COMPARISON OF SOME DETERMINISTIC AND STATISTICAL METHODS FOR PROCESS IDENTIFICATION

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The paper aims at comparing two main methods of Process Identification: identification by an ensemble of step responses and identification by the cross-correlation method with PRBSml.[1]

As the PRBSml — within one complete period of the sequence — is of deterministic nature, it is constructed from the algebraic sum of step functions occurring at the zero crossing points which are determined from the characteristic polynomial of the PRBSml. The crossing points are defined in an array m of order equal to the number of the zero crossing points. As a result, the output of the process is constructed from the algebraic sum of step responses at the elements of the array m under the assumption that the process is a linear time invariant. A noise is added to the output with different levels. The cross-correlation technique is used to estimate the impulse response of the process.

A guideline for matching the dynamic characteristic of the process with the parameters of the test signal is obtained. A periodic square wave function used in identification by an ensemble of step responses is obtained from the PRBSml, merely by changing the elements of the array m such that the process output reaches its steady state before the step function changes its level. The output contaminated with noise is ensembled and the step response obtained.

The comparison of the two methods, under the same measuring time and noise level, shows that an identification by an ensemble of step responses is preferable in the case of a low noise level in the output; for it is easy to apply and to realize.

The cross-correlation method is used when the process output is higly contaminated with noise and the ensemble of step responses method fails to provide acceptable accuracy in a reasonable measuring time.

[1] Pseudo Random Binary Sequence of maximum length.

In general, it is preferable first of all to test with a square wave function; this gives an advantageous information about the system (rising time, settling time, besides the noise level in the output). This information is helpful for the choice of the test signal parameters (PRSBml).

1. Introduction

The test signals used for identification purposes are very frequently so-called random telegraph signals formed recurrently from linear shift registers (PRBSml). The generation and the properties of (PRBSml) are intensively discussed and well-known [5, 8, 9]. The PRBSml generated by a digital method differs from the true random signal in that it is easy to generate and is determinant their autocorrelation function, if determined over an integer number of sequence periods, has no stochastic feature [9]. In addition, the requirement of zero mean value of the test signal in the (PRBSml) is, within one period, approximately fulfilled, so the relative frequency tends to 0.5 as the length of the shift register increases. Main stress is put on the correlation properties besides the zero mean value of the test signal [4, 15]. It can generally be said that the past ten years have been devoted to intensive work on the research and application of PRBS for the identification of concrete systems [9, 10, 15, 17].

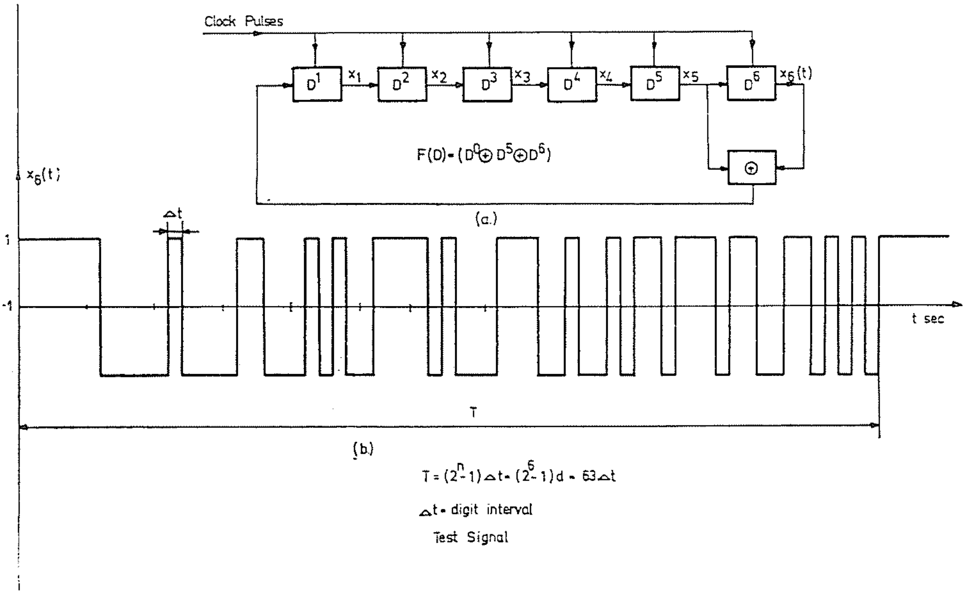


Fig. 1

In our paper, we want to make use of the deterministic nature of the PRBSml, as it switches from one level to the other only at the time intervals of the shift register in a known way. The zero crossing points can be determined from the characteristic polynomial of the shift register [8]. For example Fig. 1 demonstrates a 6-stage shift register circuit, having a certain characteristic polynomial. These points are defined in an array *m* of order equal to the number of zero crossing points. The PRBSml is regarded as the algebraic sum of step functions at the switching points of the PRBSml. The output of the process is the superposition of the step responses starting at the switching points. White noise with zero mean value and different levels are added to the outputs. In the case of PRBS a cross-correlation between the test signal and its noisy output is performed.

In the other case, namely if the input is a periodic square wave test signal, the ensemble of step responses has been accomplished.

2. Basic analysis

a) *The cross-correlation method*

From the tables of irreducible primitive polynomials of the PRBSml proposed by PATRESON [8], let us choose a PRBS of suitable order, then determine the zero crossing points in one period of the sequence and define them

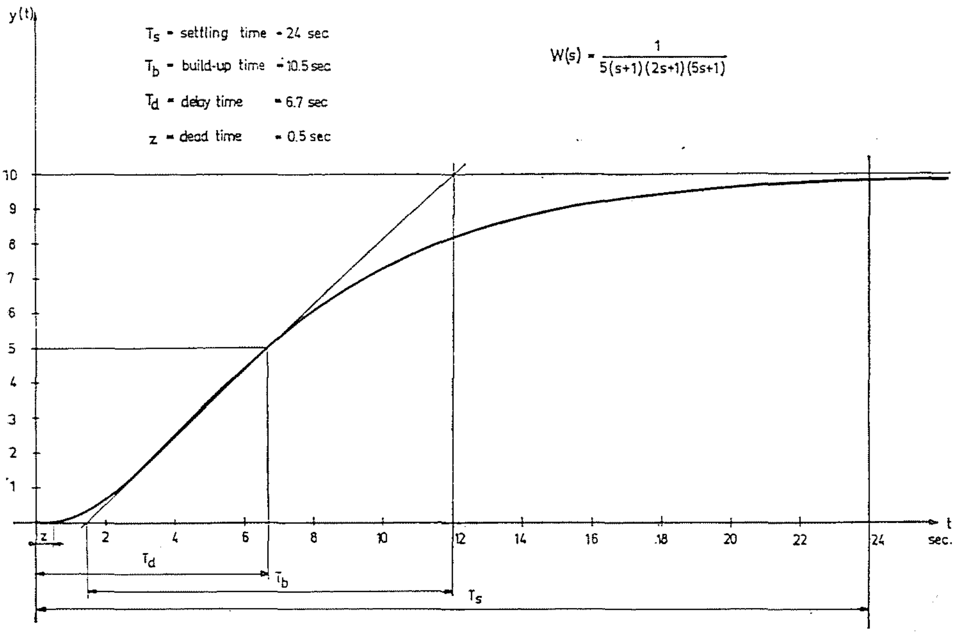
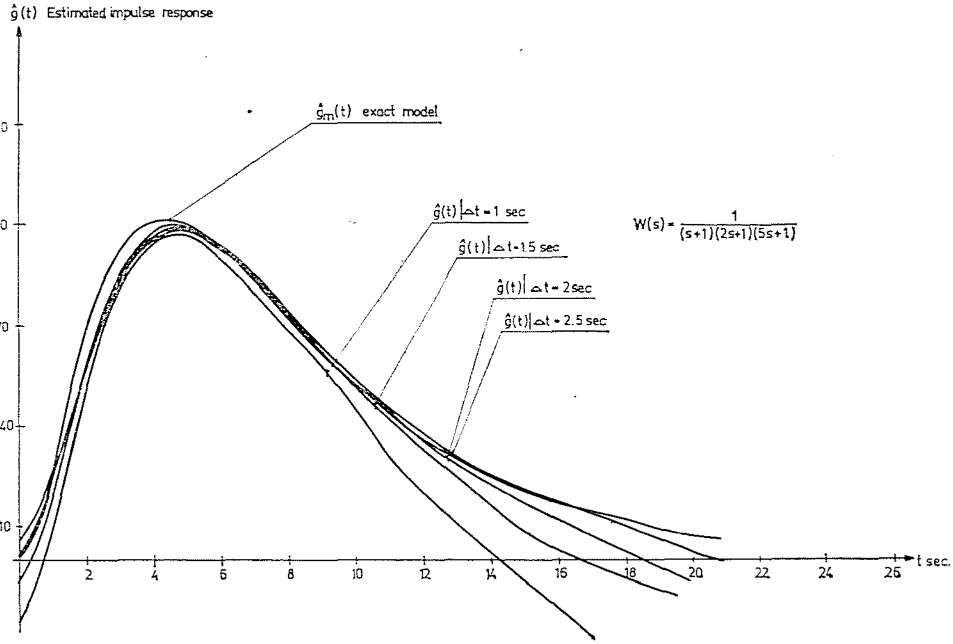
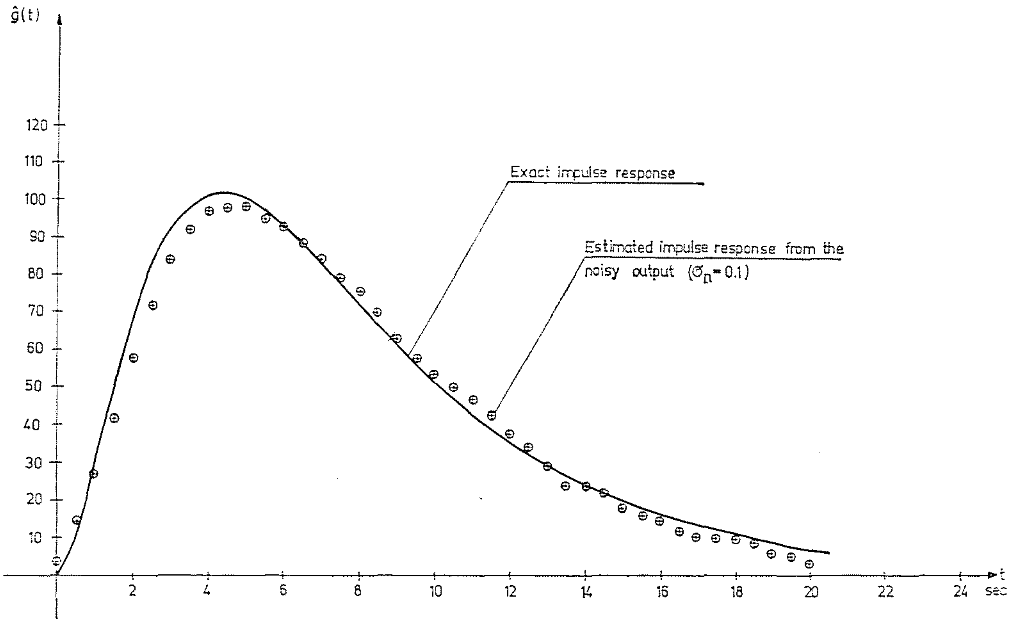


Fig. 2



ĝ(t) Estimated impulse response
$\hat{g}_m(t)$ exact model
ĝ(t)|Δt = 1 sec
ĝ(t)|Δt = 1.5 sec
ĝ(t)|Δt = 2sec
ĝ(t)|Δt = 2.5 sec
$W(s) = \frac{1}{(s+1)(2s+1)(5s+1)}$
130
100
70
40
10
2
4
6
8
10
12
14
16
18
20
22
24
26
t sec.

Fig. 3



ĝ(t)
Exact impulse response
Estimated impulse response from the noisy output ($\sigma_n = 0.1$)
120
110
100
90
80
70
60
50
40
30
20
10
2
4
6
8
10
12
14
16
18
20
22
24
t sec

Fig. 4

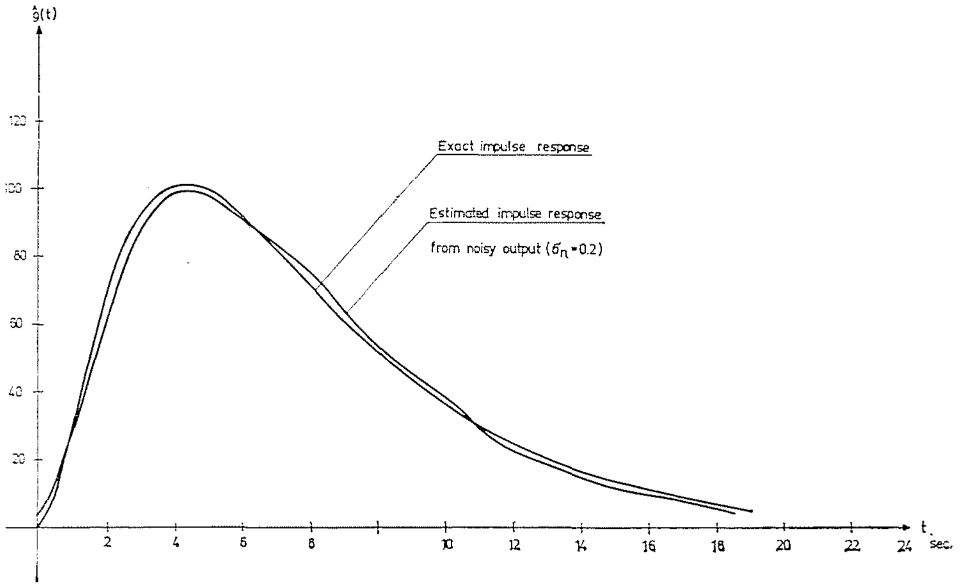

ĝ(t)
120
100
80
60
40
20
2
4
6
8
10
12
14
16
18
20
22
24
t sec.
Exact impulse response
Estimated impulse response
from noisy output (σ_n = 0.2)

Fig. 5

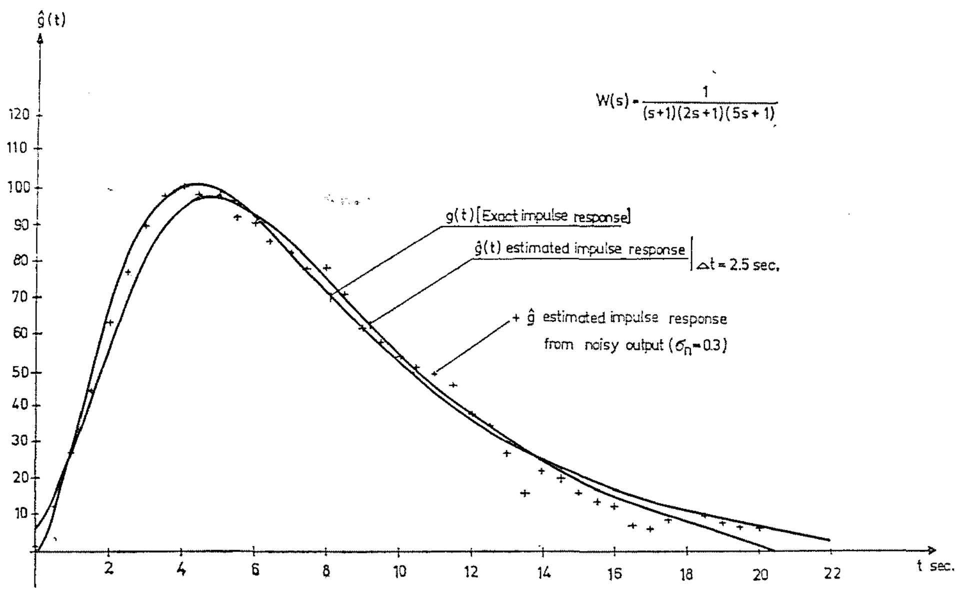

ĝ(t)
120
110
100
90
80
70
60
50
40
30
20
10
2
4
6
8
10
12
14
16
18
20
22
t sec.
$W(s) = \frac{1}{(s+1)(2s+1)(5s+1)}$
g(t) [Exact impulse response]
ĝ(t) estimated impulse response | Δt = 2.5 sec.
+ ĝ estimated impulse response
from noisy output (σ_n = 0.3)

Fig. 6

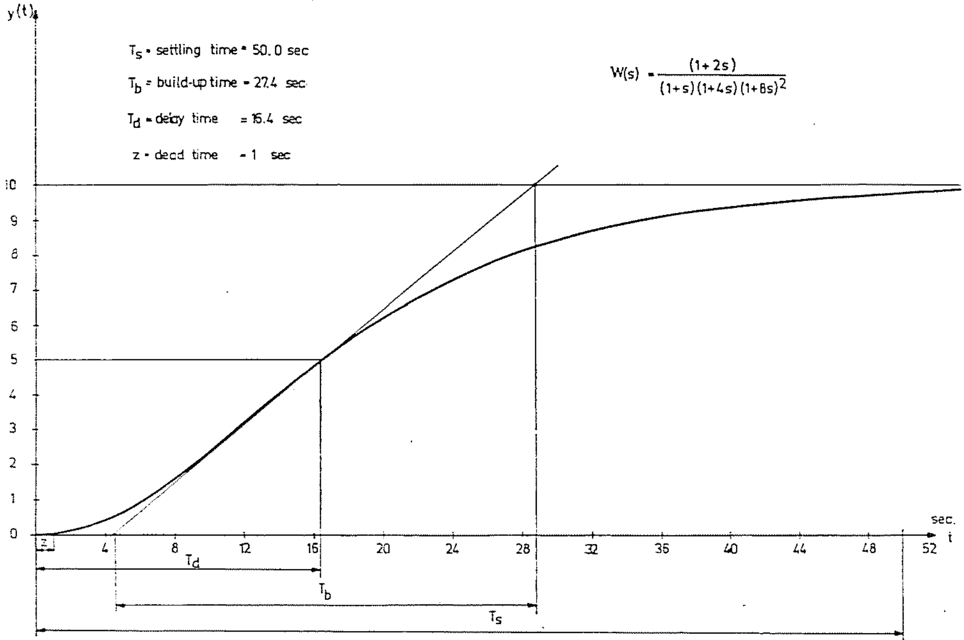


Fig. 7

in array m. The array m is composed by the time instants at which the PRBSml changes its sign. Let $u(t)$ be the unit step function and nn the order of the array m, then the input signal $x(t)$ can be written in the form:

$$x(t) = u(t) + 2 \sum_{j=1}^{nn} (-1)^j u(zt - t_j)$$

or

$$x[i\Delta t] = u[i\Delta t] + 2 \sum_{j=1}^{nn} (-1)^j u[i - m[j]\Delta t]$$

with Δt, the digit interval of the shift register. In discrete form:

$$x[i] = u[i] + 2 \sum_{j=1}^{nn} (-1)^j u[i - m[j]] \qquad (1)$$

Eq. (1) gives the PRBSml constructed from step functions.

If the process is assumed to be linear, time invariant, then the output generated by the test signal given by Eq. (1) can be expressed in the following form:

$$y_p[i] = y[i] + 2 \sum_{j=1}^{nn} (-1)^j y[i - m[j]] \qquad (2)$$

where $y[i]$ is the step response.

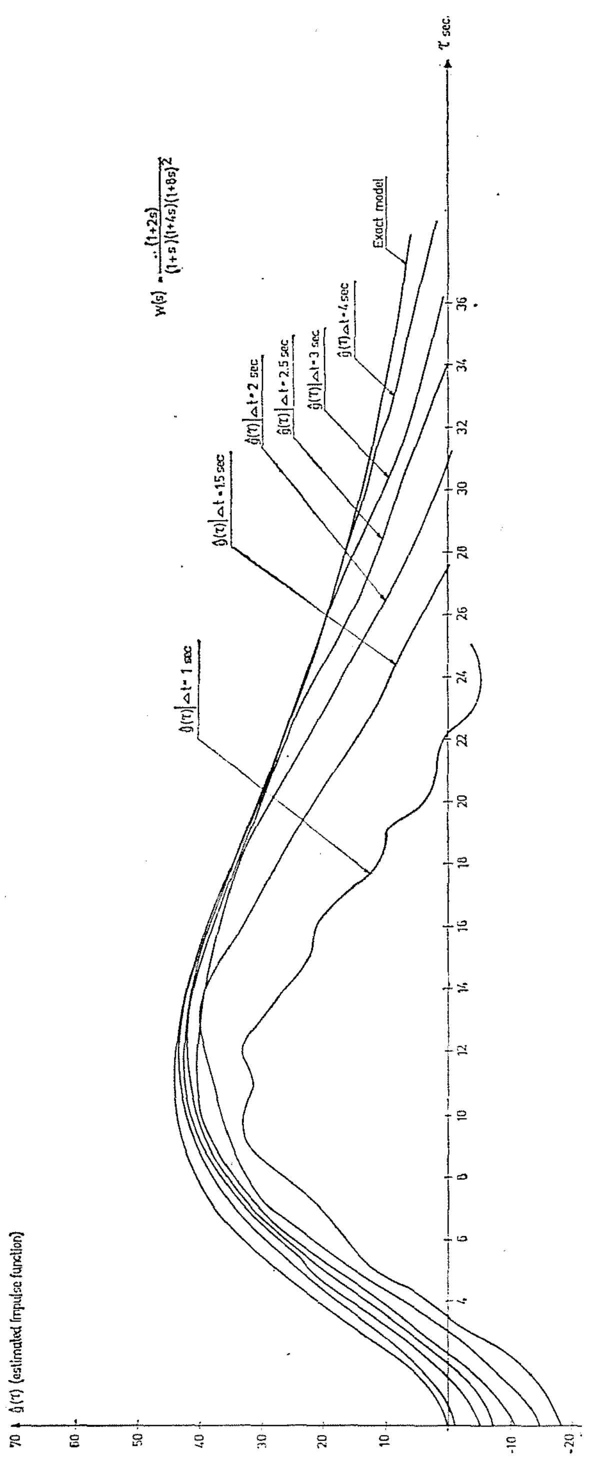

ĝ(τ) (estimated impulse function)
τ sec
Exact model
ĝ(τ)|Δt = 1 sec
ĝ(τ)|Δt = 1.5 sec
ĝ(τ)|Δt = 2 sec
ĝ(τ)|Δt = 2.5 sec
ĝ(τ)|Δt = 3 sec
ĝ(τ)|Δt = 4 sec

Fig. 8

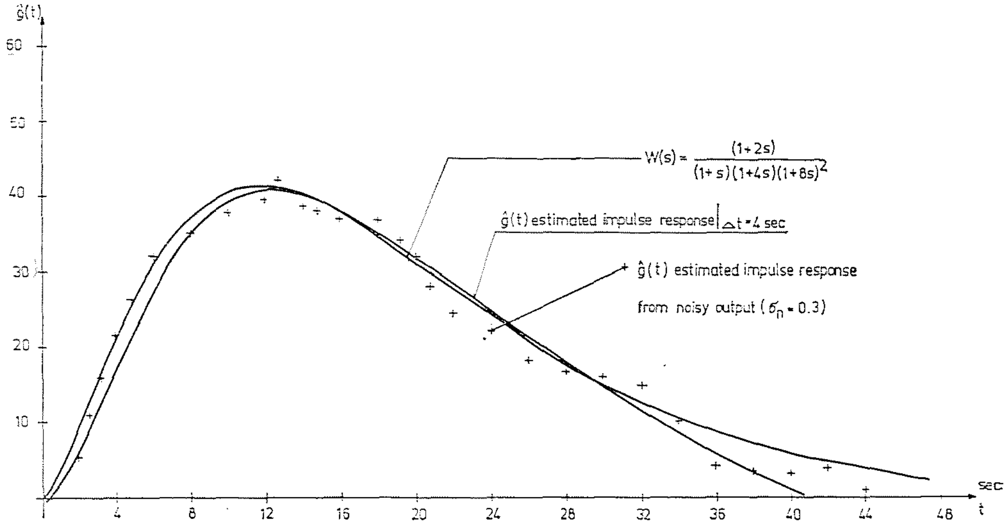

ĝ(t)
$W(s)=\frac{(1+2s)}{(1+s)(1+4s)(1+8s)^2}$
ĝ(t) estimated impulse response | $\Delta t=4$ sec
ĝ(t) estimated impulse response
from noisy output ($\sigma_n = 0.3$)
sec
t

Fig. 9

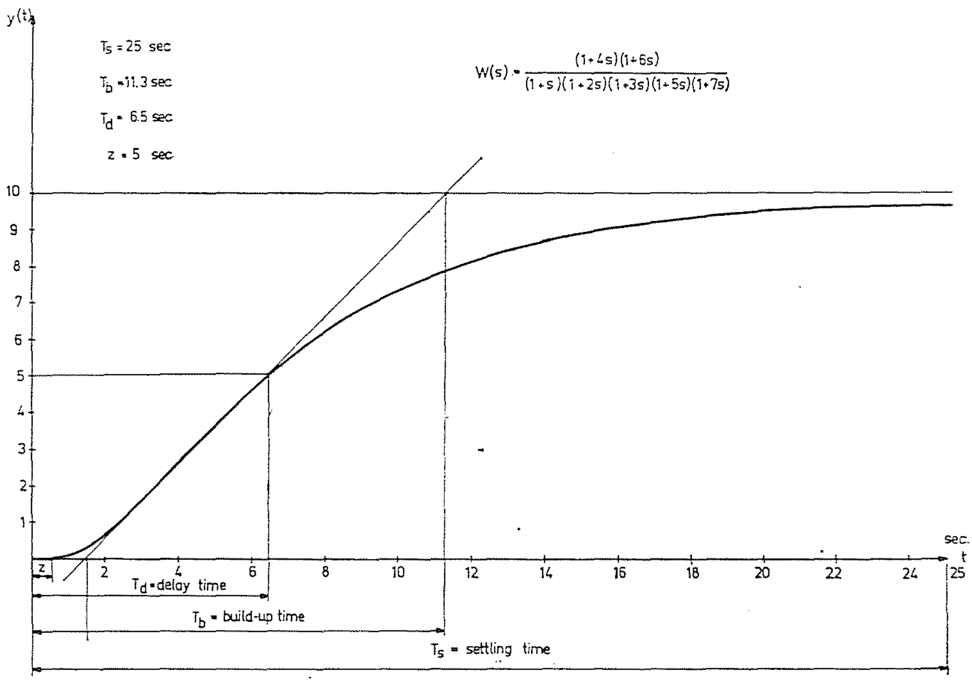

y(t)
$T_s = 25$ sec
$T_b = 11.3$ sec
$T_d = 6.5$ sec
z = 5 sec
$W(s)=\frac{(1+4s)(1+6s)}{(1+s)(1+2s)(1+3s)(1+5s)(1+7s)}$
T_d = delay time
T_b = build-up time
T_s = settling time
sec.
t

Fig. 10

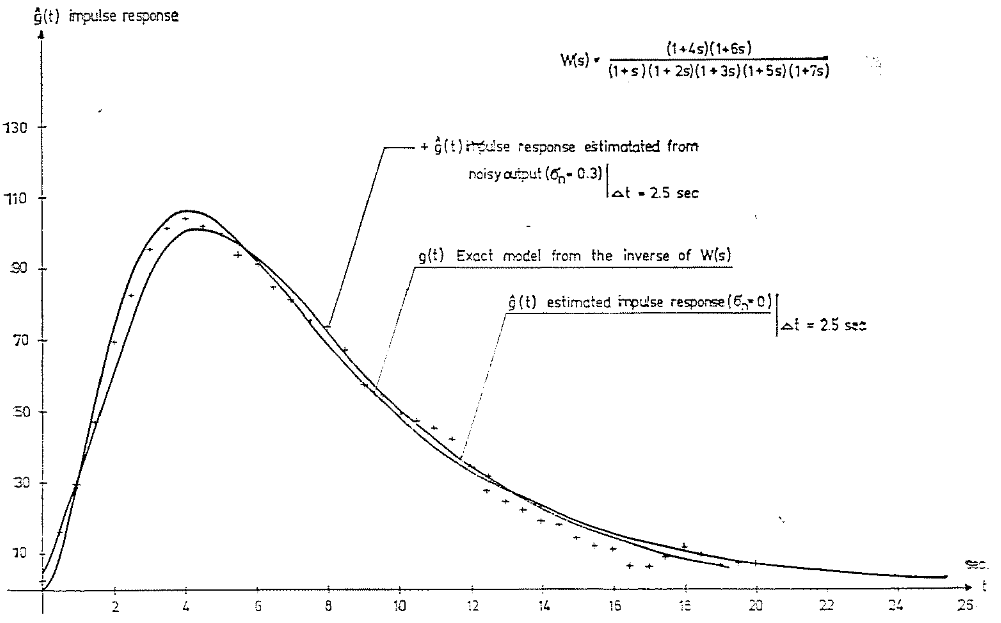


Fig. 11

As the noise is unavoidably present in a real system, it is included in the model by adding it to the output of the unknown linear process. It is assumed to be white with zero mean and standard deviation σ_n. Then the contaminated output is:

$$z[i] = y_p[i] + n[i] \tag{3}$$

Calculation of the cross-correlation function on a digital computer is usually carried out in accordance with the following equation: [11, 15]:

$$K_{xz}^{(k\Delta t)} = \frac{1}{n1 - k} \sum_{i=1}^{n1-k} x(i\Delta t)z((i + k)\Delta t)$$

or

$$K_{xz}[k] = \frac{1}{n1 - k} \sum_{i=1}^{n1-k} x[i]z[i + k] \ldots \tag{4}$$

Eq. (4) gives the cross-correlation function between the input $x[i]$ and the output $z[i]$ of the process.

It is well known that, if the test signal is a PRBSml with one period, whose autocorrelation function approximates the Dirac function, the cross-correlation function approximates the impulse response of the process, that is:

$$K_{xz}[k] = \beta\hat{h}[k],\ \hat{h} \text{ is the impulse response} \tag{5}$$

with $\beta = K_{xx}^{(o)} =$ a. c. f.[1] of the test signal at the origin, and the a.c.f. is corrected for bias term. That is: $\beta = a^2 \Delta t$, where a is the amplitude of the test signal. Then:

$$\hat{h}[k] = K_{xz}[k]/\beta$$

If the amplitude of the test signal is chosen as 1, then:

$$\hat{h}[k] = K_{xz}[k]/\Delta t \tag{6}$$

Eq. (6) gives an estimate of the impulse response.

b) *A method using the ensemble of step responses*

A periodic square wave function is used as an input test signal for a process to be identified because it is easy to realize in practice, besides it eliminates the nonlinearities in the actuators [1, 20]. The square wave function changes its level at a time approximately equal to the settling time of the process. It is interesting to note that this signal can be generated from PRBSml by changing the element of the array m and its order. The periodic square wave function is introduced by:

$$x_p(t) = u(t) + \tfrac{1}{2} \sum_{j=1}^{N_1} (-1)^j 2u(t + jT_p)$$

$$0 \leq t < T$$

or:

$$x_p[i] = u[i] + \tfrac{1}{2} \sum_{j=1}^{N_1} (-1)^j 2u[i + m[j]] \tag{7}$$

where N_1 is the number of steps: T_0/T_p.

Thus the output generated by the periodic square wave function can be written in the following form:

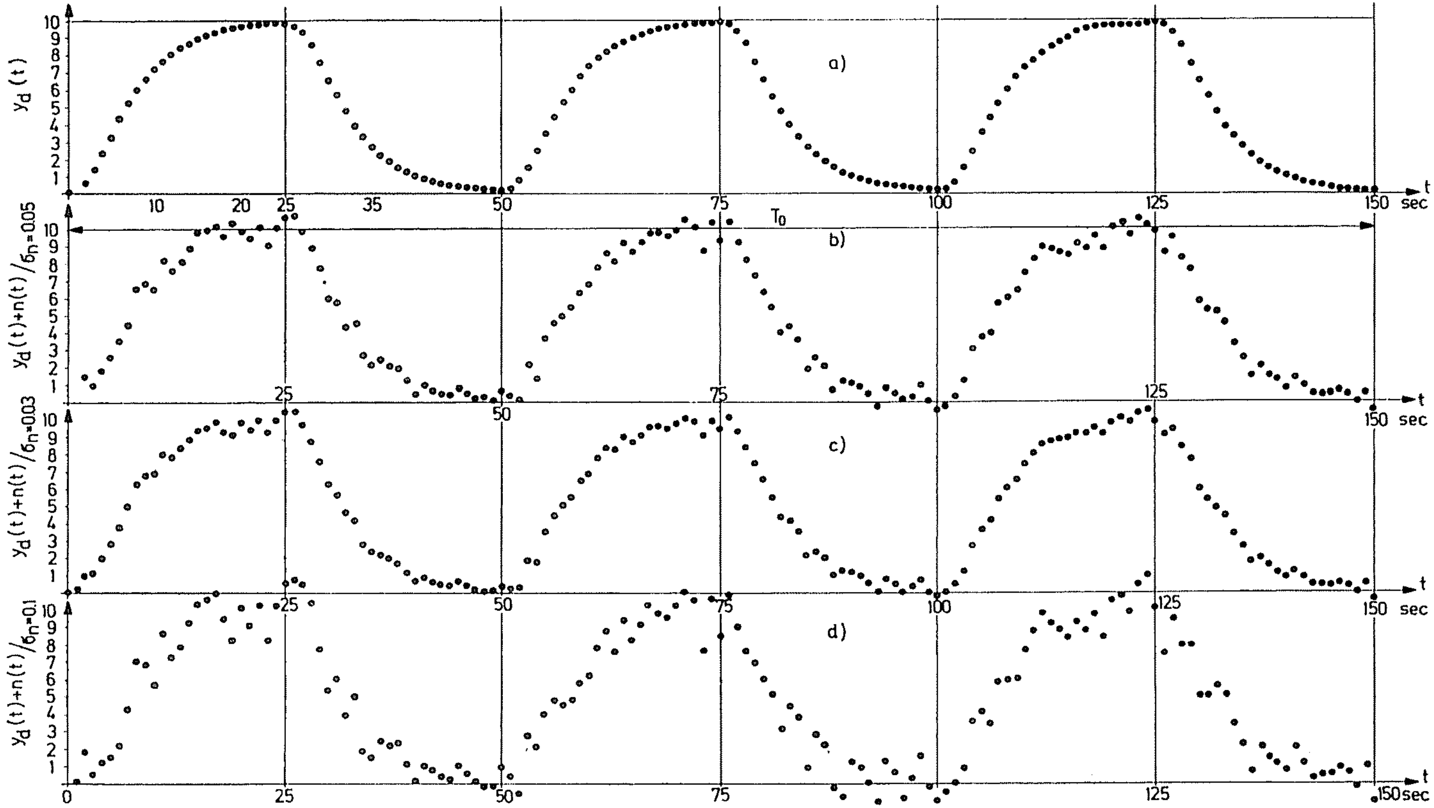
$$y_p[i] = y[i] + \tfrac{1}{2} \sum_{j=1}^{N_1-1} (-1)^j 2y[i + m[j]] \tag{8}$$

$$0 \leq i < m[1]$$

The output contamined with noise is given by:

$$z[i] = y_p[i] + n[i] \tag{9}$$

[1]a. c. f. = autocorrelation function



a) The output without nois
b) The output with noise of standard deviation = 0.05
c) The output with noise of standard deviation = 0.03
d) The output with noise of standard deviation = 0.1

Fig. 12

By averaging over the measuring time T_0, the estimated step response is given by:

$$y_z[i] = \frac{1}{N_1}\left\{\left[[z[i] + \frac{1}{2}\sum_{j=1}^{N_1-1}(-1)^j 2z[i+m[j]\right]\right\} \tag{10}$$

$$0 \leq i < m[1]$$

Namely the step response is deterministic and averaging will affect only the noise;

$$y_z[i] = y[i] + \frac{1}{N_1}\left\{n[i] + \frac{1}{2}\sum_{j=1}^{N_1-1} 2(-1)^j n[i+m[i]]\right\} \tag{11}$$

3. Numerical examples

a) *Estimating the impulse response*

The following structures are chosen in order to illustrate the procedures:

1. $w(s) = \dfrac{1}{(s+1)(2s+1)(5s+1)}$

2. $w(s) = \dfrac{1+2s}{(1+s)(1+4s)(1+8s)^2}$

3. $w(s) = \dfrac{(1+4s)(1+6s)}{(1+s)(1+2s)(1+3s)(1+5s)(1+7s)}$

A-priori information is required to choose the test signal parameters [10]. These are the settling time T_S, rising time T_R which are obtained from the step response of the process (Figs 2, 7, 10).

Let the characteristic equation of shift register be chosen from the tables arranged by PETRESON [8]: $F(D) = (D^0 \oplus D^5 \oplus D^6)$, D is shift operator and $\oplus$ modulo two additions.

The length of one period of the PRBSml is:

$$L = 2^n - 1 = 2^6 - 1 = 63 \text{ bits}$$

The number of runs in one period is given by:

$$nn = (L+1)/2 = 64/2 = 32$$

The period $T = L\Delta t = 63\Delta t$ with Δt, the clock pulse interval of the shift register.

The zero crossing points in terms of the digit interval Δt are defined in an array m which is given by:

$$m = [6, 11, 12, 16, 18, 21, 22, 23, 24, 26, 30, 31, 32, 35, 38, 40, 41, 43, 44, 45, 47, 48, 51, 52, 54, 56, 58, 59, 60, 61, 62, 63]$$

The digit interval is sampled such that:

$$\Delta t = kt_s = 10\ t_s$$

where k equals the number of samples in one digit interval and t_s is the sampling interval.

Different values of the digit interval are assumed for constant k in order to adjust the band width of the test signal to suit the band width of the process under test. The test signal is shown in Fig. (1b).

The output of the process is sampled with the same sampling interval of the test signal.

4. Results and discussion

a) *Cross-correlation method*

It appears from the different tests using different values of Δt, as shown in Figs 3, 8, 11, that: for the first example a good estimate of the impulse response is obtained at $\Delta t = 2$ sec with a period $T_0 = L\Delta t = 63 \times 2 = 126$ sec. This period is equal to the measuring time. For the second example, a good estimate is obtained at $\Delta t = 4$ sec, with a measuring time $T_0 = 63 \times 4 = 252$ sec. For the third example, $\Delta t = 2.5$ sec and $T_0 = 63 \times 2.5 = 157.5$ sec.

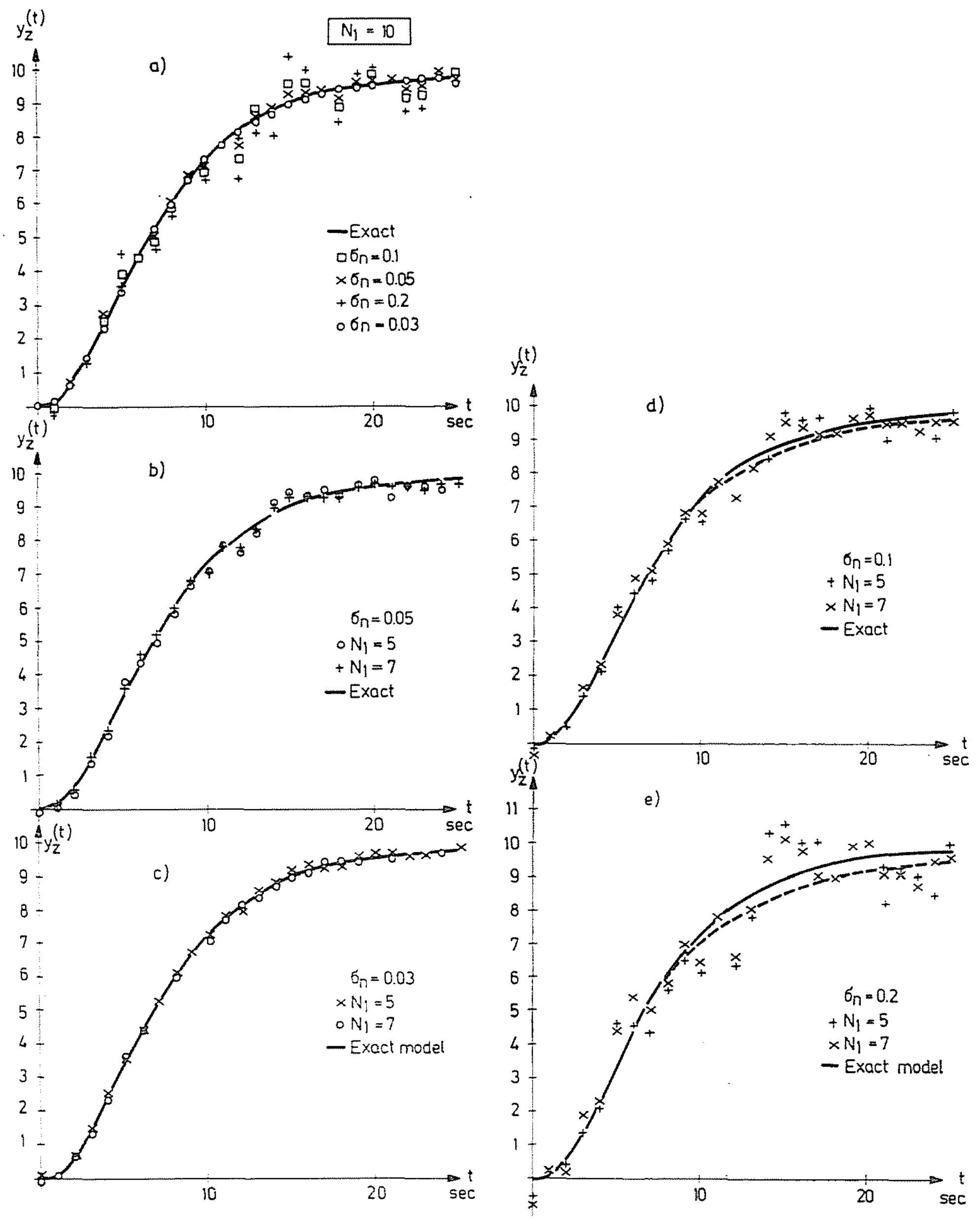
The noise with different values of the standard deviation $\sigma_n = 0.1; 0.2; 0.3$ is added to the output of each model, the cross-correlation technique is applied at the same digit interval for each example (2; 4; 2.5 sec). It appears from the different estimated impulse responses that this technique gives a good estimate of the impulse response in the presence of disturbance, too, and it is affected slowly as the noise level increases in the process output. (Figs 4, 5, 6, 8, 11).

It has been noted for the different tested structures arranged in Table (a) that good estimates are obtained if the product of the digit interval Δt by the maximum run of the test signal is not bigger than the rising time of step response of each structure ("run" of the test signal is the time between two successive switchings). The relationship between Δt and the rising time is shown in Fig. 15.

Table: (a)

No.	Structure	Rise time T_R	Settling time T_s	Digit interval Δt sec	$t_s = \frac{\Delta t}{k}$ $k = 10$	$T = N\Delta t$ $N = 63$	T_m/T_s	$T_R/\Delta t$	n_1 = Number of samples in one period T_m
1.	$\frac{1+2s}{(1+0)(1+4s)(1+8s)^2}$	24	50	4	0.4	252	5,04	6	630
2.	$\frac{1}{(1+s)^2(1+2s)(1+16s)^2}$	48	70	8	0.8	504	7.2	6	630
3.	$\frac{1}{(s+1)(1+2s)(1+5s)}$	12	24	2	0.2	126	5.24	6	630
4.	$\frac{(1+4s)(1+6s)}{(1+s)(1+2s)(1+3s)(1+5s)(1+7s)}$	11.5	25	2.5	0.25	157.5	6.3	4.6	630
5.	$\frac{(s+0.25)}{(s+0.243)(s+1\cdot1390 \pm j1.6768)}$	5	16	0.75	0.075	47.75	3	6.6	630
6.	$\frac{1}{(s+0.6208)(s+0.316 \pm j1.229)}$	3	10	0.5	0.05	31.5	3.15	6	630
7.	$\frac{1.644}{(1+s)(1+2s)(1+0.6s+s)^2}$	5	15	0.8	0.08	50.4	3.36	6	630
8.	$\frac{1}{(s+1)^4}$	6	12	1	0.1	63	5	6	630
9.	$\frac{10 \cdot 109}{(1+0.354s)^2(1+2.828s)^2}$	8	16	1.33	0.133	83.79	5.24	6	630
10.	$\frac{9.125}{(1+0.125s)(1+s+s^2)}$	2	10	0.33	0.033	20.79	2.08	6	630

* T_m = measuring time



$N_1 = 10$
a)
—Exact
□ $\sigma_n = 0.1$
× $\sigma_n = 0.05$
+ $\sigma_n = 0.2$
o $\sigma_n = 0.03$
b)
$\sigma_n = 0.05$
o $N_1 = 5$
+ $N_1 = 7$
—Exact
c)
$\sigma_n = 0.03$
× $N_1 = 5$
o $N_1 = 7$
— Exact model
d)
$\sigma_n = 0.1$
+ $N_1 = 5$
× $N_1 = 7$
— Exact
e)
$\sigma_n = 0.2$
+ $N_1 = 5$
× $N_1 = 7$
— Exact model
$y_z(t)$
t sec

Fig. 13

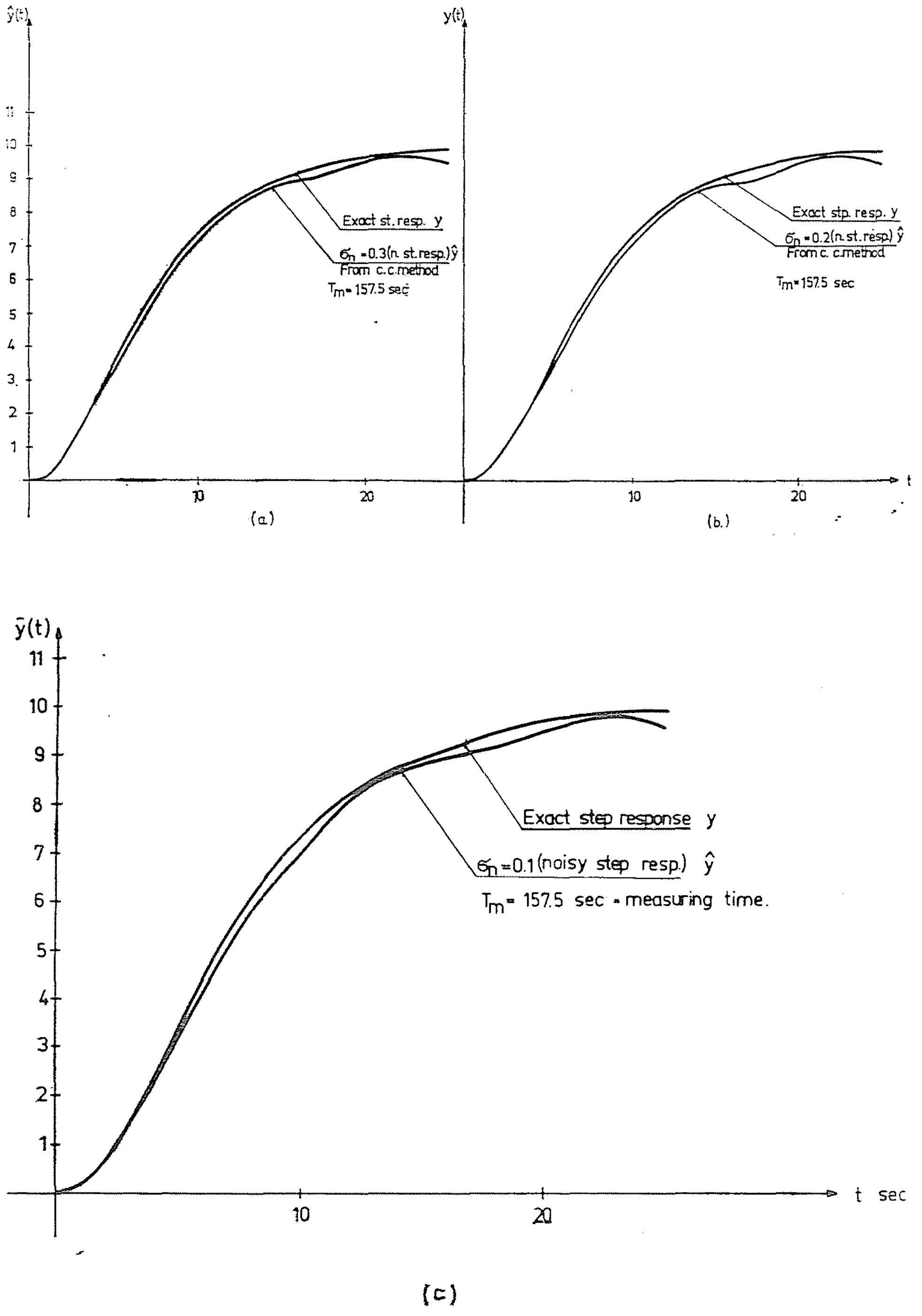

ŷ(t)
y(t)
Exact st. resp. y
σ_n = 0.3 (n. st. resp.) ŷ
From c.c. method
T_m = 157.5 sec
Exact stp. resp. y
σ_n = 0.2 (n. st. resp.) ŷ
From c. c. method
T_m = 157.5 sec
(a)
(b)
ȳ(t)
Exact step response y
σ_n = 0.1 (noisy step resp.) ŷ
T_m = 157.5 sec - measuring time.
t sec
(c)

Fig. 14

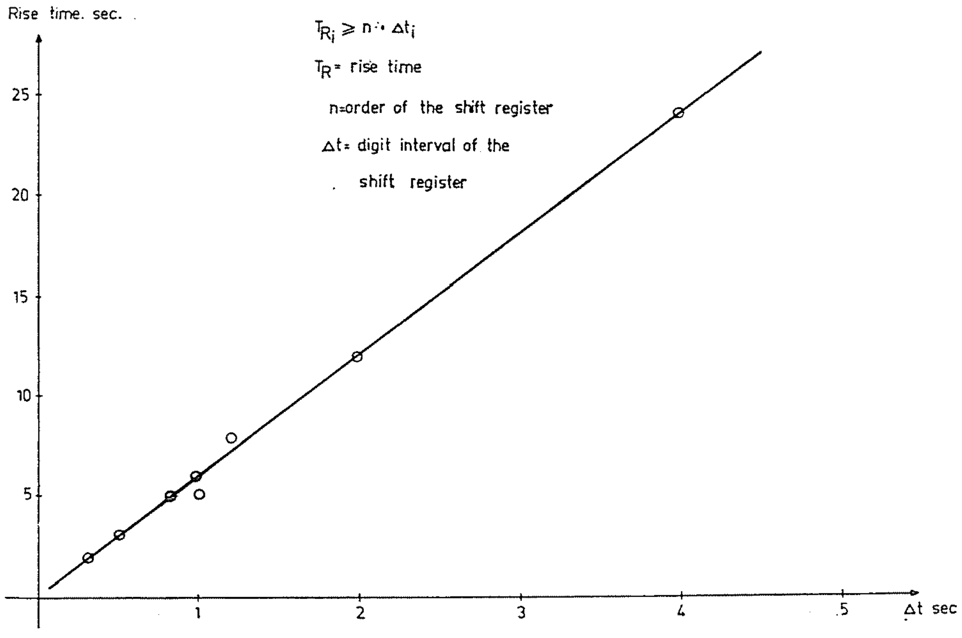


Fig. 15

This can be considered as a guideline for the choice of Δt. Since the maximum run is a constant number of digits, we can choose an appropriate value of Δt only if the rising time is approximately known. Since the period of the test signal T_0 must be chosen such that $T_0 > T_s$, the settling time, in this case the minimum length of the shift register can be chosen according to

$$L = T/\Delta t = 2^n - 1.$$

That is, the order of shift register polynomial n is determined. As it is clear from the previous discussion that an a-priori information is required (settling time, rising time) for the choice of the test signal parameters Δt, L.

An important advantage of the cross-correlation algorithm of this method is that, as the output of the process is constructed from a series of step responses (Eq. 4), the time between these steps is too much smaller than the settling time of the step response (the maximum run — as pointed out before — is not longer than the rising time); the output does not deviate too much from the operating level and saturation is avoided. So on-line identification is performed with a good safety.

b) *Ensemble of step responses*

1 For the purpose of comparison, the same structures have been chosen over the same measuring time T_0 and the same level noise for both methods.

In example 1, the total measuring time T_0 in the case of (PRBS) test signal was 126 sec and the settling time $T_s = 25$ sec.

The number of step responses is $126/25 = 5$. The outputs contaminated with noise are shown in Fig. 12b, c, d. The ensemble has been carried out in 5 steps. It is found that the method gives a good estimate of the step response only in the case of low disturbance level (see the case of $\sigma_n = .03, .05$). This is shown in Fig. 13b, c.

2. A better estimate of the step response is obtained as the number of steps N_1 increases, because the variance of the noise σ_n^2, diminishes inverse by proportion to the number of tests. (See Fig. 13a.)

3. In the rising part, the estimated step response does not deviate too much from the step response obtained from the structure, and a notable deviation (swinging) is found in the steady state part, particularly in the case of high disturbance. If the steady state part is estimated by linear averaging such that the rising part is interconnected directly with the averaged steady state part, a good estimate is obtained even for a relatively high disturbance level. (See Fig. 13c, d.)

5. Comparison between the cross-correlation method and the ensemble of step response method

— It is found that, in the case of low disturbance levels, the identification by the ensemble of step responses is preferable to the cross-correlation method, because it is easy to apply and to realize. Besides, no a-priori information is required for the identification process.

— In the case of large disturbances, the cross-correlation method is more acceptable than in the step response method. On the other hand, a-priori information is required in order to choose the parameter of the test signal. (Fig. 14a, b, c.)

— In general, it can be concluded that it is advantageous to test at first with a square wave function. This provides good information of the system under test, i.e. rising time, and settling time in addition to the noise levels in the output. When the output is highly contaminated with noise, and the step response provides unsatisfactory accuracy in a reasonable time, the cross-correlation method is performed with the help of the information obtained from the step response method.

Summary

An algorithm of estimating the impulse response of a process by cross-correlation method is obtained. Guidelines for matching the dynamic characteristic of the process (rise time, settling time) with the parameters of the test signal are given. A comparison between the cross-correlation method and the average step response method has been done with the same measuring time and noise level.

It has been concluded that when the process is higly contamined with noise and the average step response method provides unsatisfactory accuracy in a resonable time, the cross-correlation method is performed with the help of the information obtained from the average step response method.

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