A REMARK ON THE COMPLEX SHIFT OPERATOR $e^{\alpha s}$

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1. Introduction

[5] described construction of an operator field A_1 by the one-sided Laplace transform and the completion principle. This operational calculus is, however, not only a one-sided operational calculus, but in a certain meaning it is a two-sided one, too. In order to make clear this fact the operator field A_1 will be developed in a slightly different way by means of the two-sided Laplace transform (see Section 2). Another two-sided operator algebra A_2 based on the two-sided Laplace transform has been suggested in [2]. Since our problem is related to these investigations, a short introduction in Section 3 will be presented.

The differential operator s and the shift operators $e^{\lambda s}$ (λ real) are known to belong to the intersection $A_1 \cap A_2 \cap M$, where M is the Mikusinski operator field [4]. On the other hand, the shift operators $e^{\alpha s}$ for complex α's are in $A_1 \cap A_2$, but in this case $e^{\alpha s} \notin M$ [4].

In [7] the operators $e^{\alpha s}$ are explained to be complex shift operators for certain holomorphic functions embedded in A_1. But that has been done in the image space rather than in the original t-domain. In [2] the therm of a complex shifting has beeen introduced in the t-domain.

The question in which case the complex shifting of a certain classical function in the t-domain can be claimed to be again a classical one will be investigated here. Since the functions treated in [7] are irrelevant, to find an answer to the previous question is the aim of the present paper.

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2. An equivalent construction of the operator field A_1

$L_{\nu,\sigma}$ stands for the set of all complex-valued functions $f(t)$, $-\infty < t < +\infty$, having the following properties:

$$f(t) \text{ is locally integrable in the Lebesgue sense;} \tag{2.1}$$

$$\bar{f}(z) = \int_{-\infty}^{\infty} e^{-zt} f(t) dt , \tag{2.2}$$

the two-sided Laplace integral converges absolutely in the strip $\nu < Re(z) < \sigma$.

The equality in $L_{\nu,\sigma}$ is defined in the Lebesgue sense hence the Laplace transform is a bijection between the set $L_{\nu,\sigma}$, and the corresponding image set $\bar{L}_{\nu,\sigma}$ of $L_{\nu,\sigma}$. (See [3].)

Let $L = \bigcup_{-\infty \leq \nu < \infty} L_{\nu,\sigma}$. That is, for any $f(t) \in L$ there exists a right half-plane $\Delta = \{z: Re(z) > \nu\}$ (where ν depends on f) such that the Laplace integral (2.2) absolutely converges in Δ. In L the addition is defined as usual, and multiplication as convolution for

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t-u)\, g(u)\, du , \tag{2.3}$$

which is always a function in L by assumptions (2.1) and (2.2). (See [3] p. 121.)

Let M_R be the field of all functions $f(z)$ meromorphic in some right half-planes Δ_f of the complex z-plane (Δ_f may depend on f), where two functions in M_R are equal if they coincide in some right half-plane Δ, and the operations are defined pointwise. H denotes the subalgebra of all functions $h(z) \in M_R$ holomorphic in some right half-planes Δ. By the assumptions (2.2) and (2.3) the convolution theorem holds ([3] p. 121), it follows for L that the map (2.2) is an algebraic isomorphism of L onto $\bar{L} \subset H$, hence L is an integral domain.
Remark. The linear space $C(-\infty, \infty)$ under the convolution multiplication (2.3) has zero divisors therefore no operator field analogous to M can be defined.

Now let $Q(L)$ be the quotient field of L. The elements of $Q(L)$ are of the form $\frac{f}{g}$, where f and $g \neq 0$ (g is non-zero in the Lebesgue sense) belong to L and $\frac{f}{g}$ is called convolution quotient with respect to (2.3). The equality and the operations in $Q(L)$ are defined as usual.

In H the following convergence will be introduced: suppose that $(h_n(z))$ is a sequence in H and $h(z) \in H$. By definition, $\lim h_n(z) = h(z)$, if there exists a right half-plane Δ where $h_n(z)$ and $h(z)$ ($n = 1, 2, \ldots$) are holomorphic, and the sequence $(h_n(z))$ converges to $h(z)$ uniformly on every compact subdomain of Δ. This convergence is compatible with the algebraic structure of H. In analogy to [5] a sequence $\left(\frac{f_n}{g_n}\right) \subset Q(L)$ is called fundamental if there are functions $h(z)$ and $g(z)$ in H such that $\lim f_n(z) = h(z)$ and $\lim g_n(z) = g(z)$. Obviously the function

$$\tilde{L}\left\langle \frac{f_n}{g_n} \right\rangle = \frac{h(z)}{g(z)} \tag{2.4}$$

belongs to M_R. Two fundamental sequences (f_n/g_n) and (v_n/w_n) are equivalent if $\tilde{L}\langle f_n/g_n\rangle = \tilde{L}\langle v_n/w_n\rangle$ in the sense of M_R. It is easy to prove that the above relationship is an equivalence relationship. The equivalence classes are called operators, and set of all operators will be denoted by A_1. An operator $a \in A_1$ represented by a fundamental sequence (f_n/g_n) will be written in the form $a = \langle f_n/g_n\rangle$. Two operators are equal if their representatives are equivalent. The algebraic operations in A_1 will be defined as follows:

$$\begin{aligned} a+b &= \langle (f_n * w_n + v_n * g_n)/(w_n * g_n)\rangle \\ a*b &= \langle (f_n * v_n)/(g_n * w_n)\rangle \end{aligned} \tag{2.5}$$

where $a = \langle f_n/g_n\rangle$ and $b = \langle v_n/w_n\rangle$.

Theorem 1. The map $\tilde{L}[a] = \tilde{L}\langle f_n/g_n\rangle$, $a = \langle f_n/g_n\rangle$, defines an algebraic isomorphism of A_1 onto M_R.

The proof can be handled similarly as in [5].

An obvious consequence of the previous theorem is

Theorem 2. A_1 is a field under the operations (2.5).

To prepare the proof of Theorem 1 the following is needed: a sequence $(\delta_n) \subset C_c^\infty(-\infty, \infty)$ is called a δ-sequence if for all n ($n = 1, 2, 3, \ldots n$) the following properties are fufilled:

$$\left.\begin{aligned} &\operatorname{supp} \delta_n \subset \left[0, \frac{1}{n}\right]; \\ &\delta_n(t) \geqq 0 \text{ for all } t; \\ &\int_{-\infty}^{\infty} \delta_n(t)\, dt = 1. \end{aligned}\right\} \tag{2.6}$$

Since in this case the integral (2.2) for δ_n is a finite Laplace integral, $\bar{\delta}_n$ is an entire function for each n. In [5] it has been proved:

Lemma 1. $\lim \bar{\delta}_n(z) = 1$.

We should remark that the assumptions $\operatorname{supp} \delta_n \subset \left[\frac{1}{n}, \frac{1}{n}\right]$ are also sufficient for Lemma 1, and its proof involves no difficulties. For the proof of Lemma 3 we need.

Lemma 2. ([1] p. 258.) For any function $h(z) \in H$ polynomial sequences $(p_n(z))$ can be found with the property $\lim p_n(z) = h(z)$.

Now let us prove:

Lemma 3. Let $h(z) \not\equiv 0$ be any function of H. Then there are sequences $(f_n) \subset C_c^\infty(-\infty, \infty) \subset L$ where $f_n \not\equiv 0$ for each n, such that $\lim \bar{f}_n(z) = h(z)$. (For $h(z) \equiv 0$, $f_n \equiv 0$ for any n can be chosen.)

Proof: There is a sequence of polynomial functions $p_n(z)$ such that $\lim p_n(z) = h(z)$ by Lemma 2. If (δ_n) is a δ-sequence then, according to Lemma 1, $\lim \bar{\delta}_n(z)\, p_n(z) = h(z)$, since the convergence in H is compatible with the algebraic structure of H. The function $\bar{f}_n(z) = \bar{\delta}_n(z)\, p_n(z)$ is the image of a function $f_n \in C_c^\infty(-\infty, \infty)$ because of the derivation rule of the Laplace transform. Since $\bar{f}_n(z) \not\equiv 0$ we have $f_n \equiv 0$. This completes the proof.

The proof of Theorem 1. $\tilde{L}$ will be proved to be a bijection of A_1 onto M_R. If $a = \langle f_n/g_n \rangle$ is any operator in A_1 then obviously $\tilde{L}[a] = \langle \bar{f}_n/\bar{g}_n \rangle \in M_R$ is unique. Let $f(z) \in M_R$ be any function, then functions $h(z)$ and $g(z) \neq 0$ of H can be found by the theorems of Mittag—Leffler and Weierstrass [1] such that $f(z) = \dfrac{h(z)}{g(z)}$. According to Lemma 3, sequences (f_n) and (g_n) in L $(g_n \neq 0)$ exist, satisfying $\lim \bar{f}_n(z) = h(z)$ and $\lim \bar{g}_n(z) = g(z)$. Therefore $(f_n/g_n) \in Q(L)$ is a fundamental sequence which defines an operator $a = \langle f_n/g_n \rangle$. If another representation of $f(z)$ is used as a quotient of two holomorphic functions or other sequences according to Lemma 3, then always equivalent fundamental sequences are obtained; therefore the operator $a \in A_1$ has been determined uniquely.

Finally it is easy to show that the properties of an isomorphism are fulfilled by applying the compatibility of the convergence in H with respect to the algebraic structure. Hence Theorem 1 holds.

The convolution quotient field $Q(L)$ can be embedded in A_1 since the map

$$f/g \to \langle f/g \rangle, \ (f/g \in Q(L)) \tag{2.7}$$

is an algebraic isomorphism of $Q(L)$ onto a subfield of A_1. On the other hand, the map $f \to (f * g)/g$ $(g \not\equiv 0,\ g \in L)$ provides an embedding of L in $Q(L)$ such that

$$f \to \langle (f * g)/g \rangle \tag{2.8}$$

defines an embedding of L in A_1. Hence in A_1 we write f too. It is easy to see that $\tilde{L}[f] = \bar{f}(z)$. Similarly, the function $\tilde{L}[a]$, where $a \in A_1$ is any operator, is said to be the *Laplace transform* of the operator a.

Now let L_+ be the subalgebra of L consisting of all functions $f \in L$ having the property $\operatorname{supp} f \subset [0, \infty)$. Obviously for any $g \in L$ the function

$$g_+ = \begin{cases} g(t) & \text{for } t \geq 0, \\ 0 & \text{for } t < 0 \end{cases}$$

belongs to L_+. The operator

$$s = \langle f/(1_+ * f)\rangle$$

has the properties $\tilde{L}[s] = z$ and $s * f = f'$ if for f the theorem of differentation holds (see [3]). Therefore s is the differential operator in A_1. If $a \in A_1$ has the Laplace transform $\tilde{L}[a] = f(z) \in M_R$ then using Theorem 1 one can write formally $a = f(s)$.

Since L_+ is an integral domain, the quotient field $Q(L_+)$ of L_+ can be considered. It is known that $Q(L_+) \subset A_1 \cap M$ (see [5]). Other examples for operators from A_1 are found in [5] and [6].

There is a convergence structure defined on A_1 compatible with the field structure. A sequence $(a_n) \subset A_1$ converges to a A_1 if there exist quotients $\tilde{L}[a_n] = h_n(z)/g_n(z)$ and $\tilde{L}[a] = h(z)/g(z)$ in M_R h_n, g_n, h, $g \in H$, $g_n \neq 0$, such that $\lim h_n(z) = h(z)$ and $\lim g_n(z) = g(z)$.

3. The operator algebra A_2

[2] starts with the linear space B of all functions $\varphi(t)$ having the properties:

$$\varphi(t) \in L_{0,\sigma} \text{ for certain } \sigma > 0 \text{ (depending on } \varphi), \tag{3.1}$$

$$\bar{\varphi}(z) \text{ is holomorphic for } 0 < |z| < \sigma. \tag{3.2}$$

With the convolution product (2.2) B is also an integral domain, isomorphic to the image algebra $\bar{B}$ under the pointwise operations. Obviously $L \cap B \neq \emptyset$. But there are functions $\varphi \in L$ not contained in B; for example $e^{\alpha t}_+$ whenever $Re(\alpha) > 0$, and on the other hand $\varphi(t) = \begin{cases} 0 & \text{for } t \geq 0 \\ e^{\alpha t} & \text{for } t < 0 \end{cases}$ where $Re(\alpha) > 0$ belongs to B but not to L. In B the scalar products

$$(\bar{\varphi}(z), \bar{\psi}(z))_\varepsilon = \frac{1}{2\pi i} \int_{|z|=\varepsilon} \bar{\varphi}(z)\, \bar{\psi}^*(z) \frac{dz}{z}$$

are introduced, where $\bar{\psi}^*(z)$ is the complex conjugate of $\bar{\psi}(z)$ and ε is a sufficiently small positive number.

$$\|\bar{\varphi}(z)\|_\varepsilon = (\bar{\varphi}(z), \bar{\varphi}(z))_\varepsilon^{1/2}$$

defines a norm for each ε. These norms generate norms $\|\varphi(t)\|_\varepsilon = \|\bar{\varphi}(z)\|_\varepsilon$ in B too. Let A_2 and $\bar{A}_2$ be the inductive limits of B, $\bar{B}$ respectively.

Theorem 3. (See [2]) A_2 consists of all functions $\Phi(z)$ which are holomorphic for $0 < |z| < \sigma$ with a certain number σ depending on $\Phi(z)$. The algebras A_2 and $\bar{A}_2$ are isomorphic.

4. On the operators exp (αs)

From the definitions of A_2 and A_1 and from the previous results $A_1 \cap A_2 \neq \emptyset$ follows (also we hawe $A_1 \cap A_2 \cap M \neq \emptyset$). Obviously, if an operator $f(s) \in A_1$ has a Laplace transform $f(z) \in M_R$, which can be extended holomorphically to a function in $\bar{A}_2$, then $f(s)$ can be identified with an operator from A_2.

The differential operator s (with the image z) the operators $e^{\alpha s}$ (for any complex number α; $\tilde{L}[e^{\alpha s}] = e^{\alpha z}$) belong to $A_1 \cap A_2$; evidently the intersection $L \cap B$ is a subset of $A_1 \cap A_2$. For $\alpha = \lambda$, λ is a real number, $e^{\lambda s}$ is the shift operator, i.e. $e^{\lambda s} * \varphi(t) = \varphi(t + \lambda)$ follows from the shift theorem (see [3] p. 87). If $\varphi \in L \cap B$ and supp $\varphi \subset [0, \infty)$ then for all real numbers α the shifting formula also holds and $e^{\alpha s} \in M$ for all real α. But when α is a complex (not real) number, $e^{\alpha s} \notin M$. (See [4].)

The formal shifting in complex case for all generalized functions from A_2 is defined in [2], but the suppositions where $e^{\alpha s} * \varphi(t)$ is a classical function have not yet been investigated. In [7] the shift operators $e^{\alpha s}$ have been explained for certain functions different from the basic funtions $\varphi \in L$. Our purpose in this part is an investigation of the term $e^{\alpha s} * \varphi(t)$ for functions of A_1 or A_2 where α is a complex number.

For $\alpha = \lambda + i\tau$ (λ, τ real),

$$e^{\alpha s} * \varphi(t) = e^{i\tau s} * e^{\lambda s} * \varphi(t) = e^{i\tau s} * \varphi(t + \lambda)$$

holds where $\varphi(t) \in L$ (or B). Since $\varphi(t + \lambda) \in L$ (or B) it is enough to investigate the term $e^{i\tau s} * \varphi(t)$ for real τ. We wish to get (as a natural generalization of the "ordinary" shifting rule)

$$e^{i\tau s} * \varphi(t) = \varphi(t + i\tau). \tag{4.1}$$

Obviously it is necessary to have $\varphi(t + i\tau)$ defined. From the next examples this requirement will be clearly seen not to be sufficient.

Example 1. The function $\varphi(t) = \exp(-t^2) \in L \cap B$ can be extended to a complex function $\varphi(\xi) = \exp(-\xi^2)$ ($\xi = t + i\tau$) such that $\exp[-(t + i\tau)^2]$ is meaningful for all real t and τ. One can show easily that $\exp[-(t + i\tau)^2] \in L \cap B$. Since

$$\tilde{L}[e^{i\tau s} * e^{-t^2}] = e^{i\tau z} \cdot \sqrt{\pi} \cdot \exp\left(\frac{1}{4} z^2\right)$$

from a theorem of [3] (See [3], p. 87, Satz 4)

$$\tilde{L}[\exp\{-(t+i\tau)^2\}] = e^{\tau^2}\tilde{L}[e^{-2i\tau t}\cdot e^{-t^2}] = e^{\tau^2}\sqrt{\pi}\, e^{\frac{1}{4}(z+2i\tau)^2} = e^{i\tau z}\sqrt{\pi}\, e^{\frac{1}{4}z^2},$$

therefore the shift formula (4.1) holds.

Example 2. The function $\varphi(t) = e_+^{-t} \in L \cap B$ can be extended

$$\varphi(\xi) = \begin{cases} e^{-\xi} & \text{for } Re(\xi) \geq 0, \\ 0 & \text{for } Re(\xi) < 0. \end{cases}$$

The shifting $\varphi(t+i\tau) = e_+^{-(t+i\tau)}$ is meaningful for all real τ. Moreover $\varphi(t+i\tau) \in L \cap B$ since

$$\int_{-\infty}^{\infty} e^{-zt}\varphi(t+i\tau)\,dt = \int_0^{\infty} e^{-zt}e^{-(t+i\tau)}\,dt = e^{-i\tau}\frac{1}{1+z},$$

and the integral converges absolutely for $-1 < Re(z) < \infty$. On the other hand,

$$\tilde{L}[e^{-i\tau s} * e_+^{-t}] = e^{i\tau z}\frac{1}{1+z} \neq e^{-i\tau}\frac{1}{1+z};$$

hence (4.1) does not hold in this case.

Now a class of functions will be defined for which (4.1) holds. Let $L^{\gamma,\mu}$ $(0 < \gamma < \mu)$ be the set of all functions $\varphi(t) \in L$ with the properties:

$\varphi(t)$ can be extended to a complex function $\varphi(\xi)$, $\xi = t + i\tau$, which is continuous in the strip $-\gamma < \tau < \mu$; (4.2)

for all real τ, $-\gamma < \tau < \mu$, the functions $\varphi(t+i\tau)$ belong to L; (4.3)
if $-\gamma < \tau < \mu$ and z belongs to some right half-plane Δ then the equality

$$\int_{-\infty}^{\infty} e^{-zt}\varphi(t)\,dt = \int_{-\infty+i\tau}^{+\infty+i\tau} e^{-z\xi}\varphi(\xi)\,d\xi \tag{4.4}$$

holds.

The set $B^{\gamma,\mu}$ can be defined by analogy.

Remarks. (*a*) From (4.4) it follows that the extension $\varphi(\xi)$ of $\varphi(t)$ described in (4.2) is unique. In order to see this fact, $\varphi_1(\xi)$ is supposed to be an extension of $\varphi(t)$ different from $\varphi(\xi)$, then (4.4) holds for both functions. By substitution $\xi = t + i\tau$

$$e^{-i\tau z}\int_{-\infty}^{\infty} e^{-zt}\varphi(t+i\tau)\,dt = e^{-i\tau z}\int_{-\infty}^{\infty} e^{-zt}\varphi_1(t+i\tau)\,dt$$

holds because of (4.3) and the substitution rule (see [1] p. 78). Since $\varphi(t+i\tau)$ and $\varphi_1(t+i\tau)$ are continuous, $\varphi(t+i\tau)=\varphi_1(t+i\tau)$ follows for all τ in $-\gamma<\tau<\mu$.

(*b*) The property (4.4) implies that the right-side integral in (4.4) converges absolutely in the same half-plane as $\tilde{L}[\varphi(t+i\tau)]$.

Theorem 4. The shift formula (4.1) holds for all $\varphi(t)\in L^{\gamma,\mu}$ (or $B^{\gamma,\mu}$) if $-\gamma<\tau<\mu$.

Proof: $\tilde{L}[\exp(i\tau s)*\varphi(t)]=e^{i\tau z}\bar{\varphi}(z)$, and the integral converges absolutely in a half-plane Δ. On the other hand by use of (4.4) and substituting $\xi=t+i\tau$ we obtain

$$\tilde{L}[\varphi(t+i\tau)]=e^{i\tau z}\int_{-\infty+i\tau}^{+\infty+i\tau}e^{-z\xi}\varphi(\xi)d\xi=e^{i\tau z}\bar{\varphi}(z)$$

such that Theorem 4 holds.

Let us consider special cases: $L_B^{\gamma,\mu}$ stands for all functions $\varphi(t)$ which can be extended to functions $\varphi(\xi)$ ($\xi=t+i\tau$) holomorphic in the strip $-\gamma<$ $<\tau<\mu$ $(0<\gamma<\mu)$ and fulfilling the estimations

$$\begin{cases}|\varphi(\xi)|\leq K\,e^{\nu t} & \text{for } t\geq 0\,,\\ |\varphi(\xi)|\leq K\,e^{\sigma t} & \text{for } t<0\,,\end{cases}\tag{4.5}$$

in the above strip where $\nu<0<\sigma$ and $K>0$ (ν, σ, K depend on φ).

Since $L_B^{\gamma,\mu}\subset B^{\gamma,\mu}$ (see [3] p. 403, Satz 1) Theorem 4 also holds in A_2.

In order to get a similar result for an analogous class in A_1 the functions $\varphi(t)\in L$ fulfilling (4.5) will be used, and in this case it is enough to require $\nu<\sigma$ ($\exp(-t^2)$ is such a function).

Theorem 5. Suppose that $\varphi(t)$ has a holomorphic extension to an entire function $\varphi(\xi)$, $\xi=t+iu$, which satisfies the estimation

$$|\varphi(\xi)|\leq K\exp[-\lambda(t^2-u^2+\alpha ut)]$$

for all ξ, where α, λ, K ($\lambda>0$, $K>0$) are real constants. Then for all real τ the shiftings $\varphi(t+i\tau)$ belong to L and (4.1) holds.

Proof: It is shown first that $\tilde{L}[\varphi(t+i\tau)]$ converges absolutely in the whole z-plane: ($z=x+iy$)

$$\int_{-\infty}^{\infty}|e^{-zt}\varphi(t+i\tau)|\,dt\leq\int_{-\infty}^{\infty}e^{-xt}\,|\varphi(t+i\tau)|\,d\tau\leq$$

$$\leq Ke\,e^{\lambda\tau^2}\int_{-\infty}^{\infty}e^{-(x+\lambda\alpha\tau)t}\cdot e^{-\lambda t^2}\,dt\,.$$

Since the last integral exists for all x (as the Laplace integral of $\exp(-\lambda t^2)$) we obtain $\varphi(t+i\tau) \in L$. Secondly the property (4.4) is proven for all real τ. By use of the Cauchy theorem it is enough to show that for all z

$$\int_R^{R+i\tau} |e^{-z\xi}\varphi(\xi)||d\xi| \to 0 \text{ as } R \to \pm\infty .$$

$$\int_R^{R+i\tau} |e^{-z\xi}\varphi(\xi)||d\xi| = \int_0^{\tau} |e^{-(x+iy)(R+iu)}\varphi(R+iu)|\,du \leq$$

$$\leq Ke^{-\lambda R^2} e^{-xR} \int_0^{\tau} e^{yu} e^{\lambda u^2} e^{-\lambda\alpha uR}\,du \leq$$

$$\leq Ke^{-\lambda R^2} e^{-xR} e^{\lambda|\alpha\tau R|} \int_0^{\tau} e^{yu} e^{\lambda u^2}\,du .$$

Because of the last integral is bounded (for any y and x):

$$e^{-\lambda R^2} e^{-xR} e^{\lambda|\alpha\tau R|} \to 0 \text{ as } R \to \pm\infty .$$

This completes the proof.

Example. The functions $\exp(-\beta t^2)$, with $Re(\beta) > 0$, fulfil the propositions of Theorem 5. Indeed

$$|e^{-\beta\xi^2}| = \exp\left[-Re(\beta)\,(t^2 - u^2 + \alpha ut)\right]$$

where $\alpha = 2Im(\beta)/Re(\beta)$.

Problem. We have defined the complex shifting for certain subclass of L (or B). But to decide whether a function of H can be complex shifted or not is an open problem. By other words having an arbitrary function $\bar{\varphi} \in H$ in what case it can be stated that there is a function $\varphi \in L$ and that $e^{\alpha z}\bar{\varphi}(z)$ also corresponds to a function of L such that formula (4.1) holds?

Summary

In the present paper the operator field A_1 will be developed by using two-sided Laplace transform. This operator field is analogous to the operator field M of Mikusinski. These are not the same (non isomorphic fields). However, there are common operators such as the operator of differentiation and shift operators $e^{\lambda s}$ for real λ. The shift operator $e^{\alpha s}$ for complex α is in A_1 but not in M. The question in which case one can claim the complex shifting of a certain classical function on the t-domain is again classical one will be investigated here.

References

1. BEHNKE, H., SOMMER, F.: Theorie der analytischen Funktionen einer komplexen Veränderlichen. Springer-Verlag, Berlin—Göttingen—Heidelberg 1962.
2. BERG, L.: A two-sided operational calculus. Demonstratio Mathematica (in print).
3. DOETSCH, G.: Handbuch der Laplace-Transformation I., Verlag Birkhäuser, Basel 1950.
4. MIKUSINSKI, J.: Operatorenrechnung. VEB Deutscher Verlag der Wissenschaften, Berlin 1957.
5. PREUSS, W.: Eine Verallgemeinerung der Laplace-Transformation. Beiträge zur Analysis 7 (1975), 87—99.
6. PREUSS, W.: Verallgemeinerte Laplace-Transformation und Operatorenrechnung. Beiträge zur Analysis 10 (1977) 27—39.
7. PREUSS, W.: On the embedding of an operational calculus based on the $L^{(\nu)}$-transformation in a field of transformable operators. Acta Math. Acad. Sci. Hung. 30 (3—4) (1977), 259—264.

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