ZERO-SEQUENCE CURRENT DISTRIBUTION FOR DOUBLE CIRCUIT TRANSMISSION LINE WITH SINGLE GROUND WIRE*

By

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Introduction

Zero sequence current during normal and abnormal conditions is affected by various factors. STEPHEN A. SEBO [3] calculated this current when using a single circuit.

Since nowadays it is customery to use more than one circuit, the calculations are advisably made for the double circuit which is frequently used. The line is divided into spans and values are supposed for the zero-sequence and mutual impedances of lines, also for the ground wire. The calculations include also taking equivalent zero sequence impedance of ground return path, tower footing resistance and impedance of fault itself into consideration.

The zero sequence current is the same in the three phases of every circuit, so every circuit can be represented by a single wire. The values for voltages and currents are calculated at every span division of the line.

Symbols

I_p	zero sequence current flowing in phase conductor;
I_c; $(I_{c,k})$	ground wire current (in Kth span);
I_t; $(I_{t,k})$	current in (Kth span));
V_p; $(V_{p,k})$	voltage between phase conductor and ground (at the beginning of Kth span);
V_c; $(V_{c,k})$	voltage between ground wire and ground (at the beginning of Kth span);
Z_p; $(Z_{p,k})$	zero sequence self-impedance of phase conductors (in Kth span);
Z_m; $(Z_{m,k})$	zero sequence mutual impedance between phase conductors each, and, between conductors and ground wire (in Kth span);
Z_c; $(Z_{c,k})$	zero sequence self impedance of ground wire, (in Kth span);

* Based on research made at the Institute of Heavy Current Engineering.

$Z_g,(Z_{g,k})$ equivalent zero sequence impedance of ground return path upon every phase circuit (in Kth span);

Z_{FL} resulting impedance at faulty tower taking into account the driving point impedance of line section beyond fault and resistance of the faulty tower;

$R_t,(R_{t,k})$ tower resistance (of Kth span).

Relation between voltages and currents at the two ends of one span in the double-line circuit

Figure 1 gives an illustration of the double circuit line indicating direction of currents in both circuits in ground wire and return path of ground. The impedances illustrated give the final results taking the design of line with respect to conductors and configuration of erection into account.

Taking a line of n spans beginning from load end. and considering one span between two towers (K^{th} span), as shown in Fig. 1, the following relations can be deduced:

$$V_{p1,k+1} = V_{p1,k} + I_{p1}(Z_{p1,k} - Z_{g,k}) + (I_{p1} + I_{p2} - I_{c,k})\, Z_{g,k}$$
$$+ I_{p2}(Z_{m1,k} - Z_{g,k}) - I_{c,k}(Z_{m2,k} - Z_{g,k}) \quad (1)$$
$$V_{p1,k+1} = V_{p1,k} + I_{p1}\, Z_{p1,k} + I_{p2}\, Z_{m1,k} - I_{c,k}\, Z_{m2,k} \quad (2)$$

Similarly

$$V_{p2,k+1} = V_{p2,k} + I_{p1}\, Z_{m1,k} + I_{p2}\, Z_{m2,k} - I_{c,k}\, Z_{m3,k} \quad (3)$$
$$V_{c,k+1} = V_{c,k} - (Z_{c,k} - Z_{g,k})\, I_{c1,k} + I_{p1}(Z_{m2,k} - Z_{g,k})$$
$$+ I_{p2}(Z_{m3,k} - Z_{g,k}) + (I_{p1} + I_{p2} - I_{c,k})\, Z_{g,k}$$

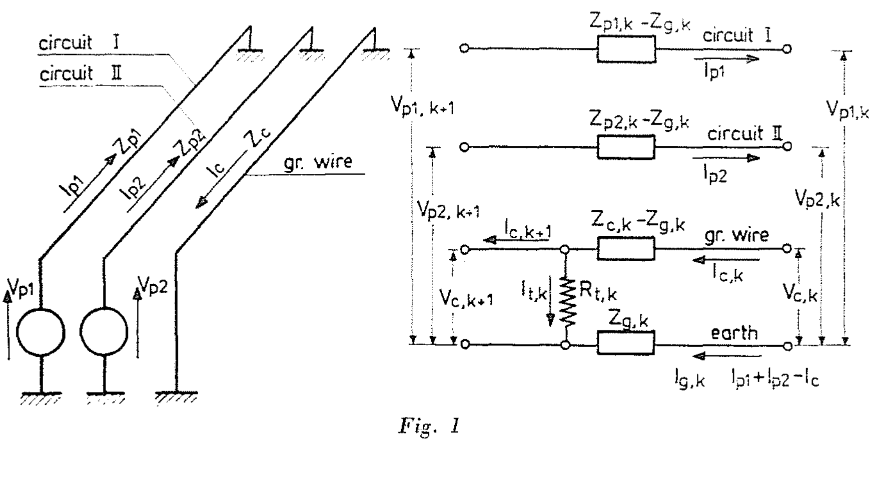

Fig. 1

$$V_{c,k+1} = V_{c,k} + I_{p1}\, Z_{m2,k} + I_{p2}\, Z_{m3,k} - I_{c,k}\, Z_{c,k} \tag{4}$$

$$I_{c,k+1} = I_{c,k} - I_{t,k}$$

$$I_{t,k} = \frac{V_{c,k+1}}{R_{t,k}}$$

$$I_{c,k+1} = I_{c,k} - \frac{V_{c,k+1}}{R_{t,k}}$$

$$I_{c,k+1} = I_{c,k} - \frac{V_{c,k}}{R_{t,k}} - \left(\frac{I_{p1}\, Z_{m2,k} + I_{p2}\, Z_{m3,k}}{R_{t,k}}\right) + I_{c,k}\frac{Z_{c,k}}{R_{t,k}}$$

$$= -\frac{V_{c,k}}{R_{t,k}} - I_{p1}\frac{Z_{m2,k}}{R_{t,k}} - I_{p2}\frac{Z_{m3,k}}{R_{t,k}} + I_{c,k}\left(1 + \frac{Z_{c,k}}{R_{t,k}}\right) \tag{5}$$

Since the zero-sequence current at the phase conductors is not changing,

$$I_{p1,k} = I_{p1,k+1} = I_{p1} \tag{6}$$

$$I_{p2,k} = I_{p2,k+1} = I_{p2} \tag{7}$$

Eqs. 2, 3, 4, 5, 6, and 7 can be written in matrix form as,

$$\begin{bmatrix} V_{p1,k+1} \\ V_{p2,k+2} \\ V_{c,k+1} \\ I_{p1} \\ I_{p2} \\ I_{c,k+1} \end{bmatrix} \begin{matrix} = \\ = \\ = \\ = \\ = \\ = \end{matrix} \begin{bmatrix} 1 & 0 & 0 & Z_{p1,k} & Z_{m1,k} & -Z_{m2,k} \\ 0 & 1 & 0 & Z_{m1,k} & Z_{m1,k} & -Z_{m3,k} \\ 0 & 0 & 1 & Z_{m2,k} & Z_{m3,k} & -Z_{c,k} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\dfrac{1}{R_{t,k}} & -\dfrac{Z_{m2,k}}{R_{t,k}} & -\dfrac{Z_{m3,k}}{R_{t,k}} & \left(1 + \dfrac{Z_{c,k}}{R_{t,k}}\right) \end{bmatrix} \begin{bmatrix} V_{p1,k} \\ V_{p2,k} \\ V_{c,k} \\ I_{p1} \\ I_{p2} \\ I_{c,k} \end{bmatrix} \tag{8}$$

Let

$$(E_{k+1}) = \begin{bmatrix} V_{p1,k+1} \\ V_{p2,k+1} \\ V_{c,k+1} \\ I_{p1} \\ I_{p2} \\ I_{c,k+1} \end{bmatrix} \qquad (E_k) = \begin{bmatrix} V_{p1,k} \\ V_{p2,k} \\ V_{c,k} \\ I_{p1} \\ I_{p2} \\ I_{c,k} \end{bmatrix}$$

$$\text{and } (S_k) = \begin{bmatrix} 1 & 0 & 0 & Z_{p1,k} & Z_{m1,k} & -Z_{m2,k} \\ 0 & 1 & 0 & Z_{m1,k} & Z_{p2,k} & -Z_{m3,k} \\ 0 & 0 & 1 & Z_{m2,k} & Z_{m3,k} & -Z_{c,k} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\dfrac{1}{R_{t,k}} & -\dfrac{Z_{m2,k}}{R_{t,k}} & -\dfrac{Z_{m3,k}}{R_{t,k}} & \left(1 + \dfrac{Z_{c,k}}{R_{t,k}}\right) \end{bmatrix}$$

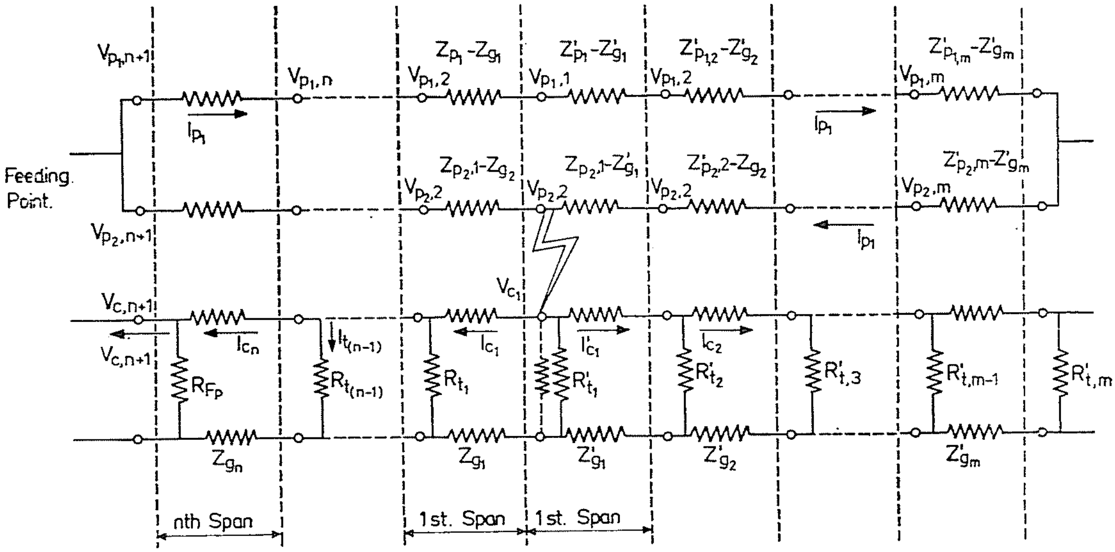

Fig. 2. Equivalent circuits of faulty line

then $$(E_{k+1}) = (S_k)\ (E_k) \tag{9}$$

or $$(E_k) = (S_k)^{-1}\ (E_{k+1}) \tag{10}$$

Now this method can be applied to a line consisting of n-spans and earth fault at one tower as shown in Fig. 2.

Distribution of zero-sequence current for double circuit transmission line when connected between a feeding point and static load

When an earth fault occurs at a point between the feeding point and the load, the zero sequence current in the sound conductor flows from the feeding point to load while in the faulty conductor a part flows from feeding point to fault location, the other in the direction from load to fault location.

$$(E_1) = \begin{bmatrix} V_{p1,1} \\ V_{p2,1} \\ V_{c1,1} \\ I_{p1} \\ I_{p2} \\ I_{c1} \end{bmatrix} = \begin{bmatrix} -I_{p1} \sum_{n=1}^{n} (Z'_{p1,k} - Z'_{g,n}) \\ Z_{FL}(I_{p1} + I_{p2} - I_{c1}) \\ Z_{FL}(I_{p1} + I_{p2} - I_{c1}) \\ I_{p1} \\ I_{p2} \\ I_{c1} \end{bmatrix} \tag{11}$$

From Eq. (9):

$$(E_2) = (S_1)(E_1)$$
$$(E_3) = (S_3)(E_2) = (S_2)(S_1)(E_1)$$
$$(E_{n+1}) = (S_n)(S_{n-1})(S_{n-2}) \dots (S_2)(S_1)(E_1) \tag{12}$$

Let

$$(S_{nn}) = (S_n)(S_{n-1}) \dots (S_2)(S_1)$$

then

$$(E_{n+1}) = (S_{nn})(E_1) \tag{13}$$

From Eqs (12) and (13):

$$\begin{bmatrix} V_{p1,n+1} \\ V_{p2,n+1} \\ V_{c,n+1} \\ I_{p1} \\ I_{p2} \\ I_{c,n+1} \end{bmatrix} = \begin{bmatrix} V_{p1,n+1} \\ V_{p2,n+1} \\ R_{FP}(I_{c,n} - I_{c,n+1}) \\ I_{p1} \\ I_{p2} \\ I_{c,n+1} \end{bmatrix}, \quad E_1 = \begin{bmatrix} -I_{p1} \sum_{n=1}^{n} (Z'_{p1,n} - Z'_{g,n}) \\ Z_{FL}(I_{p1} + I_{p2} - I_{c1}) \\ Z_{FL}(I_{p1} + I_{p2} - I_{c1}) \\ I_{p1} \\ I_{p2} \\ I_{c1} \end{bmatrix}$$

from which $V_{p1,n+1}$, $V_{p2,n+1}$ are seen to be functions of I_{p1}, I_{p2} and I_{c1} taking into account the multiplication: $(S_n) \cdot (S_{n-1}) \dots (S_3) \cdot (S_2) \cdot (S_1)$. This latter can be handled in case of a limited number of spans, but for long lines a computer must be used.

Now all the elements of E_1 are known. The values of currents and voltages can then be calculated for any desired span.

Conclusion

Esimating the zero sequence currents is usually required in power system analysis of circuits concerning unbalanced loading or unsymmetrical short circuits. In this respect the configuration of the overhead transmission lines as well as the type of erathing the corresponding towers are important factors in determining the distribution of the zero sequence currents in the transmission line. This article presents the distribution of the zero sequence current applied to double-circuit three-phase overhead transmission lines with ground wires. Similar analysis could be adopted to other overhead line systems.

Summary

Calculations are made to generalize the method for evaluating the zero-sequence current in single circuit to be used with the double circuit. The values of impedances in every circuit are considered for lines whether self or mutual, fault impedance, ground return path impedance, ground wire, tower footing and tower resistances.

The line is divided into spans and during normal conditions the relation between voltages and currents at the two ends of the double line is calculated. A fault is considered at a point on the line and the spans are arranged beginning from fault location. Values for line voltage earth wire voltage, currents in both lines and ground wire current are obtained. The results are written in a matrix form easy to be computed.

References

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