TOPOLOGICAL ANALYSIS OF TRANSMISSION NETWORKS

By

I. PÁVÓ

Research Group on Mathematical Logic and Theory of Automata of the Hungarian Academy of Sciences, Szeged

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Presented by Prof. Dr. I. VÁGÓ

Circuit equations of transmission networks

Let us consider a transmission network consisting of transmission lines and two-terminals with concentrated parameters [5]. Let b be the number of the transmission lines connected in n nodes with each other. Let the ith network line be given by its admittance matrix

$$\begin{pmatrix} p_i & r_i \\ r_i & p_i \end{pmatrix},$$

where

$$p_i = y_{0i} \ \text{cth}\ \gamma_i l_i \,,$$

$$r_i = -y_{0i} \frac{1}{\text{sh}\ \gamma_i\, l_i}\,,$$

y_{0i} is the wave admittance, γ_i the transmission coefficient and l_i the length of the ith line. Assume the two-terminal between the vertices of the jth node of the transmission network is given with passive admittance y_j together with voltage and current (ideal) generators u_{gj} and i_{gj}, resp., as usual. Figure 1 shows the closure of the jth node where j and j' mark the vertices of the node in question.

Suppose that none of the nodes is short circuited and mark by u_j the voltage between the vertices of the jth node (taking the direction in Fig. 1 into consideration). Marking the column vector of size n of the node voltages by U, if det $(\mathbf{Y}_c + \mathbf{Y}_g) \neq 0$, so according to [5] we can write:

$$U = (\mathbf{Y}_c + \mathbf{Y}_g)^{-1} (\mathbf{Y}_g\, U_g + I_g) \tag{1}$$

In formula (1) $\mathbf{Y}_g$ is a diagonal matrix of size n consisting of admittances of the passive two-terminals connected to the nodes, U_g and I_g are column vectors of size n of the source voltages and source currents, and $\mathbf{Y}_c$ is the node admittance matrix of the transmission network.

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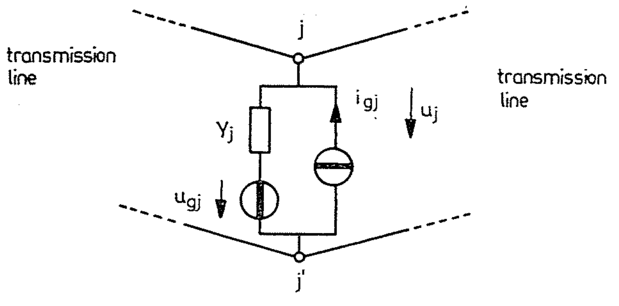


Fig. 1

It is known from [5] that:

$$\mathbf{Y}_c = \frac{1}{2}\,\mathbf{A}_{t0}(\mathbf{P} + \mathbf{R})\,\mathbf{A}_{t0}^{+} + \frac{1}{2}\,A_t(\mathbf{P} - \mathbf{R})\,\mathbf{A}_t^{+}\ , \tag{2}$$

where $\mathbf{P} = \langle p_1, \ldots, p_b \rangle$, $\mathbf{R} = \langle r_1, \ldots, r_b \rangle$ are diagonal matrices consisting of the parameters of the transmission lines, $\mathbf{A}_t$ and $\mathbf{A}_{t0}$ are the directed, and the undirected (non-reduced) incidence matrices resp., of the network graph. In formula (2) the superscript $+$ refers to the matrix transposition.

If the network graph, the two-terminal admittances connected to the nodes together the driving generators, and the parameters of the transmission lines are known, then using (2), from (1) each node voltage can be determined. We remark that from computer aspects, in using formula (1) it is necessary to determine the elements of an inverse matrix, that is, to calculate determinants and cofactors.

Further we shall point out that the node voltage calculation of a transmission network is possible in another way as well. Namely a substituting model to the network in question can be given (often a ladder network), whose node potentials are equal to the node voltages of the original network in that order. This model consists purely of passive elements and (ideal) generators. For calculating the node potentials of the model a topological formula can be given, so it is possible to use a purely topological method to determine the node voltages of transmission networks.

The concentrated element model of the network

Mark the nodes of the transmission network with natural numbers $1, \ldots, n$. Order a model to the network obtained from the original network by replacing the ith transmission line with a π-network term, with length and cross adminttances $-r_j$ and $p_i + r_i$, respectively. The parameter admittance matrix of the substituting π-network trivially equals the admittance matrix of the ith line [1]. We must take care that the π-network terms, substituting the lines joining the same vertex are connected in a common node.

The network model trivially holds the following two properties

1. The model often is a ladder network with $n+1$ nodes.
2. The cross admittances connected with the jth node can be reduced to a two-terminal of admittance parameter

$$s_j = \sum_k (p_k + r_k),$$

where subscript k refers to all subscripts of the transmission lines which are incident in the jth node of the original network.

We agree that the nodes of the model have the same mark as the nodes of the network, and the reference node of the model is marked by $n+1$.

Now we prove that the node potentials of the model are equal to the node voltages of the original network in that order.

According to [5], the model equation system of the model network is

$$V = \mathbf{Y}^{-1}\mathbf{A}(\mathbf{Y}'_g U'_g + I'_g)\,, \tag{3}$$

where V is the vector of the node voltages, Y is the node admittance matrix, $\mathbf{A}$ is the (reduced) incidence matrix of the model, Y'_g is the (diagonal) admittance matrix of the passive elements, U'_g and I'_g are vectors consisting of source voltage and current generators of the passive edges.

First let us consider the node admittance matrix of the model in question. For the purpose of calculating it let us introduce a direction of the network graph as follows: Let the direction of the passive edge with parameter $-r_i$ be the same as that of the corresponding graph edge in the transmission network. Direct each of the other passive edges towards the reference node. For writing down the reduced incidence matrix of the model, let the edges with admittance $-r_i$ occur in the first b columns of the matrix, further n columns contain the admittances s_j in that order, finally the last n columns refer to the edges of admittances Y_j. Then partly

$$\mathbf{A} = [\mathbf{A}_i\, \mathbf{1}\, \mathbf{1}]\,, \tag{4}$$

partly the passive edge admittance matrix of the model is:

$$\mathbf{T} = \begin{bmatrix} -\mathbf{R} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Y}_g \end{bmatrix} \tag{5}$$

where $\mathbf{S} = \langle s_1, \ldots, s_n \rangle$, $\mathbf{1}$ and $\mathbf{0}$ are unit and zero matrices of size n.

It is known from [3]:

$$\mathbf{Y} = \mathbf{A}\,\mathbf{T}\,\mathbf{A}^{+}. \tag{6}$$

Taking into account (4) and (5), from (6):

$$\mathbf{Y} = [\mathbf{A}_t\ \mathbf{1}\ \mathbf{1}] \cdot \begin{bmatrix} -\mathbf{R} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Y}_g \end{bmatrix} \cdot \begin{bmatrix} \mathbf{A}_t^+ \\ \mathbf{1} \\ \mathbf{1} \end{bmatrix} = [-\mathbf{A}_t\,\mathbf{R}\ \mathbf{S}\ \mathbf{Y}] \cdot \begin{bmatrix} \mathbf{A}_t^+ \\ \mathbf{1} \\ \mathbf{1} \end{bmatrix} =$$

$$= -\mathbf{A}_t\,\mathbf{R}\,\mathbf{A}^+ + \mathbf{S} + \mathbf{Y}_g. \tag{7}$$

After some calculation we get

$$\mathbf{S} = \frac{1}{2}\mathbf{A}_t\,\mathbf{P}\,\mathbf{A}_t^+ + \frac{1}{2}\mathbf{A}_{t0}\,\mathbf{P}\,\mathbf{A}_{t0}^+ + \frac{1}{2}\mathbf{A}_t\,\mathbf{R}\,\mathbf{A}_t^+ + \frac{1}{2}\mathbf{A}_{t0}\,\mathbf{R}\,\mathbf{A}_{t0}^+ \,. \tag{8}$$

Considering (7) and (8) we can write from (2)

$$\mathbf{Y} = \mathbf{Y}_c + \mathbf{Y}_g \,. \tag{9}$$

Because of det $(Y) \neq 0$, $\mathbf{Y}^{-1}$ really exists.

Secondly let us consider the factor of $\mathbf{Y}^{-1}$ in the right-hand side of Eq. (4). We can write:

$$\mathbf{A}(\mathbf{Y}_g'\,U_g' + I_g') = [\mathbf{A}_t\,\mathbf{1}\,\mathbf{1}]\left(\begin{bmatrix} -\mathbf{R} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Y}_g \end{bmatrix} \cdot \begin{bmatrix} O \\ O \\ U_g \end{bmatrix} + \begin{bmatrix} O \\ O \\ I_g \end{bmatrix}\right) \tag{10}$$

where O are zero vectors of size b or n. Write the right side of (10) as follows:

$$[\mathbf{A}_t\ \mathbf{1}\ \mathbf{1}]\left(\begin{bmatrix} O \\ O \\ \mathbf{Y}_g\,U_g \end{bmatrix} + \begin{bmatrix} O \\ O \\ I_g \end{bmatrix}\right)$$

After multiplying we obtain

$$\mathbf{A}(\mathbf{Y}_g'\,U_g' + I_g') = \mathbf{Y}_g\,U_g + I_g \,. \tag{11}$$

Having compared (9) and (11) to (1), our statement is proved.

For the sake of further applications, the property of the model proved above will be written as:

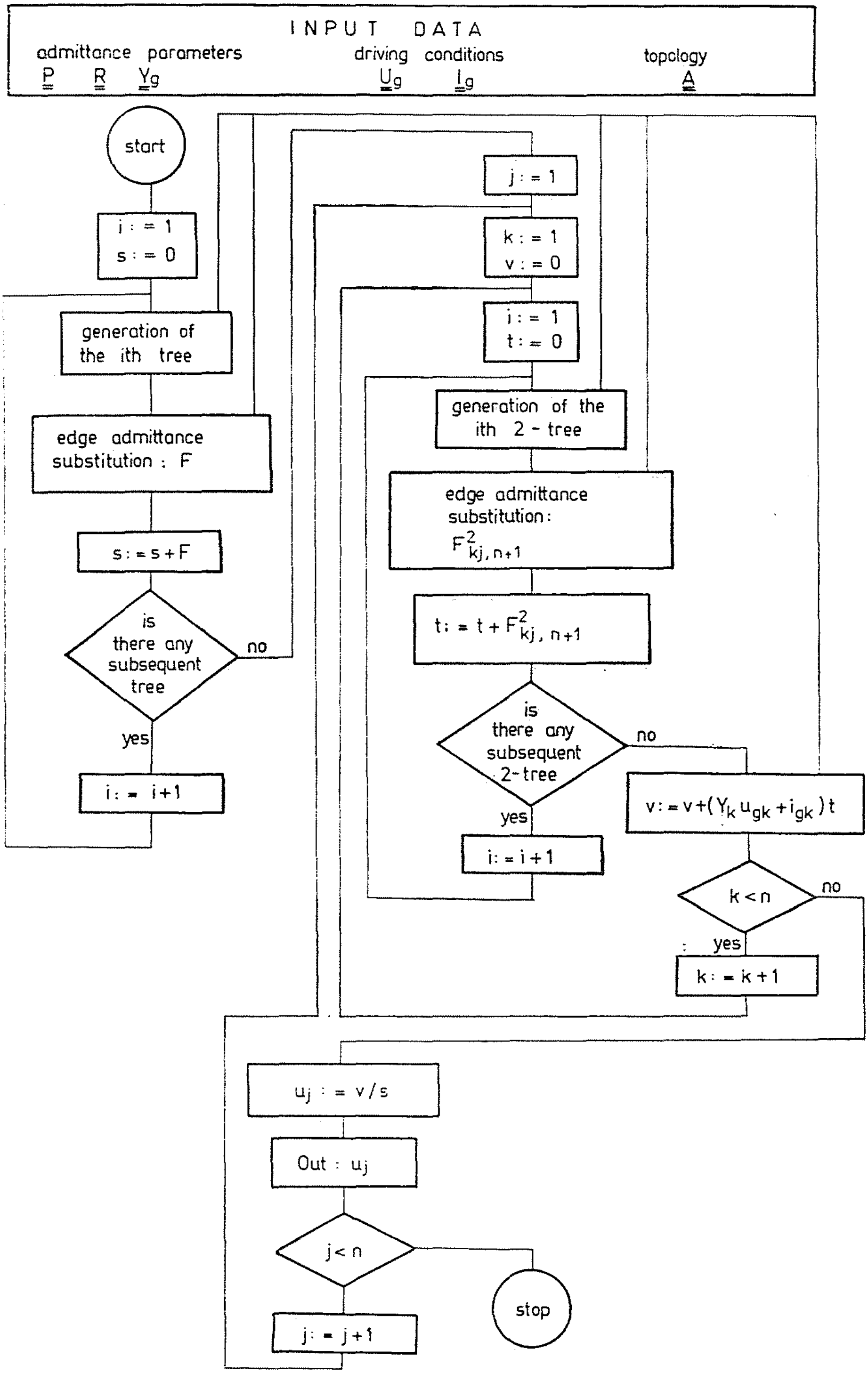
$$U = \mathbf{Y}^{-1}(\mathbf{Y}_g\,U_g + I_g)\,. \tag{12}$$

(12) is another form of (1). Formula (12) contains only the node admittance matrix from the model.

Finally, (12) will also be given terms of the topological formula. Therefore let us consider the jth equation of the system (12):

$$u_j = \frac{1}{\det(\mathbf{Y})}\sum_{k=1}^{n} \operatorname{adj}\ \mathbf{Y}_{kj}(Y_k\,u_{gk} + i_{gk}) \tag{13}$$

where det $(\mathbf{Y})$ marks the node determinant, and adj $\mathbf{Y}_{kj}$ the cofactor concerning the element Y_{kj} of matrix $\mathbf{Y}$.



INPUT DATA
admittance parameters
P R Yg
driving conditions
Ug Ig
topology
A
start
j: = 1
s: = 0
generation of the ith tree
edge admittance substitution : F
s: = s + F
is there any subsequent tree
no
yes
i: = i+1
j: = 1
k: = 1
v: = 0
i: = 1
t: = 0
generation of the ith 2 - tree
edge admittance substitution:
F²kj, n+1
t: = t + F²kj, n+1
is there any subsequent 2-tree
no
yes
i: = i+1
v: = v+(Yk ugk + igk) t
k < n
no
yes
k: = k+1
uj : = v/s
Out : uj
j < n
stop
j: = j+1

Fig. 2

It follows from [3]:

$$\det(\mathbf{Y}) = \Sigma F, \text{ and } \operatorname{adj} \mathbf{Y}_{kj} = \Sigma F^2_{kj,n+1}, \tag{14}$$

where F is a tree, $F^2_{kj,\,n+1}$ is a 2-tree of the model graph, the latter separates the vertices k and j from vertex $n+1$, and the summation affects all suitable tree and 2-tree edge admittance products.

According to (14), from (13) we get the topological formula of the node voltages:

$$u_j = \frac{1}{\Sigma F} \sum_{k=1}^{n} \left((Y_k\, u_{gk} + i_{gk})\, \Sigma F^2_{kj,n+1} \right) \qquad j = 1, \ldots, n. \tag{15}$$

For the numerical calculation of the voltage u_j see the detailed flow chart in Fig. 2 which can be used to make a computer program as well. We remark that calculating u_j by such a program one can practically use the k-trees generation method in [2].

Topological formula for transmission network with extreme closures

Formula (15) is not valid if some of the nodes are closed by short-circuit. Without violating the general case it is assumed that the short-circuit closure contains a source voltage generator but no parallel source current generator exists. To extend formula (15) for the general case our method is the following: To the transmission network in question another network is formed by substituting a (real) admittance Y for each short-circuit closure. Formula (15) is valid for this new network, if $\det(\mathbf{Y}) \neq 0$, which is supposed. Arrange the edge admittance polynomials in both the numerator and denominator of (15) according to the decreasing order of exponents Y. Let the degree of polynomial ΣF be m. At the same time m is the number of the short-circuit closures in the original network. It is clear that the degree of the numerator of (15) is not higher than m. If $Y \to \infty$ both the new network becomes the original one and the limit of (15) gives the node voltages of the transmission network we are interested in. As the right-hand side of (15) is a rational fractional function of Y, it is sufficient to determine only the coefficients of the largest exponent of Y both in the numerator and in the denominator.

For this reason mark the nodes of short-circuit closure by $n-m+1, \ldots, n$, the other nodes by $1, \ldots, n-m$, and apply formula (15), so we get:

$$u_j = \frac{1}{\Sigma F} \left(\sum_{k=1}^{n-m} (Y_k\, u_{gk} + i_{gk})\, \Sigma F^2_{kj,n+1} + \sum_{k=n-m+1}^{n} Y u_{gk}\, \Sigma F^2_{kj,n+1} \right), \tag{16}$$

$$j = 1, \ldots, n.$$

According to the remark concerning the coefficients, the generalized formula for calculating the node voltages is:

$$u_j = \frac{1}{\Sigma F^{m+1}} \left(\sum_{k=1}^{n-m} (Y_k \cdot u_{gk} + i_{gk}) \, \Sigma F^{m+2}_{kj,n+1} + \sum_{k=n-m+1}^{n} u_{gk} \cdot \Sigma F^{m+1}_{kj,n+1} \right), \qquad (17)$$

$$j = 1, \ldots, n \, .$$

In formula (17) the meanings of the k-trees are:

F^{m+1} marks an $(m + 1)$-tree of the model graph which becomes a connected circuitless graph after fusing one by one the vertices of the short-circuited nodes; $F^{m+2}_{kj,n+1}$ is an $(m + 2)$-tree which separates vertices k and j from $n + 1$ after the former fusion; finally $F^{m+1}_{kj,n+1}$ means an $(m + 1)$-tree which also separates vertices k and j from $n + 1$ but now the fusion does not affect the kth node.

Observe that from (17):

$$u_j = u_{gj}$$

if $j = n - m + 1, \ldots, n$, that is, the voltage of the short-circuited node is trivially equal to the voltage of the corresponding source generator, moreover in case $m = 0$, formula (17) turns into formula (15), so (17) is really more general than (15).

Topological formula for the driving-point impedance matrix of the transmission network

For the determination of characteristic matrices of a transmission network see reference [6]. Now use (17) for determining elements of the driving-point impedance matrix of a transmission network by a topological formula.

Let us consider a transmission network with nodes closed by passive two-terminals. Let the nodes be ordered in three groups. Nodes belong to the first group if they can be joined to other networks, i.e. they are driving-point nodes; nodes of the second group are closed by finite admittance; and the other nodes closed by short-circuit belong to the third group. We agree that nodes of the first group are marked by $1, \ldots, l$, those of the second group by $l + 1, \ldots, n - m$, and of the third group by $n - m + 1, \ldots, n$ where n is the number of the nodes, m is the number of the short-circuits in the transmission network.

For determining the matrix elements connect source current generators i_{gk} with the driving-point nodes, where k is the mark of the node $k = 1, \ldots, l$. Taking into account that the network contains no source voltage generators, all $u_{gk} = 0$, where $k = 1, \ldots, n$. Referring to (17) we can write:

$$u_j = \frac{1}{\Sigma F^{m+1}} \sum_{k=1} i_{gk} \cdot \Sigma F^{m+2}_{kj,n+1} + \sum_{k=l+1}^{n-m} (Y_k \cdot 0 + 0)\, \Sigma F^{m+2}_{kj,n+1} +$$
$$+ \sum_{k=n-m+1}^{n} 0 \cdot \Sigma F^{m+2}_{kj,n+1}, \quad j = 1, \ldots, l\,.$$

So we have got:

$$u_j = \frac{1}{\Sigma F^{m+1}} \sum_{k=1}^{l} i_{gk} \cdot \Sigma F^{m+2}_{kj,n+1}, \qquad j = 1, \ldots, l\,. \tag{18}$$

From the definition of the driving-point impedance matrix and from (18) taking $F^{m+2}_{kj,n+1} = F^{m+2}_{jk,n+1}$ into account it follows:

$$\mathbf{Z} = (z_{ij}) = \left(\frac{\Sigma F^{m+2}_{ij,n+1}}{\Sigma F^{m+2}}\right), \qquad i, j = 1, \ldots, l\,. \tag{19}$$

Again we suggest the use of the method in [2] for the production of elements z_{ij} of the impedance matrix by topological formula.

Applications

1. Consider the transmission network in Fig. 3. Let us calculate node voltages u_j $(j = 1, 2, 3)$ by a topological formula.

The network has two transmission lines with given admittance parameters (see Fig. 3).

Figure 4 shows the network model and its graph, edges of which are indicated by the corresponding edge admittances from the model.

According to formula (15) in the present situation we can write:

$$u_j = \frac{i_{g3} \cdot \Sigma F^2_{3j,4}}{\Sigma F}, \quad j = 1, 2, 3. \tag{20}$$

In order to calculate u_j we need k-trees F, $F^2_{31,4}$ $F^2_{32,4}$ and $F^2_{3,4}$ of the model graph.

Observing the model graph in Fig. 4 and the meaning of the edge admittances:

$$\Sigma F = (Y_1 + s_1)\ s_2 s_3 + (Y_1 + s_1)\ s_2(-r_1 - r_2) + (Y_1 + s_1)\ s_3(-r_2) +$$
$$+ s_2 s_3(-r_1) + (Y_1 + s_1) r_1 r_2 + s_2 r_1 r_2 + s_3 r_1 r_2\,,$$
$$\Sigma F^2_{31,4} = r_1 r_2 - r_1 s_2,\ \Sigma F^2_{3,2,4} = r_1 r_2 - r_2(Y_1 + s_1)\,,$$

and finally:

$$\Sigma F^2_{3,4} = (Y_1 + s_1) s_2 + r_1 r_2 - r_1 s_2 - r_2(Y_1 + s_1)\,.$$

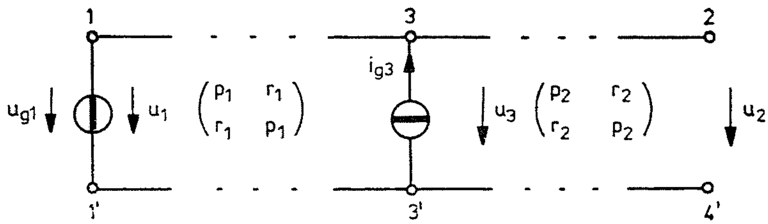


Fig. 3

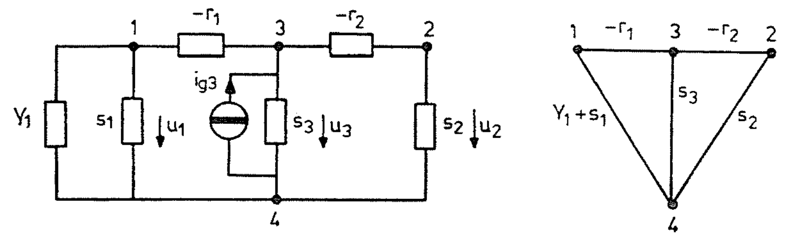


Fig. 4

After some calculation we have the following formulas for the edge admittance products:

$$\Sigma F = p_1 p_2 Y_1 + (p_2^2 - r_2^2)(p_1 + Y_1) + p_2(p_1^2 - r_1^2)\,, \tag{21}$$

$$\Sigma F^2_{31,4} = -r_1 p_2, \; \Sigma F^2_{32,4} = -r_2(Y_1 + p_1)\,, \tag{22}$$

and $\quad \Sigma F^2_{3,4} = p_2(p_1 + r_1 + Y_1)\,.$

Taking into account (21) and (22) we have got from (20):

$$u_1 = \frac{-r_1 p_2 i_{g3}}{p_1 p_2 Y_1 + (p_2^2 - r_2^2)(p_1 + Y_1) + p_2(p_1^2 - r_1^2)}$$

$$u_2 = \frac{-r_2(p_1 + Y_1) i_{g3}}{p_1 p_2 Y_1 + (p_2^2 - r_2^2)(p_1 + Y_1) + p_2(p_1^2 - r_1^2)}$$

$$u_3 = \frac{p_2(p_1 + r_1 + Y_1) i_{g3}}{p_1 p_2 Y_1 + (p_2^2 - r_2^2)(p_1 + Y_1) + p_2(p_1^2 - r_1^2)}\,.$$

2. Consider the transmission network in Fig. 5. Calculate the node voltages u_j ($j = 1, 2, 3$).

The network in question has extreme closures in the 1st and the 2nd node. However, network in Fig. 5 differs from that calculated earlier only by the closure of the 1st node (see Fig. 3). But now, formula (15) cannot be used because of the short-circuit closure. Now $m = 1$, and taking (17) into account we can write:

$$u_j = \frac{1}{\Sigma F^2}(i_{g3} \cdot \Sigma F^3_{3j,4} + u_{g1} \cdot \Sigma F^2_{1j,4}) \tag{23}$$

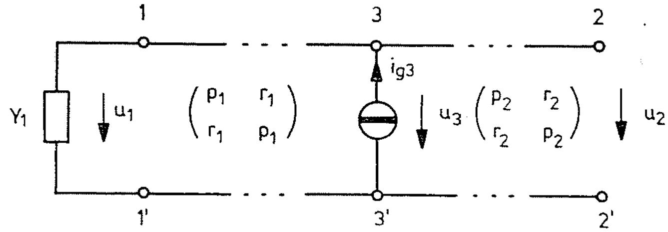


Fig. 5

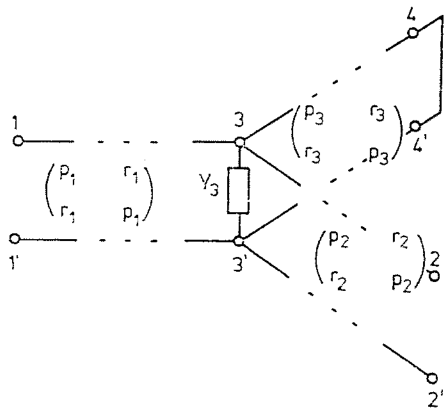


Fig. 6

where the model graph in Fig. 4 can be used again, for calculating the k-tree admittance products if you write Y instead of Y_1.

In this case the meanings of the corresponding k-tree products are as follows:

$$\Sigma F^2 = \Sigma F^2_{\bar{1},4} = r_1 r_2 - s_2(r_1 + r_2) + s_3(s_2 - r_2) = p_2^2 + p_1 p_2 - r_2^2 , \quad (24)$$

$$\Sigma F^2_{1,4} = \Sigma F^2, \ \Sigma F^2_{12,4} = r_1 r_2, \ \Sigma F^2_{13,4} = -r_1 s_2 + r_1 r_2 = -r_1 p_2 ,$$

$$\Sigma F^3_{31,4} = 0, \ \Sigma F^3_{32,4} = -r_2 \text{ and } \Sigma F^3_{3,4} = -r_2 + s_2 = p_2 .$$

From (23) and (24) we have obtained:

$$u_1 = u_{g1}$$

$$u_2 = \frac{r_1 r_2 u_{g1} - r_2 \dot{i}_{g3}}{p_2^2 + p_1 p_2 - r_2^2}$$

$$u_3 = \frac{-r_1 p_2 u_{g1} + p_2 \dot{i}_{g3}}{p_2^2 + p_1 p_2 - r_2^2}$$

3. Calculate the driving-point impedance matrix of the transmission network in Fig. 6 by the topological method.

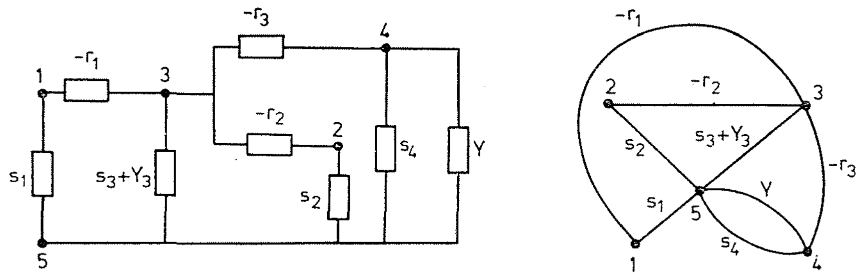


Fig. 7

The model of the network is seen in Fig. 7 together with its graph. Using formula (17), at present $l = 2$ and $m = 1$. So we can write:

$$z_{ij} = \frac{\Sigma F^3_{ij,5}}{\Sigma F^2}, \qquad i,j = 1,\ 2. \tag{25}$$

To final the k-trees for (25) we shall use the method [2].

From Fig. 7, the modified adjacency matrix of the graph is:

$$\mathbf{M}_g = \begin{bmatrix} 0 & 0 & 3 & 0 & 5 \\ 0 & 0 & 3 & 0 & 5 \\ 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 3 & 0 & \textcircled{5} \\ 1 & 2 & 3 & \textcircled{4} & 0 \end{bmatrix} \tag{26}$$

The circled symbols in (26) refer to the admittance Y in the network graph.

To produce the F^2 2-trees for (25) all circuitless representations should be calculated from (26) which contain the symbol *0* only in the 4th and 5th components. Now, to produce ΣF^2 let us substitute the corresponding edge admittances. The substitution is made easier by Table 1.

Table 1

The symbol							
3	5	3	5	1	2	4	5
occurring in the							
first		second		third			
component of the k-tree representation corresponds to the factor							
$-r_1$	s_1	$-r_2$	s_2	$-r_1$	$-r_2$	$-r_3$	s_3+Y_3
in the admittance product							

Table 2

Type of the 2-trees	Possible 2-tree representations	Circuit-less	Corresponding edge admittance products
	3 3 1 0 0	n	—
	2	n	—
	4	y	$(-r_1)(-r_2)(-r_3)$
	5	y	$(-r_1)(-r_2)(s_3+Y_3)$
	3 5 1 0 0	n	—
	2	y	$(-r_1)s_2(-r_2)$
	4	y	$(-r_1)s_2(-r_3)$
F^2	5	y	$(-r_1)s_2(s_3+Y_3)$
	5 3 1 0 0	y	$s_1(-r_2)(-r_1)$
	2	n	—
	4	y	$s_1(-r_2)(-r_3)$
	5	y	$s_1(-r_2)(s_3+Y_3)$
	5 5 1 0 0	y	$s_1s_2(-r_1)$
	2	y	$s_1s_2(-r_2)$
	4	y	$s_1s_2(-r_3)$
	5	y	$s_1s_2(s_3+Y_3)$

Calculation of ΣF^2 is summarized in Table 2. In the 1st column we wrote the type of the k-tree, the 2nd column contains all 2-tree representations in question, while marks "y" and "n" in the 3rd column indicate whether the completed cycle check performed on the representation has a finite outcome or not, i.e. whether the corresponding graph is circuitless or not. The 4th column of the table contains the edge admittance product for the circuitless case. Hence, ΣF^2 is the sum of data in the 4th column. After some simple calculation we have got:

$$\Sigma F^2 = r_1\big(r_2(s_3 - r_1 + Y_3) + s_2(r_2 + r_3 - s_3 - Y_3)\big) + \\ + s_1\big(r_2(r_1 + r_3 - s_3 - Y_3) + s_2(s_3 + Y_3 - r_1 - r_2)\big)\,.$$

Concerning the production of $F^3_{ij,5}$ 3-trees see Table 3. Similarly to Table 2, the 1st column in Table 3 contains the type of the k-trees, the 2nd column consists of the 3-tree representations (at present the 1st, 4th and 5th components of the representation must be 0), the 3rd column informs us about

Table 3

Type of the 3-trees	Possible 3-tree representations	Circuit-less	Corresponding edge admittance products
$F^3_{1,5}$	0 3 1 0 0	y	$(-r_2)(-r_1)$
	2	n	—
	4	y	$(-r_2)(-r_3)$
	5	y	$(-r_2)(s_3+Y_3)$
	0 5 1 0 0	y	$s_2(-r_1)$
	2	y	$s_2(-r_2)$
	4	y	$s_2(-r_3)$
	5	y	$s_2(s_3+Y_3)$
$F^3_{12,5}$	0 3 1 0 0	y	$(-r_2)(-r_1)$
$F^3_{2,5}$	3 0 1 0 0	n	—
	2	y	$(-r_1)(-r_2)$
	4	y	$(-r_1)(-r_3)$
	5	y	$(-r_1)(s_3+Y_3)$
	5 0 1 0 0	y	$s_1(-r_1)$
	2	y	$s_1(-r_2)$
	4	y	$s_1(-r_3)$
	5	y	$s_1(_3+Y_3)$

the result of the completed cycle checks, finally the edge admittance products needed for the calculation are in the 4th column of the table.

Taking Table 3 into account we have arrived at the driving-point impedance matrix of the network:

$$\mathbf{Z} = \frac{1}{\Sigma F^2} \cdot \begin{pmatrix} r_2(r_1+r_3-s_3-Y_3)+s_2(s_3+Y_3-r_1-r_2-r_3) & r_1 r_2 \\ r_1 r_2 & r_1(r_2+r_3-s_3-Y_3)+s_1(s_3+Y_3-r_1-r_2-r_3) \end{pmatrix}.$$

Summary

In this paper the author deals with networks consisting of transmission lines, nodes of which are closed by passive or active two-terminals and by break or short-circuit (extreme closures). First the model network is introduced, ordered to the transmission network, which contains only passive admittances and (ideal) source generators. It is proved that the node potentials of the model are equal to the node voltages of the original network. In order to calculate the node voltages, topological formulas are deduced. A method elaborated earlier by the author is suggested for, searching k-trees needed for using these formulas. Another topological formula is also given for the driving-point impedance matrix of the transmission network, and finally the topological analysis is shown on concrete examples.

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Dr. Imre PÁVÓ, H-6720 Szeged, Somogyi u. 7. Hungary

* In Hungarian.