence system if the system rotates in the positive direction, the phases will have an (sequential) order of $a, b, \ldots m$.

The values of $\varepsilon_{i(k)}$ expressed in terms of $\gamma_{i(i+1)}$ are:

$$\varepsilon_{i(k)} = \Delta\varepsilon_i + 2\pi k/m \tag{4}$$

where the additive part $\Delta\varepsilon_i$ is due to angle asymmetry:

$$\Delta\varepsilon_i = \begin{cases} \dfrac{2\pi}{m} - \gamma_{i(i+1)} & k \neq m - 1 \\[2ex] \dfrac{-2\pi}{m} + \gamma_{i(i+1)} & k \neq 1 \end{cases} \tag{5}$$

If a symmetrical 3-phase system is assumed $m = 3$, $\gamma_{1(2)} = \gamma_{2(3)} = \gamma_{3(1)} = 120°$, the k^{th} component of phases a, b, c are

$$I_{b(1)} = I_{a(1)} \exp\left[-j\left(\frac{2\pi \cdot 2}{3} - 120\right)\right] = a^2 I_{a(1)}$$

$$I_{b(2)} = I_{a(2)} \exp\left[-j\left(\frac{2\pi}{3} + 120\right)\right] = a I_{a(2)}$$

$$I_{c(1)} = a^2 I_{b(1)}; \qquad I_{c2} = a I_{b(2)}; \qquad I_{c(0)} = I_{a(0)}$$

$$I_{a(1)} = a^2 I_{c(1)}; \qquad I_{a(2)} = a I_{c(2)}; \qquad I_{a(0)} = I_{c(0)}$$

where $a = \exp(j\,120°)$, so, considering Eq. (1):

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix}. \tag{6}$$

From Eq. (3) it is obvious that for an m-phase angle asymmetrical system the l-th-phase current expressed in terms of the modified symmetrical components is:

$$I_l = I_{l(1)} + I_{l(2)} + \dots I_{l(m)} = I_{a1} e^{-j \sum_{1}^{l-1} \varepsilon_{i(1)}} + I_{a1} e^{-j \sum_{1}^{l-1} \varepsilon_{i(2)}} + \dots \tag{7}$$

It follows that all the phase currents expressed in terms of the modified symmetrical components of phase a will be

$$I = \mathbf{C}_{3m} I_{a(k)} \tag{8}$$

where

$$I = [I_a, I_b, \dots, I_m]_t \qquad (t \text{ denotes transpose}) \tag{8a}$$

and

$$I_{a(k)} = [I_{a(1)}, I_{a(2)}, \dots I_{a(m)}]_t \tag{8b}$$

The generalized symmetrical component transformation is:

$$\mathbf{C}_{3m} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \varepsilon_m^* \cdot e^{-j\Delta\varepsilon_1} & \varepsilon_m^{*2} e^{-j\Delta\varepsilon_2} & \dots & \varepsilon_m^{*m} e^{-j\Delta\varepsilon_1} \\ \vdots & \vdots & & \vdots \\ (\varepsilon_m^{*1})^{m-1} \cdot e^{-j \sum_{i=1}^{m-1} \Delta\varepsilon_i} & (\varepsilon_m^{*2})^{m-1} \cdot e^{-j \sum_{1}^{m-1} \Delta\varepsilon_i} & \dots & \varepsilon_m^{*m} e^{-j \sum_{1}^{m-1} \Delta\varepsilon_i} \end{bmatrix} \tag{8c}$$

(asterisk denotes the conjugate), where $\varepsilon_m^k = \exp [2\pi k/m]$. The symmetrical components are obtained from the phase variables by inverse transformation. In a system where all the ν-th harmonics are present in mmf, Eq. (8c) can be regarded as the transformation holding in case of fundamental harmonic components, the transformation for the ν-th harmonic is, however, similar to that of Eq. (8c).

It is easy to show that for a m-phase system without angle-asymmetry:

$$[\mathbf{C}_{3m}]_{symm} = \begin{bmatrix} 1 & \dots & 1 & 1 \\ \varepsilon_m^{-1} & \dots & \varepsilon_m^{-(m-1)} & 1 \\ \vdots & & & \\ \varepsilon_m^{-(m-2)} & \dots & \varepsilon_m^{-(m-2)(m-1)} & 1 \\ \varepsilon_m^{-(m-1)} & \dots & \varepsilon_m^{-(m-1)(m-1)} & 1 \end{bmatrix} \tag{9}$$

in agreement with that known from the general electrical machine theory (5, 7). Transformation matrix (8c) can be directly, a-priori derived mathematically by calculating the modal-matrix of an impedance matrix which can be expressed as a power series of a (mxm) primitive cyclic matrix, where the members of the series are multiplied by $k_0 = 1$, $k_1 = \exp[-j\Delta\varepsilon_1]$, $k_2 = \exp[-j(\Delta\varepsilon_1 + \Delta\varepsilon_2)] \ldots, k_m = \exp\left[-j\sum_1^{m-1}\Delta\varepsilon_i\right]$.

Therefore, the eigenvalues are:

$$\lambda_i = c_0 + c_1\varepsilon_m^i e^{-j\Delta\varepsilon_1} + \ldots c_{m-1}\varepsilon_m^{i(m-1)} e^{-j\sum_1^{m-1}\Delta\varepsilon_i} \tag{10}$$

($c_0, c_1, \ldots c_{m-1}$ are the elements of the symmetrical system's impedance matrix).

The eigenvectors (generalized symmetrical components) are:

$$s_i = [s_{i1}, s_{i2}, \ldots s_{im}] \qquad i = 0, 1, \ldots m-1 \tag{11}$$

where

$$s_{im} = \varepsilon_m^{*i(m-1)} \cdot e^{-j\sum_1^{m-1}\Delta\varepsilon_i} \tag{11a}$$

(conjugate is present to get the usual form).

The m eigenvectors are linearly independent as the determinant of matrix $\mathbf{C}_{3m}$ consisting of the eigenvectors

$$\mathbf{C}_{3m} = [s_0, s_1, s_2, \ldots s_{m-1}] \tag{12}$$

is non-zero ($\det \mathbf{C}_{3m} \neq 0$). Therefore, the system of s_i eigenvectors can be considered as the base-vectors of a m-dimensional reference frame, and all m-dimensional x can be resolved into components parallel to the eigenvectors

$$x = \mathbf{C}_{3m}x' \tag{13}$$

where the co-ordinates of vector x in the new reference frame are the symmetrical components:

$$x = [x_a, \ldots x_m]_t; \qquad x' = [x'_0, \ldots x'_{m-1}]_t. \tag{14}$$

Using Eqs (11), (11a) and (12), the generalized symmetrical component transformation is:

$$\mathbf{C}_{3m} = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ \varepsilon_m^{*m} e^{-j\Delta\varepsilon_1} & \varepsilon_m^{*1} e^{-j\Delta\varepsilon_1} & \ldots & \varepsilon_m^{*m-1} e^{-j\Delta\varepsilon_1} \\ \vdots & \vdots & & \vdots \\ (\varepsilon_m^{*m})^{m-1} e^{-j\sum_1^{m}\Delta\varepsilon_i} & (\varepsilon_m^{*1})^{m-1} e^{-j\Delta\varepsilon_i} & \ldots & (\varepsilon_m^{*m-1})^{m-1} e^{-j\sum_1^{m-1}\Delta\varepsilon_i} \end{bmatrix}.$$

Transformation given by Eq. (15) is the same as that in Eq. (8c), only now the last and first rows have been exchanged, as in Eq. (14) the zero-sequence components stand in the first row of x'. General transformation is easy to reduce for the more practical two and three phases as shown in the following.

2.1 *Three-phase modified symmetrical component transformation*

From Eq. (15) the generalized three-phase symmetrical component transformation is directly derived, and

$$\mathbf{T} = \begin{bmatrix} 1 & 1 & 1 \\ \exp\left[-j\left(\frac{4\pi}{3} - \gamma_{1(2)}\right)\right] & \exp\left[\ j\left(\frac{2\pi}{3} - \gamma_{1(2)}\right)\right] & \exp\left[-j\left(\frac{8\pi}{3} - \gamma_{1(2)}\right)\right] \\ \exp\left[-j\left(\frac{8\pi}{3} - \gamma_{1(2)} - \gamma_{2(3)}\right)\right] & \exp\left[-j\left(\frac{4\pi}{3} - \gamma_{1(2)} - \gamma_{2(3)}\right)\right] & \exp\left[-j\left(\frac{16\pi}{3} - \gamma_{1(2)} - \gamma_{2(3)}\right)\right] \end{bmatrix} \tag{16}$$

holds. This can be further simplified by considering $\gamma_{1(2)} + \gamma_{2(3)} + \gamma_{3(1)} = 360°$. From Eq. (16) in case of a symmetrical three-phase machine, the well-known [5] symmetrical component transformation is derived.

2.2 *Two-phase modified symmetrical component transformation*

As the generally used two-phase system can be considered as a semi-four-phase system, several considerations must be made in deriving the generalized two-phase symmetrical component transformation from Eq. (15).

Let the displacement angle between the main and auxiliary phases (designated by 1 and 2) be $\Delta\alpha$. As the system is a semi-four-phase system, the displacement of the 2-nd and 3-rd winding is $(180 - \Delta\alpha)$. If b-winding (main-winding) is designated by 1, and a-winding (auxiliary winding) by 2, the inspection equations for the currents are easy to write:

$$I_b = I_1 = -I_3$$

$$I_a = I_2 = -I_4\ . \tag{17}$$

Considering Eqs (17) and (15) as well as the displacement angles discussed in the foregoing, the $k = 1, 2, 3, 4$ symmetrical components of phases 2,3 and 4 can be expressed in terms of the symmetrical components of phase 1.

$$\begin{aligned}
&k = 1 \quad I_{2(1)} = I_{1(1)} e^{-j(\pi - \Delta\alpha)}; \quad && I_{3(1)} = I_{1(1)}; \quad && I_{4(1)} = -I_{2(1)} \\
&k = 2 \quad I_{2(2)} = I_{1(2)} e^{-j\left(\frac{3\pi}{2} - \Delta\alpha\right)}; \quad && I_{3(2)} = I_{1(2)}; \quad && I_{4(2)} = I_{2(2)} \\
&k = 3 \quad I_{2(3)} = I_{1(3)} e^{j(\pi - \Delta\alpha)}; \quad && I_{3(3)} = -I_{1(3)}; \quad && I_{4(3)} = -I_{2(3)} \\
&k = 4 \quad I_{2(4)} = I_{1(4)} e^{-j\left(\frac{\pi}{2} - \Delta\alpha\right)}; \quad && I_{3(4)} = I_{1(4)}; \quad && I_{4(4)} = I_{2(4)}.
\end{aligned} \tag{18}$$

As defined in Eq. (1) the first subscript refers to the phase, and the second to the order of symmetrical components. It follows that phase currents "b" and "a" are:

$$\begin{aligned}
I_b &= I_{1(1)} + I_{1(2)} + I_{1(3)} + I_{1(4)} \\
I_a &= I_{2(1)} + I_{2(2)} + I_{2(3)} + I_{2(4)}
\end{aligned} \tag{19}$$

and by considering the inspection equations (17):

$$\begin{aligned}
I_b &= -(I_{3(1)} + I_{3(2)} + I_{3(3)} + I_{3(4)}) \\
I_a &= -(I_{4(1)} + I_{4(2)} + I_{4(3)} + I_{4(4)}).
\end{aligned} \tag{20}$$

From Eqs (19) and (20):

$$I_{1(2)} = I_{1(4)} = I_{2(2)} = I_{2(4)} = 0 \tag{21}$$

and from Eq. (19), by considering Eq. (21):

$$\begin{aligned}
I_a &= I_{2(1)} + I_{2(3)} = I_{a(1)} + I_{a(2)} \\
I_b &= I_{1(1)} + I_{1(3)} = I_{b(1)} + I_{b(2)}
\end{aligned} \tag{22}$$

where $I_{a(1)}, I_{a(2)}, I_{b(1)}, I_{b(2)}$ are the symmetrical components of phases "a" and "b". Considering Eq. (19), if $k = 1$, in Eq. (22):

$$\begin{aligned}
I_{a(1)} &= I_{b(1)} \exp\left[-j(\pi - \Delta\alpha)\right] \\
I_{a(2)} &= I_{b(2)} \exp\left[j(\pi - \Delta\alpha)\right]
\end{aligned} \tag{23}$$

From Eqs (22) and (23) the symmetrical components of phases "a" and "b" expressed in terms of the symmetrical components of phase currents "a" and "b":

$$\begin{aligned}
I_{a(1)} &= -j\,\frac{(I_a \exp[j\Delta\alpha]) + I_b}{2\sin\Delta\alpha}; \qquad & I_{a(2)} &= j\,\frac{(I_a \exp[-j\Delta\alpha]) + I_b}{2\sin\Delta\alpha} \\
I_{b(1)} &= j\,\frac{(I_b \exp[-j\Delta\alpha]) + I_c}{2\sin\Delta\alpha}; \qquad & I_{b2} &= -j\,\frac{(I_b \exp[j\Delta\alpha]) + I_a}{2\sin\Delta\alpha}
\end{aligned} \tag{24}$$

Eq. (24) leads to

$$I' = \mathbf{T}^{-1} I \tag{25}$$

where

$$I' = [I_{a(1)}, I_{a(2)}]_t \ ; \qquad I = [I_a, I_b]_t$$

and the inverse of the new modified symmetrical transformation is:

$$\mathbf{T}^{-1} = \begin{bmatrix} \dfrac{-j \exp [j\Delta\alpha]}{2 \sin\Delta\alpha} & \dfrac{-j}{2 \sin\Delta\alpha} \\ \dfrac{j \exp [-j\Delta\alpha]}{2 \sin\Delta\alpha} & \dfrac{j}{2 \sin\Delta\alpha} \end{bmatrix} \tag{16}$$

so

$$\mathbf{T} = \begin{bmatrix} 1 & 1 \\ \exp (-j\Delta\alpha] & \exp [j\Delta\alpha] \end{bmatrix} \tag{27}$$

in agreement with the transformation presented in [4].

The transformations for the ν-th harmonic are similar, only that $\Delta\alpha$ has to be replaced by $\nu\Delta\alpha$ in the transformations. Therefore

$$\mathbf{T}_\nu = \begin{bmatrix} 1 & 1 \\ \exp [-j\nu\Delta\alpha] & \exp [j\nu\Delta\alpha] \end{bmatrix} \tag{28}$$

and

$$\mathbf{T}_\nu^{-1} = \begin{bmatrix} \dfrac{1}{1 - e^{-j\nu\Delta\alpha}} & \dfrac{j}{2} \dfrac{1}{\sin \nu\Delta\alpha} \\ \dfrac{1}{1 - e^{j\nu\Delta\alpha}} & -\dfrac{j}{2} \dfrac{1}{\sin \nu\Delta\alpha} \end{bmatrix} .$$

The derived two-phase transformations are in agreement with those of STEPINA [6], who has, however, not given a general treatment of the derivation of m-phase modified symmetrical component transformations.

3. Compatibility with earlier publications

In [4] it was shown how the new transformation can be applied for calculating two-phase induction machines where the stator windings were not in strict quadrature. It is not the purpose of present paper to derive general equations for $m-n$-phase machines with stator and/or rotor winding asymmetries using the derived general transformation.

This will be discussed in a following paper. It can be shown, however, to exist a close relationship between the generalized symmetrical component equations and those obtained by the theory of BROWN and VAS [8]. The positive sequence field operator applied on the phase quantities will lead to the new modified symmetrical components derived above. Therefore, the version of the voltage equation by BROWN and JHA [1] — holding for two-phase machines — extended to m-phase winding asymmetrical machines gives extended rotating field equations which are the most general rotating field equations and include the newly derived symmetrical components, too. The general equation in terms of the rotating field components for a machine with m-phase on the stator is [8], [9], [10]:

$$U = \mathbf{z}I + \sum_{\nu} \mathbf{F}_\nu Z_{\nu f} I + \Sigma \mathbf{F}_\nu^* Z_{\nu b} I \tag{29}$$

where

$$U = [U_a, U_b, \dots U_m]_t ; \quad I = [I_a, I_b, \dots I_m]_t ;$$

$$z = \mathrm{diag}\,(z_a, z_b, \dots z_m) .$$

The parameters are the same as defined in [2], and

$$\frac{1}{m}\mathbf{F}_0 = \mathbf{S}_0$$

$$\vdots$$

$$\frac{1}{m}\mathbf{F}_{m-1} = \mathbf{S}_{m-1}$$

holds for the forward fields operators $(\mathbf{F}_0, \dots \mathbf{F}_{m-1})$ but $\mathbf{S}_0, \dots \mathbf{S}_{m-1}$ are the newly defined sequence operators.

Subsequently, at a later paper, equations governing the behaviour of general asymmetrical induction machines will be given by using the new, modified symmetrical component transformations. Analytical treatment will also be given for a single-phase motor, with asymmetrical arrangement of the stator winding. Equations will be compared to those derived by generalized rotating field theory.

Summary

New, modified symmetrical component transformation has been derived a-priori on a fully mathematical basis, giving also a physical interpretation. Using this transformation, m-phase machines with general winding displacement angles can be studied. It is pointed out that application of such a theory will lead to one with close relationship to that using the general revolving field theory. It was proved that by adequate definition of sequence operators, the general rotating field theory involves even the generalized symmetrical component theory.

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