ASYMMETRICAL THREE-PHASE AND SINGLE-PHASE INDUCTION MACHINES

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Introduction

Due to their reliability and simple manufacturing technology, squirrel cage induction motors are one of the most generally applied electrical machines. Squirrel cage rotors for high performance are usually manufactured with copper rotor bars and end rings, while machines of lower performance are being manufactured by die-casting technologies. Examination of die-cast rotors have shown impurities, asymmetries due to technological difficulties.

Presence of asymmetries in the squirrel cage rotor is however not always of stochastical nature, but they can be intentionally produced. In case of a fractional hp. single-phase induction motor, for light-load starting applications a motor can be used, where the stator has a single winding and different types of asymmetries exist in the bars or end-rings. In this case the starting torque can be produced without using any auxiliary phase impedance. Asymmetry in the rotor circuit is very detrimental to the performance of the machine, so it is important to study its effect.

In technical literature there are few papers on the problem of single-phase and three-phase squirrel cage induction motors with rotor asymmetries. A main reason is that all the papers deal with symmetrical types of asymmetries where only one asymmetrical place is present, or the asymmetrical places are symmetrically spaced on the rotor. But investigations showed that most of the practically encountered asymmetries due to technological reasons are not symmetrical types. For these cases steady state equivalent circuits can be derived, which contain also controlled generators, and the only attempt to introduce them in the theory of asymmetrical electrical machines is due to the author.

WEICHSEL [1] presented a theoretical analyzis of a three-phase induction motor where the end rings had cuts 360 electrical degrees apart, and cuts in the front ring were displaced by 180 electrical degrees against those in the back ring. An approximate calculation of current distribution was made for the steady state, but no equivalent circuit was derived. Also the current distribution of a squirrel cage was determined by SCHUISKY [2] with a model where the rotor was slotless and had thin currentsheet on its surface.

By the application of transmission-line theory, squirrel cage induction motors with asymmetrical rotor circuit were analyzed by JORDAN and SCHMITT [3] for the case of a broken rotor bar. HILLER [4] applied the same theory as [3] without reference to the work of JORDAN and SCHMITT. His model does not take the additive currents of rotor bars neighbouring the broken one correctly into consideration, when stating that the absolute values of these currents are equal. It can be shown that this holds only if the machine is at standtill, as only in this case are the positive and negative sequence symmetrical component impedances of the machine equal.

Special squirrel-cage asymmetry of single-phase induction motor has been discussed by GROTSTOLLEN and SCHROEDER [5], where some of the rotor slots are not filled with conducting material. The derived "general" equivalent circuit is erronous as it omits the higher stator and rotor time harmonics, which actually exist due to the two-side asymmetry. An intentionally caused rotor assymmetry of single-phase induction motors has been discussed by SUBBA RAO, TRIVEDI and DESAI [6, 7] applying the crossfield theory.

PEWEZ DE VERA and PAGANO applied the symmetrical component theory [8] for analyzing the behaviour of a three-phase squirrel cage induction motor where some of the adjacent rotor bars were broken. In the derived equivalent circuit, the coupling network which connects the positive and negative sequence impedances of the machine holds only for symmetrical type of asymmetries. Also higher symmetrical component impedances were neglected. VAS [9] extended the foregoing theory for such cases where the neighbouring rotor bars had different impedances, and TŐKE and VAS [10] derived all the symmetrical component impedances of the m-rotor phase machine.

Operational characteristics of wye-connected slip-ring induction motors with rotor asymmetries have been discussed vy J. VAS and P. VAS applying symmetrical component and crossfield theories. Iron losses and impedance of the supply network were also taken into consideration. A new equivalent circuit for steady state and constant speed transient operation was derived, equations of currents and powers were given in a ready-to-calculate form. Also the d, q operator impedances of the machine were presented.

Transient and steady state operation of induction motors with general stator, rotor and two-side asymmetries were discussed by J. VAS and P. VAS [12, 13] and [14] deriving new general steady state equivalent circuits. For the first time in electrical machine theory, the coupling impedances also consisted controlled generators, to cope with the general type of asymmetry. Differential equations of asymmetrical machine were given in state-variable form. At present it will be shown how this can be applied for three-phase and single-phase induction motors, for a squirrel cage machine with general type of asymmetries. In this case the rotor asymmetries can be such, that the broken rotor bars are not adjacent. No noise and vibration analysis of asymmetrical squirrel cage induction motors will be made but reference is given to HANAFI and JORDAN [16] whose work is based on the Doctors Theses by HANAFI [15].

In the followings an induction motor with general type of rotor asymmetry will be discussed where arrangement of the broken rotor bars can be arbitrary. Asymmetries with rotor bars not broken but having different impedances — due to die-casting — and asymmetrically displaced will be discussed in a subsequent paper. As the general type of assymetry involves a simultaneous fault in the rotor circuit, fault location method — using symmetrical components — could lead to very complicated equivalent circuits, therefore it is not used. This method can however be effectively used in case of a small degree of rotor asymmetry [10].

Assumptions will be the same as in general electrical machinery theory [21], interbar currents are also neglected. A two-pole machine with m-phase rotor with symmetrical and identical end rings is assumed. In this case the rotor is equal with a wye-connected m-phase system. Relationships between the symmetrical component rotor quantities permit the derivation of the coupling impedance network due to asymmetry. The symmetrical component equation of rotor currents is

$$I'_r = \mathbf{Y}'_r\, U'_r \tag{1}$$

where I'_r is the column vector of symmetrical component rotor currents, $\mathbf{Y}'_r$ is the symmetrical component admittance matrix, and U'_r is the symmmetric component rotor voltage column vector:

$$I'_r = \begin{bmatrix} I'_0 \\ I'_1 \\ \vdots \\ I'_{m-1} \end{bmatrix} \quad U'_r = \begin{bmatrix} U'_0 \\ U'_1 \\ \vdots \\ U'_{m-1} \end{bmatrix} \quad Y' = \begin{bmatrix} Y'_0 & Y'_{m-1} & Y'_{m-2} \dots & Y'_1 \\ Y'_1 & Y'_0 & Y'_{m-1} \dots & Y'_2 \\ \vdots & & & \vdots \\ Y'_{m-1} & Y'_{m-2} & Y'_{m-3} \dots & Y'_0 \end{bmatrix} \tag{2}$$

It should be emphasized that in (2) the symmetrical component admittances Y'_i are not the inverses of the symmetrical component impedances of the machine, but the symmetrical component admittances of the m-phase rotor circuit. Equation (1) was intentionally written in the given form, using the symmetrical component admittance matrix, as the elements of this matrix $(Y'_i\ i = 0, 1 \dots m-1)$ are easy to realize in case of general type of rotor asymmetries. The symmetrical components are obtained from the phase-co-ordinates by applying the m-phase symmetrical component transformation:

$$\mathbf{T}^{-1} = \frac{1}{\sqrt{m}} \begin{bmatrix} 1 & 1 \dots\dots & 1 \\ 1 & \varepsilon_m & \varepsilon_m^{(m-1)} \\ \vdots & \vdots & \vdots \\ 1 & \varepsilon_m^{(m-1)} & \varepsilon_m^{(m-1)(m-1)} \end{bmatrix} \tag{3}$$

where $\varepsilon_m = \exp [j2\pi/m]$. The inverse transformation(matrix) of Eq. (3) is a modal matrix of an m-phase impedance matrix showing cyclic symmetry. The columns of the modal matrix are eigenvectors in a reference frame of eigendirections of the cyclic symmetrical matrix. Therefore the co-ordinates in the eigendirections (symmetrical components) expressed in terms of the real phase co-ordinates are:

$$Y'_i = \frac{1}{m} \sum_{k=1}^{m} Y_k \, \varepsilon_m(k-1)i \qquad i = 0, 1 \ldots\ldots m-1 \tag{4}$$

For a stator winding not sinusoidally distributed, application of Eq. (1) and of the 0, 1,... $m-1$ sequence symmetrical component impedances of the induction motor [10] permits to calculate the performance of the machine but only a very complicated equivalent circuit is derived. If the stator is sinusoidally distributed, since no zero zero sequence currents flow in the wye-connected rotor, from Eq. (1):

$$\begin{bmatrix} 0 \\ I'_1 \\ I'_{m-1} \end{bmatrix} = \begin{bmatrix} Y'_0 & Y'_{m-1} & Y'_1 \\ Y'_1 & Y'_0 & Y'_2 \\ Y'_{m-1} & Y'_{m-2} & Y'_0 \end{bmatrix} \begin{bmatrix} U'_0 \\ U'_1 \\ U'_{m-1} \end{bmatrix} \tag{5}$$

Solution of (5) for the symmetrical component voltages separating the zero sequence equation leads to

$$\begin{bmatrix} U'_1 \\ U'_{m-1} \end{bmatrix} = \begin{bmatrix} Z'_0 & Z'_2 \\ Z'_{m-1} & Z'_0 \end{bmatrix} \begin{bmatrix} I'_1 \\ I'_{m-1} \end{bmatrix} \tag{6}$$

Here the symmetrical component impedances–elements of the coupling network connecting the positive and negative sequence impedances of the machine–are:

$$Z'_0 = R'_0 + jX'_0 = \frac{Y'^2_0 - Y'_1 Y'_{m-1}}{\det \mathbf{Y}'} \tag{7}$$

$$Z'_2 = R'_2 + jX'_2 = \frac{Y'_2 Y'_{m-1} - Y'_0 Y'_1}{\det \mathbf{Y}'}$$

and Z'_{m-1} differing from Z'_2 in case of general asymmetry

$$Z'_{m-1} = R'_{m-1} + jX'_{m-1} = \frac{Y'^2_{m-1} - Y'_0 Y'_{m-2}}{\det \mathbf{Y}'}$$

The determinant of the symmetrical component admittance matrix of (5) is:

$\det \mathbf{Y}' = Y'^3_0 - Y'_0 Y'_2 Y'_{m-2} - Y'_0 Y'_1 Y'_{m-1} + Y'^2_1 Y'_{m-2} + Y'^2_{m-1} Y'_2 - Y'_0 Y'_1 Y'_{m-1}$.

The admittances Y'_i can be obtained from Eq. (4).

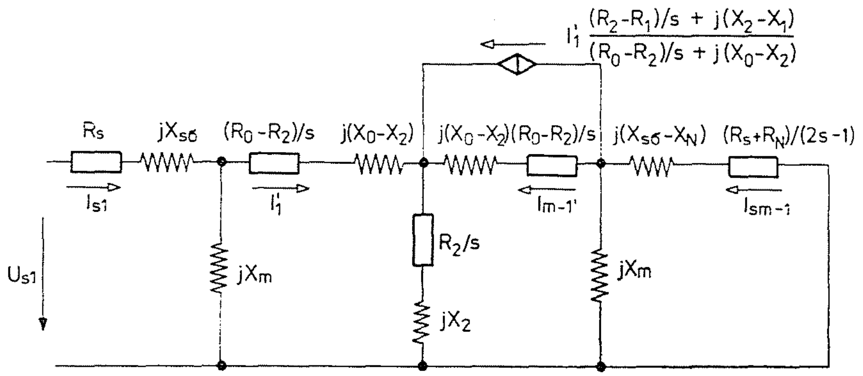

Fig. 1. General steady state equivalent circuit of three-phase induction motor with symmetrical squirrel cage (realization with impedance parameters)

Equation (6) may yield a general steady stateequ ivalent circuit of the machine. The coupling network may consist of one or two controlled generators, due to general type of asymmetry, the former case will be examined. Forty different connections can be realized by symmetrical or asymmetrical π or T networks. Figure 1 shows the general steady state equivalent circuit of the asymmetrical machine, stator quantities and network parameters have subscripts s and N, respectively. All rotor quantities are referred to the stator, X_m is the magnetizing reactance, the generator is a current controlled one.

The coupling network can be realized also by directly using the admittance components, from (5), separating the zero sequence equation:

$$\begin{bmatrix} I_1' \\ I_{m-1}' \end{bmatrix} = \begin{bmatrix} Y_0' & Y_2' \\ Y_{m-2}' & Y_0' \end{bmatrix} \begin{bmatrix} U_1' \\ U_{m-1}' \end{bmatrix} \tag{9}$$

and a coupling equivalent circuit is shown in Fig. 2.

If the asymmetry is of the symmetrical type, and broken rotor bars are neighbouring, the controlled generators will disappear and equivalent circuits will be similar to that in [9]. If the rotor circuit consists of N_r rotor

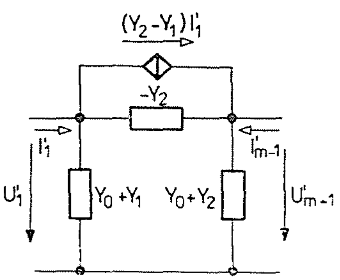

Fig. 2. Coupling network realized by symmetrical component admittances

bars, and x adjacent bars are broken, for e.g. an odd number of broken rotor bars:

$$Y_1' = \frac{Y_B}{m}\left[1 + \sum_{i=1}^{(N_r-x-1)/2} \exp\,(ji2\pi/m) + \sum_{i=1}^{(N_r-x-1)/2} \exp\,(-j2\pi/m)\right]$$

$$Y_{m-1}' = \frac{Y_B}{m}\left[1 + \sum_{i=1}^{(N_r-x-1)/2} \exp\,(ji4\pi/m) + \sum_{i=1}^{(N_r-x-1)/2} \exp\,(-j4\pi/m)\right] \quad (10)$$

$$Y_0' = \frac{Y_B}{m}(N_r - x)$$

where Y_B is the admittance of one rotor bar (taking into account the end ring segments).

For a small number of broken rotor bars:

$$Y_1' \approx 0$$

$$Y_{m-1}' \approx 0 \quad (11)$$

and

$$Y_0' \approx \frac{Y_B}{m} N_r$$

trivial in case of a three-phase rotor as then

$$Y_0' = \frac{Y_B}{3}(1 + 1 + 1) = Y_B$$

$$Y_1' = \frac{Y_B}{3}(1 + a + a^2) = 0 \quad (12)$$

$$Y_2' = \frac{Y_B}{3}(1 + a^2 + a) = 0$$

(now Y_B is the admittance of a rotor phase).

Considering Eq. (7) and applying Eq. (11)

$$Z_0' \approx \frac{1}{Y_0'} = \frac{m}{Y_B(N_r - 1)} \quad (13)$$

so the simplified equivalent circuit of the machine will be analogue to that of a symmetrical machine (in agreement with [8]), however, now Z_0 is greater

$$Z_0' = \frac{m}{Y_B(N_r - 1)} > \frac{m}{N_r\, Y_B} \quad (14)$$

to be placed in the rotor branch of the equivalent circuit. The foregoing theory can also be applied in case of general type asymmetry and if the machine is a double-cage induction motor.

Rotor asymmetry of double-cage machine

If the cage system has a single end-ring at both ends of the bars, then

$$Y_B = Y_{Be} + Y_{Bi} \tag{15}$$

and in Eq. (4)

$$Y_k = Y_r = \frac{(Y_{Be} + Y_{Bi})\, Y_{s\sigma}}{Y_{Be} + Y_{Bi} + Y_{s\sigma}} \tag{16}$$

where Y_{Be} and Y_{Bi} are the admittances of the outer and inner rotor bars, and $Y_{s\sigma}$ is the admittance due to the leakage flux coupling the outer and inner rotor bars. For inner and outer rotor bars connected to different end rings, a symmetrical component voltage equation has to be set up for both the outer and inner cage system. After the necessary substitutions, following the method of [8]

$$\begin{bmatrix} U'_1 \\ U'_{m-1} \end{bmatrix} = \begin{bmatrix} Z'_{D0} & Z'_{Dm-1} \\ Z'_{D2} & Z'_{D_m0} \end{bmatrix} \begin{bmatrix} I'_1 \\ I'_{m-1} \end{bmatrix} \tag{17}$$

similar to Eq. (6) –so a coupling network analogous to that in Fig. 1. can be realized, only the symmetrical component impedances due to the double-cage structure are:

$$\begin{aligned} Z'_{D0} &= \frac{1}{\det \mathbf{Z}_D} \{Z'_{0e}[Z'_{0e} + Z'_{0i} + Z'^2_{0i} - Z'_{1i}(Z'_{2e} + Z'_{2i})] + \\ &\quad + Z'_{2e}[Z'_{1i}(Z'_{0e} + Z'_{0i}) - Z'_{0i}(Z'_{1e} + Z'_{1i}) + jX_{ei}\} \\ Z'_{Dm-1} &= \frac{1}{\det \mathbf{Z}'_D} \{Z'_{0e}[Z'_{2i}(Z'_{0e} + Z'_{0i}) - Z'_{0i}(Z'_{2e} + Z'_{2i})] + \\ &\quad + Z'_{2e}[Z'_{0e}\ Z'_{0i} + Z'^2_{0i} - Z'_{2i}(Z'_{1e} + Z'_{1i})]\} \\ Z'_{D2} &= \frac{1}{\det \mathbf{Z}'_D} \{Z'_{0e}[Z'_{1i}(Z'_{0e} + Z'_{0i}) - Z'_{0i}(Z'_{1e} + Z'_{1i}) + Z'_{1e}[Z'_{0e} Z'_{0i} + \\ &\quad + Z'^2_{0i} - Z'_{1i}(Z'_{2e} + Z'_{2i})]\} \end{aligned} \tag{18}$$

and Z'_{D0} gets modified as:

$$\begin{aligned} Z'_{D_m0} &= \frac{1}{\det \mathbf{Z}_D} \{Z'_{0e}[Z'_{0e} Z'_{0i} + Z'^2_{0i} - Z'_{2i}(Z'_{1e} + Z'_{1i})] + \\ &\quad + Z'_{1e}[Z'_{2i}(Z'_{0e} + Z'_{0i}) - Z'_{0i}(Z'_{2i} + Z'_{2e})] + jX_{ei}\} \end{aligned}$$

in matrix (17) otherwise analogues to Eq. (6). Here det $\mathbf{Z}_D$ is the determinant of the impedance matrix (17) and X_{ei} is a reactance due to the mutual leakage flux coupling the inner and outer rotor bars. Z'_{0e}, Z'_{0i}, Z'_{1e}, Z'_{1i}, and Z'_{2e}, Z'_{2i}

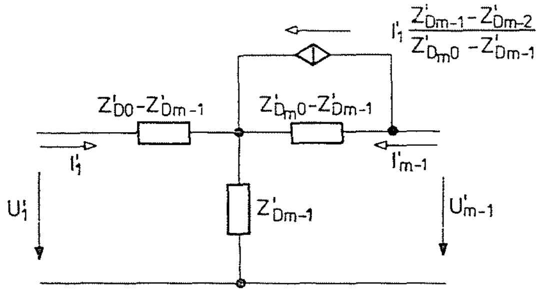

Fig. 3. Coupling network for double-cage machine with asymmetrical rotor

are the zero, positive and negative sequence component impedances of the outer and inner rotor bars. As $Z'_{D0} \neq Z'_{D_m0}$, the coupling network cannot be realized by a symmetrical T or π network containing controlled generators, but only by asymmetrical T or π networks. A possible network is shown in Fig. 3, helping to realize the total equivalent circuit of the asymmetrical double-cage induction motor.

If the degree of asymmetry is low, a simplified equivalent circuit can be derived, also analogous to the steady state equivalent circuit of the symmetrical machine. All the foregoing equivalent circuits are easy to extend for the case of constant speed operation by introducing the operator impedances applying the method of [11]. The currents and hence the performance of the machine can be calculated also for this case from the derived equivalent circuit.

Steady state operation of single-phase induction motors with rotor asymmetry in the squirrel cage

A new equivalent circuit was derived [14] for an induction motor with general two-side asymmetry, by using the symmetrical components, including all the higher time harmonics. In case of single-phase induction motors, asymmetries are sometimes introduced intentionally, in these cases symmetrical type of asymmetries exist. If the degree of asymmetry is low, the simplified equivalent circuit [14] can be changed to involve the symmetrical component rotor impedances, derived from Eq. (4). This new equivalent circuit is shown in Fig. 4. It is analogous to the steady state equivalent circuit of a single-phase induction motor with symmetrical rotor, only now in the rotor circuit the zero sequence rotor parameters have to be used. This is analogous to the fact that in case of three-phase slip-ring motors-if a small resistance asymmetry exists in the rotor — an average resistance $R = (R_a + R_b + R_c)/3$ can be used in the equivalent circuit of the symmetrical machine, R is seen to be the zero sequence resistance.

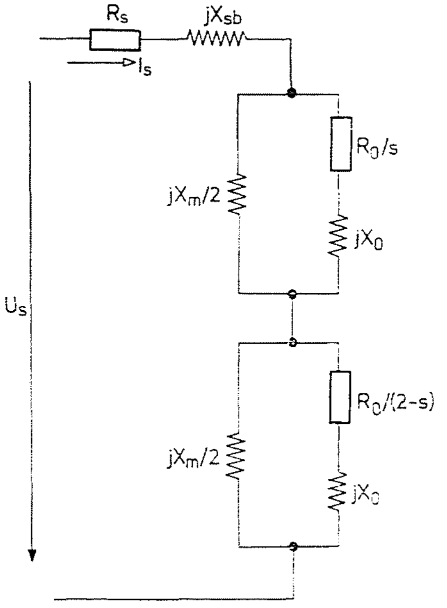

Fig. 4. Simplified steady state equivalent circuit of single-phase induction motor with asymmetrical squirrel cage if small degree of asymmetry exists

If the degree of asymmetry is not low, the general equivalent circuit of [14] has to be used taking our Eq. (4).

Transient operation of induction motors with motors general two-side impedance asymmetry

The general transient equations of the machine with general asymmetrical stator and rotor circuit can be derived, by application of the state-variable Park vector method first presented in [20]. In [14] the transient equations of a machine with two-side asymmetry were derived, but only resistance asymmetry existed in both sides. Now reactance asymmetry is also assumed, the machine is wye-connected on the stator, and the rotor can be of the slip-ring or squirrel cage type. Using the well-known assumptions [18, 21], Park vector equations of a three-phase symmetrical ac machine, rotating in a reference frame at an arbitrary varying ω_a speed:

$$\begin{aligned} \bar{u}_s &= R_s \bar{i}_s + \frac{d\bar{\psi}_s}{dt} + j\omega_a \bar{\psi}_s \\ \bar{u}_r &= R_r \bar{i}_r + \frac{d\bar{\psi}_r}{dt} + j(\omega_a - \omega_r)\, \bar{\psi}_r \end{aligned} \tag{19}$$

where $\bar{u}_s$, $\bar{u}_r$, $\bar{i}_s$, $\bar{i}_r$ and $\bar{\psi}_s$ and $\bar{\psi}_r$ are the stator and rotor voltage and flux vectors.

The fluxes are

$$\bar{\psi}_s = L_s \bar{i}_s + L_m \bar{i}_r$$
$$\bar{\psi}_r = L_m \bar{i}_s + L_r \bar{i}_r \tag{20}$$

here L_s and L_r are the stator and rotor inductances, L_m is the mutual inductance between the stator and rotor. The torque is

$$m = \frac{3}{2} p(\bar{\psi}_s \, x\bar{i}_s) \tag{21}$$

where p is the number of pole pairs. Assuming asymmetrical stator and rotor, these equations can be resolved into d,q component equations. As the voltage equations consist of the derivative of both stator and rotor currents, for sake of simplicity the differential equations are solved for the fluxes. Thus the state variable equation in a reference frame rotating at a speed ω_k is

$$\dot{x} = \mathbf{A}x + \mathbf{B}u \tag{22}$$

where x is the state vector of fluxes, $\mathbf{A}$ is the transition matrix and $\mathbf{B}u$ the forced voltages:

$$x = \begin{bmatrix} \psi_{sd} \\ \psi_{sq} \\ \psi_{rd} \\ \psi_{rq} \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; \quad u = \begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} -1/T'_{sd} & \omega_k & k_{rd}/T'_{sd} & 0 \\ -\omega_k & -1/T'_{sq} & 0 & k_{rq}/T'_{sq} \\ k_{sd}/T'_{rd} & 0 & -1/T'_{rd} & \omega_k - \omega_r \\ 0 & k_{sq}/T'_{rq} & -\omega_k + \omega_r & -1/T'_{rq} \end{bmatrix} \tag{23}$$

where ω_r is the speed of the rotor. The following constants are for the first time introduced in the theory of induction motors:

$$k_{rd} = L_m/L_{rd} \quad ; \quad k_{rq} = L_m/L_{rq}$$
$$k_{sd} = L_m/L_{sd} \quad ; \quad k_{sq} = L_m/L_{sq} \tag{24}$$

and the time constants are

$$T'_{sd} = L'_{sd}/R_{sd} \quad ; \quad T'_{sq} = L'_{sq}/R_{sq}$$
$$T'_{rd} = L'_{rd}/R_{rd} \quad ; \quad T'_{rq} = L'_{rq}/R_{rq} \tag{25}$$

where the stator and rotor transient, d, q inductances are

$$L'_{sd} = L_{sd} - L_m^2/L_{rd}$$
$$L'_{sq} = L_{sq} - L_m^2/L_{rq}$$
$$L'_{rd} = L_m^2/L_{sd} - L_{rd}$$
$$L'_{rq} = L_m^2/L_{sq} = L_{rq} \tag{26}$$

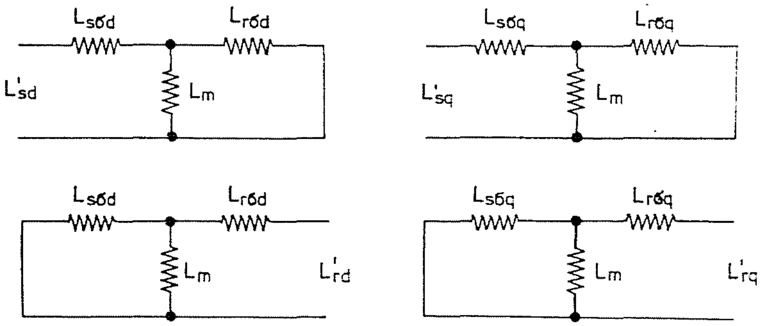

Fig. 5. Transient d,q stator and rotor inductances of induction motor

Figure 5 shows the physically realized equivalent circuits of the transient inductances d,q denoting

$$L_{sd} = L_{s\sigma d} + L_m$$
$$L_{sq} = L_{s\sigma q} + L_m \tag{27}$$
$$L_{rd} = L_{r\sigma d} + L_m$$
$$L_{rq} = L_{r\sigma q} + L_m$$

If the equation of motion is also included in the state-variable differential equation (20), and also load torque and friction torque are neglected (although they were easy to consider), an other state-variable equation is derived:

$$\dot{x}' = \mathbf{A}'x' + \mathbf{B}'u' \tag{28}$$

where

$$x' = [x, \omega_r, \alpha]_t$$

(α is the rotor angle and t holds for the transpose)

$$\mathbf{B}' = [\mathbf{B}, \mathbf{0}_{2t}, \mathbf{0}_{2t}]_t$$

here $\mathbf{0}_2$ means a zero matrix of second order, and

$$\mathbf{A} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{M} & \mathbf{A}_1 \end{bmatrix} .$$

The submatrices are:

$$\mathbf{0} = [\mathbf{0}_2, \mathbf{0}_2]_t \quad ; \quad \mathbf{A}_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

and

$$\mathbf{M} = \begin{bmatrix} -1.5p\,\theta^{-1} L_{sq}\, k_{rq}\, \psi_{rq} + \psi_{sq}(L'_{sd} - L'_{sq}) & 1.5p\,\theta^{-1}\, k_{rd}\, \psi_{rd}\, L'^{-1}_{sd} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} .$$

The derived Eq. (22) and (28) can be directly solved by a digital computer, using routine technics for solving differential equations. The components of d and q axes are ready to obtain from the symmetrical components in case of single-phase and three-phase induction motors with asymmetrical rotor if the machine has three-phase or m-phase (squirrel cage) rotor. For example, if the machine is of the slip-ring type and general rotor asymmetry exists- all the rotor impedances will differ (hence $R_a \neq R_b \neq R_c$ and $X_a \neq \neq X_b \neq X_c$);

$$R_d = R_0 + R_1 \quad ; \quad X_d = X_0 + X_1$$
$$R_q = R_0 - R_1 \quad ; \quad X_q = X_0 - X_1 \tag{29}$$

here the symmetrical components are

$$R_0 = (R_a + R_b + R_c)/3$$
$$R_1 = \left[R_a^2 + R_b^2 + R_c^2 - (R_a R_b + R_b R_c + R_a R_c)\right]^{1/2}/3$$

and

$$X_0 = (X_a + X_b + X_c)/3$$
$$X_1 = \left[X_a^2 + X_b^2 + X_c^2 - (X_a X_b + X_b X_c + X_a X_c)\right]^{1/2}/3 .$$

In case of rotor asymmetry of the squirrel cage, the symmetrical components can be calculated by the method discussed in the first part of this paper.

The derived equations hold for all types of two-side asymmetries of induction motors, if there is no angle asymmetry. The equations hold for single-phase machines too, but saturation effects were neglected. A following paper will show generalization of the equations for the case of saturation of both the main and the leakage flux paths. The state-variable differential equations will be extended for salient pole synchronous motors and for motors with asymmetrical airgap. Also extension of the derived equations will be given for saturated two-side asymmetrical induction or asynchronous machines if thyristor connections are in the stator or rotor or in both.

Summary

A general method is presented for calculating the steady state behaviour of three-phase and single-phase induction motors with general type for rotor asymmetries. Rotor asymmetry of single and double-cage machines is also discussed and new steady state equivalent circuits are derived which also contain controlled generators.

A general state-variable differential equation has been derived for calculating the transients of an induction motor with general two-side asymmetry. Application of the model for a slip ring machine with general rotor impedance asymmetry, and in case of asymmetrical squirrel cage is presented.

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