THE DYNAMIC BEHAVIOUR OF SYNCHRONOUS AND ASYNCHRONOUS MACHINES WITH TWO-SIDE ASYMMETRY CONSIDERING SATURATION

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Introduction

In recent years a new effort was made to calculate the transient operation of electrical machines with the help of the state variable Park-vector component method. Problems were solved in connection with control theory of thyristor connections in electrical machines [1, 2, 3]. In the present paper it will be shown how the method can be used for calculating the transient operational characteristics of synchronous and asynchronous machines with two-side asymmetry when saturation is also present.

The steady state analysis of an induction motor with asymmetrical cage has been discussed by VAS [4], from the paper the quadrature and direct axis rotor operator impedances can be derived, and be of use if the transients of asymmetrical squirrel cage motors are calculated by the method of this paper. Steady state and transient behaviour of the induction motors with asymmetrical rotor has been discussed in [5], [6], [7], but with no asymmetry in the stator side. Saturation of the main flux paths was also neglected, to be considered here.

Dynamical equations of asymmetrical machines neglecting saturation

Using the well-known assumptions [9], the Park-vector equations of a three-phase symmetrical ac. machine — lacking zero sequence components — in a reference-frame rotating at an arbitrarily varying speed ω_a

$$\bar{u}_s = R_s\bar{i}_s + \frac{d\bar{\psi}_s}{dt} + j\omega_a\bar{\psi}_s \tag{1}$$

$$\bar{u}_r = R_r\bar{i}_r + \frac{d\bar{\psi}_r}{dt} + j(\omega_a - \omega_r)\bar{\psi}_r \tag{2}$$

where $\bar{u}_s, \bar{u}_r, \bar{i}_s, \bar{i}_r, \bar{\psi}_s$ and $\bar{\psi}_r$ are the stator and rotor voltage, current and flux vectors. The fluxes are

$$\bar{\psi}_s = L_s\bar{i}_s + L_m\bar{i}_r \tag{3}$$

$$\bar{\psi}_r = L_m\bar{i}_s + L_r\bar{i}_r \tag{4}$$

here L_s and L_r are the stator and rotor inductances, L_m is the mutual inductance between stator and rotor. The torque is

$$m = \frac{3}{2} p(\bar{\psi}_s \times \bar{i}_s) \tag{5}$$

where p is the number of pole pairs. Assuming asymmetrical stator and rotor, these equations can be resolved into d, q components. However, as the voltage equations consist of the derivatives of both stator and rotor currents, for sake of simplicity the differential equations are solved for the fluxes. Thus the state vector equation in a reference-frame rotating at an arbitrary speed ω_a is:

$$\dot{x} = \mathbf{A}x + \mathbf{B}u \tag{6}$$

where x is the state vector of fluxes, $\mathbf{A}$ is the transition matrix, $\mathbf{B}$ is a unity matrix order four, and u is the column vector of the stator and rotor voltage components

$$\mathbf{A} = \begin{bmatrix} -\frac{1}{T'_{sd}} & \omega_a & \frac{k_r}{T'_{sd}} & 0 \\ -\omega_a & -\frac{1}{T'_{sq}} & 0 & \frac{k_r}{T'_{sq}} \\ \frac{k_s}{T'_{rd}} & 0 & -\frac{1}{T'_{rd}} & \omega_a - \omega_r \\ 0 & \frac{k_s}{T'_{rq}} & -\omega_a + \omega_r & -\frac{1}{T'_{rq}} \end{bmatrix}$$

$$x = [\psi_{sd}, \psi_{sq}, \psi_{rd}, \psi_{rq}]_t ;$$

$$u = [u_{sd}, u_{sq}, u_{rd}, u_{rq}]_t \tag{7}$$

where ω_r is the speed of the rotor, and k and T are constants given in Appendix 1.

For an induction motor the rotor voltages are $u_{rd} = u_{rq} = 0$ as the rotor circuits are short-circuited, and for a synchronous generator $u_{rd} = U_e$

where U_e is the voltage of the exciting coil, and $u_{r_g} = 0$. The stator voltages for an asymmetrically fed induction motor are:

$$
\begin{aligned}
u_{sd} &= 0{,}666(u_a - 0{,}5u_b - 0{,}5u_c)\\
u_{sq} &= 0{,}666(0{,}866u_b - 0{,}866u_c)
\end{aligned}
\tag{8}
$$

where

$$
\begin{aligned}
u_a &= U_{sa} \sin(\omega_1 t)\\
u_b &= U_{sb} \sin(\omega_1 t - 2\pi/3)\\
u_c &= U_{sc} \sin(\omega_1 t - 4\pi/3)
\end{aligned}
$$

(U_{sa}, U_{sb}, U_{sc} are the peak values of the a, b, c stator phase voltages, and for the synchronous generator it can be shown that

$$
\begin{bmatrix} U_{sd} \\ U_{sq} \end{bmatrix} = U_s \begin{bmatrix} \sin\delta \\ \cos\delta \end{bmatrix}
\tag{9}
$$

where U_s is the peak value of the stator voltage, and δ is the load angle.) In Eq. (7) for synchronous machines it must be considered that in case of a generator

$$
\delta = \int_0^t (\omega_r - \omega_1)\, dt + \delta_0
\tag{10}
$$

where ω_1 is the angular frequency of the stator.

The equation of motion, using (5) for an induction motor is:

$$
\frac{d\omega_r}{dt} = \frac{1}{J}\left[\frac{3}{2} p \frac{k_r}{L_s'} (\psi_{sq}\psi_{rd} - \psi_{rq}\psi_{sd}) - T_L - f_r\omega_r\right]
\tag{11}
$$

where J is the moment of inertia, T_L is the load torque, and f_r is the coefficient of viscous damping. For the synchronous generator, using (10) and the equation of motion, a similar equation can be derived:

$$
\frac{d^2(\delta/p)}{dt^2} = \frac{1}{J}\left[T_{\mathrm{mech}} - p \frac{3}{2} \frac{k_r}{L_s'} (\psi_{sq}\psi_{rd} - \psi_{rq}\psi_{sd}) - K \frac{d(\delta/p)}{dt}\right]
\tag{12}
$$

where T_{mech} is the mechanical torque of the turbine generator, and K is the coefficient of damping. From (5) it is seen if the rotor resistances of an induction motor differ from each other ($R_{ra} \neq R_{rb} \neq R_{rc}$), the d, q resistances of the rotor can be expressed as

$$
\begin{aligned}
R_{rd} &= \frac{1}{3}\left[R_a + R_b + R_c + \sqrt{R_a^2 + R_b^2 + R_c^2 - (R_aR_b + R_aR_c + R_bR_c)}\right] \\
R_{rq} &= \frac{1}{3}\left[R_a + R_b + R_c - \sqrt{R_a^2 + R_b^2 + R_c^2 + (R_aR_b + R_aR_c + R_bR_c)}\right]
\end{aligned}
\tag{13}
$$

In case of a squirrel cage machine with asymmetrical rotor, equations for the d, q rotor impedances can be derived from paper [4]. If the stator resistances of the synchronous or asynchronous machine differ, the d and q stator resistances can be obtained analogue to Eq. (13).

The general equations considering saturation

In the following only the saturation of the main flux paths will be considered, such a case may also occur with a symmetrical motor. From Eq. (6) — assuming that the magnetizing curve $|u_m|$ ($|i_m|$) is known, where $\bar{i}_m$ is the magnetizing current Park-vector ($\bar{i} \;\; = \bar{i}_s + \bar{i}_r$) and the stator and rotor reactances are $X_s = X_m + X_{s\sigma}$, $X_r = X \;\; + X_{r\sigma}$, — we get:

$$\dot{y} = \mathbf{A}_s y + \mathbf{B}_s u \tag{14}$$

where the current state vector is y_s (subscript s refers to saturation), A_s is the transition matrix and **B** a voltage coefficient matrix:

$$\mathbf{A}_s = C\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

where the submatrices are:

$$\mathbf{A}_{11} = \begin{bmatrix} -C_1R_{sd} & \omega_a(C_1X_s - C_2X_m) \\ \omega_a(C_2X_m - C_1X_s) - \omega_rC_2X_m & -C_1R_{sq} \end{bmatrix}$$

$$\mathbf{A}_{12} = \begin{bmatrix} C_2R_{rd} & \omega_a(C_2X_r - C_1X_m) \\ \omega_a(C_2X_r - C_1X_m) & C_2R_{rq} \end{bmatrix}$$

$$\mathbf{A}_{21} = \begin{bmatrix} C_4R_{sd} & \omega_a(C_3X_m - C_4X_s) - \omega_rC_3X_m \\ \omega_a(C_4X_s - C_3X_m) & C_4R_{sq} \end{bmatrix}$$

$$\mathbf{A}_{22} = \begin{bmatrix} -C_3R_{rd} & \omega_a(C_3X_r - C_4X_m) - \omega_rC_3X_r \\ \omega_a(C_4X_m - C_3X_r) + \omega_rC_3X_r & -C_3R_{rq} \end{bmatrix}$$

Coefficients k and C are found in Appendix 2.

$$y = [i_{sd}, i_{sq}, i_{rd}, i_{rq}]_t \;; \qquad u = [u_{sd}, u_{sq}, u_{rd}, u_{rq}]_t;$$

$$\mathbf{B}_s = c \begin{bmatrix} c_1 & 0 & -c_2 & 0 \\ 0 & c_1 & 0 & -c_2 \\ c_4 & 0 & -c_2 & 0 \\ 0 & c_4 & 0 & -c_2 \end{bmatrix}$$

Eq. (14) with the equation of motion can be solved step-wise by Runge-Kutta or other well-known methods solving differential equations. In every step the d, q components of the stator and rotor currents can be calculated to yield the absolute value of the magnetizing current:

$$|i_m| = \sqrt{(i_{sd} + i_{rd})^2 + (i_{sq} + i_{rq})^2} \tag{15}$$

Using (15), the C coefficients, functions of the first derivative of the magnetized curve, can be calculated.

The developed model suits to calculate the dynamical behaviour of symmetrical or asymmetrical synchronous or asynchronous machines, without or with saturation. The derived equations are of a form to be directly solved by a digital computer. Detailed results of computerized calculation will be shown in a following paper, for asymmetrical cases and for the case of a symmetrical three-phase step induction motor where the saturation cannot be neglected, as the stator voltage is not constant but greatly variable. Also time harmonic analysis will be presented.

APPENDIX 1

The constants k are:

$$k_r = L_m/L_r \;; \quad k_s = L_m/L_s \tag{1}$$

The stator and rotor transient time constants:

$$T'_{sd} = L'_s/R_{sd} \;; \qquad T'_{sq} = L'_s/R_{sq} \tag{2}$$

$$T'_{rd} = L'_r/R_{rd} \;; \qquad T'_{rq} = L'_r/R_{rq} \tag{3}$$

where L'_s and L'_r are the stator and rotor transient inductances.

APPENDIX 2

The stator and rotor leakage coefficients are:

$$k_{s\sigma} = X/X_{s\sigma} \;; \qquad k_{r\sigma} = X/X_{r\sigma} \tag{4}$$

where $X_{s\sigma}$ and $X_{r\sigma}$ are the stator and rotor leakage reactances, $X = X(i_m)$ is known from the magnetizing curve, X being its first derivative

$$\left(X = \frac{d\,|u_m|}{d\,|i_m|}\right)$$

The coefficients c are:

$$C = [(1 + k_{s\sigma})\,(1 + k_{r\sigma}) - k_{s\sigma}k_{r\sigma}]^{-1} \tag{5}$$

$$C_1 = (1 + k_{r\sigma})/L_{s\sigma}$$

$$C_2 = k_{s\sigma}/L_{s\sigma}$$

$$C_3 = (1 + k_{s\sigma})/L_{s\sigma}$$

$$C_4 = k_{r\sigma}/L_{r\sigma}$$

Summary

Differential equations in state-variable form have been given to calculate the behaviour of asynchronous and synchronous machines during transient operation assuming two-side asymmetry. Also saturation of the main flux paths was present. The equations are ready to solve on a digital computer. A following paper will discuss the results of several asymmetrical and symmetrical cases obtained by a digital computer. It will be also shown how the method can be applied for the calculation of electric machines with thyristor circuits if saturation is also considered.

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