A METHOD OF TIME-TABLE CONSTRUCTION BY COMPUTER

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1. Introduction

There are several computer methods available for time-table construction. Some well-known computer producing companies (IBM, UNIVAC, etc.) have their own programs for time-table construction. Furthermore, such programs have been made at several foreign universities (see: References) and attempts have been made in our country, too. Great part of these programs and algorithms have not been published or were only published in the form of summaries outlining just the principles of the operation.

These programs are either not available or the requirements of our university cannot be met and no modifications can be made to satisfy them.

In our concept the requirements are to be satisfied by utilizing the classical and modern results of operational research, by formulating the objective function and by using optimizing techniques.

At the present stage of computer and computer techniques setting up a linear model and utilizing linear programming methods seems to be the only way of solution.

At first, the application of simplex method was attempted. Its advantage is that all of the boundary conditions listed (or even more) can be considered in this model. In this case optimal solution can be obtained within the frame of the model. However, its huge memory and long computational requirements are disadvantageous. The above model has been tested on an oversimplified example. The obtained results satisfied the requirements set by us. but it could be pointed out by means of a simple estimation that the complete time-table of the university cannot be made on any of the available computers.

Second, an attempt has been made to adapt the transportation method for this task. The advantage of this model is the far less required memory capacity and computational time. But its disadvantage is that only special boundary conditions can be considered by the method. Thus, it cannot *directly* be applied for the complete solution of this task. The method constructed and proposed by us is an appropriate modification of the transportation task. Optimum is searched by subphases and time units, thus the time schedule is to be built up step by step.

2. Setting the problem

Our aim is to construct a procedure which can be used by all the faculties (of Budapest) and educational forms (graduate, evening, corresponding and post-graduate courses) of the Technical University, Budapest, for the purpose to make time schedule. The procedure itself satisfies the requirements below:

1. The best utilization of classroom capacity and uniformly distributed lunch time.

2. The requirements according to place, time or bottleneck capacity should be considered: e.g. laboratories, gymnasia, special lecture rooms (physical, chemical projections, etc.).

3. Opportunity should be provided that the deans and the corresponding authorities of the university could rank several factors, and special requirements could be satisfied (e. g. the subjects should be ranked according to their levels of difficulty).

4. The lectures and the subjects should be distributed in a didactically proper manner: lectures should be held in the morning hours, exercises (seminaries) in the afternoon; if possible they should uniformly be distributed along the week; lectures of the same subject should not be on consecutive days; lectures and exercises should directly follow each other if possible; etc.

5. Free days or occasionally, afternoons for the students should be provided for the desired times.

6. If possible, no break (idle hours) occurs in the time-table.

7. The transfer of students should be kept on a minimum.

8. In the time-table lecturers should not overlap.

9. Unifying of branches (in the case of common lectures) and dividing into smaller clusters (in the case of specialist lessons and if the number of the exercise leaders is less than that of the groups) should be provided.

10. The program should maintain the desired distribution of each subject and the differences between the even and odd-number weeks (e.g. three hours a week should be held according to the requirement of the lecturer, i.e. either 2 + 1 hours a week or 4 in even-number weeks and 2 in odd-number weeks could be maintained).

11. Free afternoons or mornings or eventually free days (but in limited number) should be provided for the departments, for the purposes of department meetings, administration and post-graduation, etc.

3. The mathematical model

The following notations have been introduced:
n — number of branches,
T — number of times.
k — number of classroom types,
Capacity$_j$ (t): the number of rooms of type j available at time t $(1 \leq j \leq k, 1 \leq \leq t \leq T)$,

Demand$_{ij}$: the demand of branch i for the room type j in a week

$$(1 \leq i \leq n,\ 1 \leq j \leq k),$$

$x_{ij}(t)$ — the demand of branch i for the room type j at time t

$$(1 \leq i \leq n,\ 1 \leq j \leq k,\ 1 \leq t \leq T).$$

The value of $x_{ij}(t)$ may be 0 or 1.

$c_{ij}(t)$ — the coefficient of the objective function belonging to $x_{ij}(t)$; $c_{ij}(t) > 0$.

Using the above notations the problem can be interpreted as an integer linear programming. Under the constraints

$$\sum_{j=1}^{k} x_{ij}(t) \leq 1\,, \tag{1}$$

$$\sum_{i=1}^{n} x_{ij}(t) \leq \text{capacity}_j\,(t)\,. \tag{2}$$

$$\sum_{t=1}^{T} x_{ij}(t) \geq \text{demand}_{ij}\,, \tag{3}$$

the maximum of the objective function

$$\sum_{i=1}^{n} \sum_{j=1}^{k} \sum_{t=1}^{T} c_{ij}(t)\, x_{ij}(t) \tag{4}$$

is to be found.

The meaning of the conditions given by inequalities (1), (2) and (3):

(1) only one room type can be given to the branch i at time t,

(2) at time t the number of rooms allotted from among rooms type j cannot exceed the available full capacity of this type,

(3) all the demands have to be satisfied.

This problem could be solved e.g. by the simplex method. Practically, however, it cannot be solved because of its large dimensions.

A heuristic algorithm is suggested for finding the quasi-optimum by operating a series of transportation problems. For each time $t(1 \leq t \leq T)$ the following transportation problem has to be solved.

Using the former notations for expressing the conditions

$$\sum_{j=1}^{k} x_{ij}(t) \leq 1 , \tag{1}$$

$$\sum_{i=1}^{n} x_{ij}(t) \leq \text{capacity}_j (t) , \tag{2}$$

the maximum of the objective function

$$\sum_{i=1}^{n} \sum_{j=1}^{k} k_{ij}(t) \, x_{ij}(t) \tag{5}$$

is to be found, where

$$k_{ij}(t) = k_{ij}\big(t, x_{ij}(1), \ldots, x_{ij}(t-1)\big). \tag{6}$$

Here, the modified coefficients $k_{ij}(t)$ of the objective function are affected by outcomes for the previous time. Thus, a transportation problem results from the original problem, omitting condition (3).

The function generating the $k_{ij}(t)$ value has to satisfy the conditions below.

1. At time T

$$\sum_{t=1}^{T} x_{ij}(t) \geq \text{demand}_{ij} . \tag{3}$$

2. The value of the objective function (4) should be quasi-optimum.

3. The value of time T should be as low as possible. The function generating the $k_{ij}(t)$ value has been chosen as:

$$k_{ij}(t) = c_{ij}(t) + \alpha \cdot \frac{m_j(t) - \xi_{ij}(t)}{s_j(t)} , \tag{7}$$

$$\xi_{ij}(t) = \frac{1}{\text{demand}_{ij}} \sum_{\tau=1}^{t} x_{ij}(\tau) , \tag{8}$$

where

α — coefficient of the deviation from the theoretical value ($\alpha > 0$)
$\xi_{ij}(t)$ — satisfactory level (the ratio of satisfied to total required hours),
$s_j(t)$ — 'standard deviation',
$m_j(t)$ — 'expected value'.

Remarks

1. T can be chosen so that the results of series of transportation problems positively satisfy the conditions (3). This means that the solution by heuristic algorithm satisfies the *original* inequalities (1), (2) and (3).

If at every time there is a single $x_{ij}(t) \neq 0$ (it is possible by properly selecting $k_{ij}(t)$) then is a proper value:

$$T_{\max} = \sum_{i=1}^{n} \sum_{j=1}^{k} \text{demand}_{ij} \,. \tag{9}$$

2. Iterating the complete procedure with modified coefficients $c_{ij}(t)$ and considering only solutions where the values of the objective function are increasing, then an increasing upper bounded series is obtained (the real optimum is an upper bound), and so the series surely converge.

4. The computer program proposed

The proposed program consists of four phases. The first and the fourth one are heuristic, while the second and the third one use a sequential transportation method setting statistically the objective function, described previously.

First Phase

The first phase is to develop the proper data base, on the one hand, and to maintain the occupations with defined time or place and those given in advance, on the other hand.

A. *Data basis*

The steps of developing the data basis are:

1. Data input. If the data basis has been set, then only the modifications, corrections are to be read in subsequently, for a significant part of the data — number of rooms, occupations by the time-table — generally do not change.
2. Data control. (Such a design of data basis can be considered as practical, where most part of control can be done by a computer).
3. Sorting and ordering of data.
4. Compiling and printing of tables.

The data basis should practically be stored on random access file.

In the followings, the educational units (essentially) equivalent by the time-table will be termed as branch, independent of whether it officially is the branch X of year Y or a section Z within.

B. *Occupations with defined time or place and that given in advance*

The corresponding data sorted and collected from the data basis are processed as follows:

1. Allocation of lectures with defined time. (If the room is allocated simultaneously, then its disposal, too).

2. Allocation in time of lectures, exercises, laboratories, post-graduate 'B' courses with defined places but undefined in time; and allocating the rooms.

3. Often, occupations can only be provided for a part of the teams in branches by steps 1 and 2 (in the case of laboratories, drawing rooms, etc). In this case occupations should be provided for the remaining teams in the time-table. Problems may arise for many of the lab-exercises held in odd-numbered hours but in step 2, they can be set either to the beginning or to the end of the given day.

4. Allocation of the joint lectures of several branches and of the central departments (social sciences, etc.)

5. Satisfying special requirements.

The requirements under 2 and 9 should be satisfied here among the requirements listed.

Second phase

Rooms are provided here to meet requirements unsatisfied in phase 1. The term 'room' does not really mean a place but one element of any room type. The following room types are reckoned with: large, medium and small lecture rooms, exercise and drawing rooms, and dining hall — but the number of types can be modified according to the requirements. The dining rooms are actually used from 12 to 3 h p.m., in other periods they are used for representing idle hours.

The allocation of rooms is made on time unit basis. The first interval is Monday morning from 8 to 9 h a. m., the subsequent ones are Tuesday, from 8 to 9 h a. m., Wednesday from 8 to 9 h a. m., etc.

This should be based on pedagogical considerations disposing first the most efficient time units to get the greatest efficiency from the educational point of view. At a given time, all the active branches are involved in the allocation process.

By *active* is meant a branch considered to have still an unsatisfied demand and this can be satisfied at the given time. (e.g. the branch has no free day).

It should be noted here again that the lectured units which are (essentially) equivalent by the time-table are termed as branches.

The applied allocation method is an appropriate modification of the transportation problem well known in operation research. Now, in the so-called transportation table the room types are the 'transporters' (supply) and the branches are the 'receivers' (demand).

The room demands of a branch are generally one unit or exercise rooms, etc. as many as there are groups within the branch. In our concept, the statistical laws for the high number of branches (about 200) permit the appropriate utilization of the rooms available, on the one hand, and prevent from allocating more rooms then available.

In the objective function table, of decisive importance for the optimization procedure, those room types will 'transport' with relative high benefit for which types there is real need in the branch, while the other types are considered as 'useless' in this case. The final transport costs are affected by the following factors.

The second phase consists of the main parts below:

1. Input,
2. Preparation,
3. Setting up the objective function,
4. 'Transportation',
5. Modification,
6. Output.

These steps must be performed for each time, involving the tasks:

1. The subprogram INPUT selects the data of the next time and constructs the tables needed in the subsequent calculations.

2. The subprogram PREPARATION prepares the matrices needed in the transport problem; sorting active branches, defining the requirements either satisfied or unsatisfied; compiling room capacities. Besides, the kind of lesson allocated to the given branch at the previous time is also examined here; the lunch time; the single, double, triple classes, etc.; to avoid idle hours and other special requirements are considered here, too. The conditions are considered either by composing the requirements, capacities and active phases, by designating some branches or by setting the coefficients of the objective function table in advance.

3. The subprogram SETTING the final object function table considers the level of satisfaction. The level of satisfying a branch by a given room type actually is the quotient of the number of allocated hours (lessons) by that of the total hours needed. (See (8).) The basic equality among the branches is unconditional, and the quantities allotted from all the room types to all the branches should follow — if possible — in didactically correct order (e.g. lecture rooms in the morning hours). This should be treated in detail since this is a rather important point in the operation and since setting up the objective function of the third phase is done in the same way.

The basis of constructing the objective function coefficients is Figure 1:

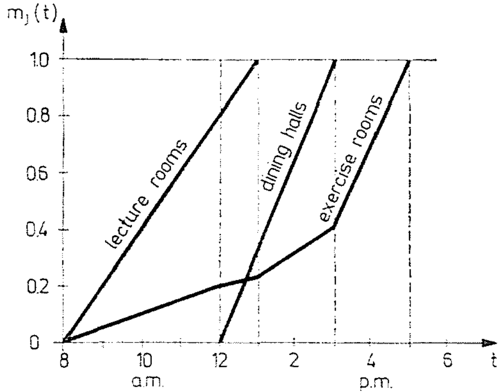

Fig. 1

By drawing an arbitrary vertical line, the theoretical percentage of the satisfied demands in lecture rooms, dining rooms, drawing rooms can be determined, both for the days of the week and for the branches in the average.

If any of the branches are below the average level of supply from any of the room types the 'transportation' from these room types becomes more economical, while in the opposite case it is uneconomical.

The objective function coefficients are expressed as

$$k_{ij}(t) = c_{ij}(t) + \alpha \frac{m_j(t) - \xi_{ij}(t)}{s_j(t)}, \alpha > 0,$$

where t — time, i — serial number of the given branch, j — serial number of the given room type, $c_{ij}(t)$ — objective function in the preparation phase, α — weighting factor, $\xi_{ij}(t)$ — the level of satisfaction of branch i according to room type j at time t, $m_j(t)$ — the theoretical expected value (see fig. 1), and $s_j(t)$ the standard deviation.

4. The subprogram TRANSPORTATION allocates the rooms optimally among the branches at a given time, taking the given capacities, the requirements, and the objective function into consideration by using the well-known transportation algorithm (distribution algorithm, 'stepping stone' method).

5. The part of the time-table for the corresponding time is made by the subprogram MODIFICATION, and the room capacities, the level of satisfaction of the branches and other tables of descriptive character are accordingly modified.

6. At each time after constructing the tables an output is made to the data carrier containing the data basis in the subprogram OUTPUT.

The second phase is essentially a series of transportation problems. The objective function coefficients are constructed at each time by statistically evaluating the current state. Among the requirements listed above, the 1st, the 5th, the 6th and partly the 4th are to be satified in this phase.

Third phase

This phase defines the concrete lessons or exercises at different times according to the requirements. Each time is processed consecutively, as mentioned above. To avoid great dimensions, not the complete curriculum of the university is treated simultaneously, but *groups* are formed of branches, with members strongly related to each other (either by the lectures or by the lecturers). For example, Faculty of Chemical Engineering, grades I and II.

For the above *groups* several transportation problems are set and solved at each time. Those branches are selected for a *group* and for a time which have lectures lasting 1, 2, 3, etc. hours, then those which have exercises beginning at the same time and lasting 1, 2, 3, etc. hours.

To illustrate the procedure let us consider, as an example, lectures lasting two hours. Thus, the *active* branches are to be selected for a given *group* and for a given time. (Branches have two hours lesson beginning at a given time.)

Then the *active* lecturers are allocated to the active branches so as to be in charge of any of the branches in question, to have unsatisfied lectures and to be free at the given time for the two hours to come, but not yet appointed to lecture to the branch in question that day. The 'receivers' in the transportation problem are the active branches, the 'transporters' are the active lecturers, according to the above interpretation.

Next, the coefficients of the objective function have to be constructed for the given time. Setting up the coefficients of the objective function basically depends on the requirement for the given subject to reach a satisfactory level to the time of examination (similarly as in Phase 2). This means that the subjects are basically equivalent and the provided allocations are made at a didactically proper time. In setting the coefficients of the objective function, numerous requirements can be considered. In the previous 'transportation table' a lecturer does not give lectures to all the branches (and in that this is not a typical transportation problem for the 'transporters' are not equivalent). Thus, transportation is made at low benefit to the 'improper' branches, but economically to the 'proper' branches.

The transportation problem for the exercises is more cumbersome than that. Here, the branches may have to be divided into smaller units for the departments cannot accomodate all the study groups at the same time. Therefore, the seminar leaders have to be appointed to the groups personally.

In the case where no active lecturers are available to a certain active branch because of coincidences at a given time (at a rather low possibility but no impossibility) then Phase 3 and occasionally Phase 2 should be repeated with altered data set.

Evidently, this phase also begins with sorting the corresponding data from the data basis and terminates with feeding back the output to the data basis. However, Phase 3 is essentially a series of transportation problems. The objective function is reconstructed in each step. Among the requirements listed in Section 2, items 3, 8, 10, 11 are completely, items 4 and 9 are partly satisfied here.

Fourth phase

The actual rooms are allocated to the branches in this phase—except those allocated in Phase 1. The allocation is based on the completed time-table. It should be noted here that Phase 2 does not deal with real rooms but with room types.

The basic concept of allocation is as follows:

A room basis — i.e. a group of exercise rooms and lecture rooms, not too far from each other — is allocated to a group of accordingly selected branches. This basis should be created so as to satisfy only about 70—80 per cent of the requirement of the group. So it can be provided that all the groups fully utilize their room basis at a high probability and rooms currently in excess are exceptional. Buffer zones are created from the remainder (excess) rooms. All the groups can get one or more rooms from the nearest undisposed buffer zone, if needed.

In establishing lectures for the branches, the basic point is as follows. If there are several lectures consecutively — with no conflicting requirement — they may be held in the very same room. In the case of study groups and exercise rooms, the situation is similar.

The function of this phase and of the program ends by printing the time-tables and other output tables. Among the requirements listed, item 7 is satisfied here. The method of this phase is of heuristic character.

Conclusions

The advantages of making time-tables by computer are as follows:

— Opportunity for the University Board and for the deans to make ranking according to different points to be considered in making time-table.

— Maximum utilization of capacities.

— Consideration of different didactical points of view.

— Free time both for the students and the teachers.
— Time-tables in several varieties according to different points of view.
— Possibility for greater faculties to reckon with requirements under 1 to 11, which is impossible by manpower. This advantage is less for smaller faculties.
— The data basis established for making time-tables can be used for a University registration system.

In using this program, lots of preparing, data processing and controlling jobs are needed, especially in the very first year of introduction, till all the data needed are stored in the proper data carrier. In the subsequent years only the modifications have to be applied.

Summary

A procedure has been developed for creating time-tables for all educational units of the Technical University, Budapest, with certain stipulations (the maximum utilization of room capacities, didactical points of view, interest of students, etc.).

A linear modell has been set up (linear boundary conditions, linear objective function). The great dimensions imposed a decomposition. Relatively small transportation problems have been solved for each phase and time, making up the time-table. The proposed program consists of four phases.

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