MINIMUM VARIANCE CONTROL
A REVIEW AND OUTLOOK

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1. Introduction

The need to control optimally a system influenced by stochastic disturbances has arisen as a real practical problem. The urgent solution of this problem has primarily been desired by such industrial technologies where the economy, the reliability, the stability, as well as the security of the whole system depend to a great extent on the performance of the regulation. This is the main reason of the huge progress which can be observed in the field of stochastic control engineering. The utilization of the theoretical results and their application for industrial and laboratorial processes have been achieved in areas where in addition to the proper hardware and software conditions (suitable real-time system and powerful real-time language) simple and powerful control algorithms have been adopted.

One of the most successful method of the last few years for designing optimal control loops is the minimum variance control, first discussed in a detailed form by ÅSTRÖM (1970). Up to now, this method has proved its effectiveness by numerous industrial applications (Borisson et al. 1973, Borisson et al. 1974), and it really has nice properties, viz.:

— it considerably decreases the deviations of the controlled signal around its desired value;
— it takes into consideration disturbances occurring in practical situations;
— it is very simple, thus it hardly needs core memory and is not time consuming;
— it is easy to implement both in the case of known and unknown system parameters;
— it hardly requires any a priori knowledge about the process to be controlled (time delay and the order of the process);
— it can follow the changes of the system parameters and that of the desired value of the controlled signals.

The fact that such a stochastic control method of nice properties has been developed has to stimulate the researchers to extend the principle of the minimum variance control and the so-called self-tuning principle to that sort of structures, by which further industrial processes could be controlled. Up to now the self-tuning principle is elaborated for single input-single output (SISO) systems and is based on the following considerations. On the basis of the order and the time delay of the process to be controlled, the structure of the regulator can be determined. By means of the control and controlled signal observed in the closed loop the optimal value of the regulator parameters can easily be estimated by a simple least squares estimation which is unbiased (ÅSTRÖM and WITTENMARK 1973). The main advantage of this method is not to require any further information about the process and disturbance dynamics.

In our paper extensions of the original minimum variance control strategy will be considered which involve industrial application problems.

2. Minimum variance control of SISO systems

Let us consider a SISO system described by the well-known ÅSTRÖM (1970) model (see Fig. 1):

$$y(t)=\frac{B(z^{-1})}{A(z^{-1})}u(t-\mathrm{d})+\lambda\frac{C(z^{-1})}{A(z^{-1})}e(t), \tag{1}$$

where $z^{-d}B(z^{-1})/A(z^{-1})$ is the pulse transfer function of the process, $\lambda C(z^{-1})/A(z^{-1})$ is the pulse transfer function of the disturbance referred to the process output, $u(t)$ is the control signal, $y(t)$ is the output of the noisy system, $e(t)$ is a sequence of independent normal variables with zero mean value and variance one (white noise). Further

$$A(z^{-1})=1+a_1z^{-1}+\ldots+a_nz^{-n}$$
$$B(z^{-1})=b_0+b_1z^{-1}+\ldots+b_mz^{-m}$$
$$C(z^{-1})=1+c_1z^{-1}+\ldots+c_kz^{-k}$$
$$t=0, 1, 2, \ldots$$

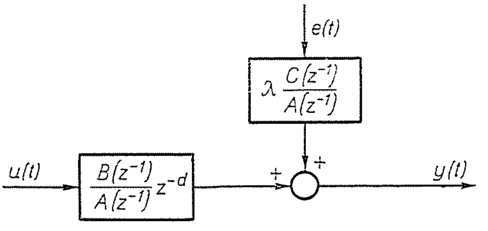

Fig. 1. Flow chart of the ÅSTRÖM-model

and d is the time delay of the process. The purpose of the minimum variance control is to determine the control signal $u(t)$ in such a way that the loss function

$$V=E\{[y(t+d)-y_r]^2\} \tag{2}$$

is as small as possible. Here y_r denotes the desired value of the controlled signal (reference value). It is well known [*Åström* (1970 and *Peterka* (1972)] that the optimal control strategy minimizing the loss function above is given by

$$u(t)=\frac{C(z^{-1})y_r-G(z^{-1})y(t)}{B(z^{-1})F(z^{-1})}, \tag{3}$$

where polynomials $G(z^{-1})$ and $F(z^{-1})$ are given by the following separation equation:

$$C(z^{-1})=A(z^{-1})F(z^{-1})+z^{-d}G(z^{-1}) \tag{4}$$

$$F(z^{-1})=1+f_1z^{-1}+\ldots+f_{d-1}z^{1-d}$$

$$G(z^{-1})=g_0+g_1z^{-1}+\ldots+g_{n-1}z^{1-n}.$$

The flow chart of the optimal control is seen in Fig. 2.

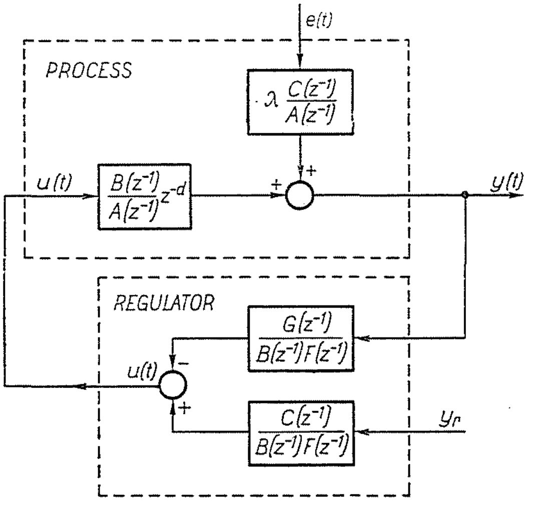

Fig. 2. Flow chart of the optimal control of SISO systems

3. Minimum variance control in the case of delayed observations

In many practical cases the controlled signal cannot be directly observed, because the sensor or measuring device has a considerable time delay. Then the ÅSTRÖM-model can be extended as:

$$y(t)=\frac{B(z^{-1})}{A(z^{-1})}u(t-d_1)+\lambda\frac{C(z^{-1})}{A(z^{-1})}e(t),$$

$$y_m(t)=z^{-d_2}y(t)=y(t-d_2),$$

where d_1 and d_2 denote the time delays of the process and of the sensor, respectively, $y(t)$ and $y_m(t)$ denote the controlled and the measured output signal, respectively. The purpose of the control is to minimize the loss function

$$E\{[y(t+d_1)-y_r]^2\}=\min_{u(t)}$$

on the basis of the observations

$$y_m(t),\ y_m(t-1),\ \ldots,\ u(t-1),\ u(t-2),\ \ldots$$

Using the separation

$$C(z^{-1})=A(z^{-1})F(z-1)+z^{-d_1-d_2}G(z^{-1}),$$

where

$$F(z^{-1})=1+f_1z^{-1}+\ldots+f_{d_1+d_2-1}z^{1-d_1-d_2},$$

$$G(z^{-1})=g_0+g_1z^{-1}+\ldots+g_{n-1}z^{1-n}$$

we get

$$y(t+d_1)=\frac{B(z^{-1})F(z^{-1})}{C(z^{-1})}u(t)+\frac{G(z^{-1})}{C(z^{-1})}y(t-d_2)+\lambda F(z^{-1})e(t+d_1),$$

thus the optimal control law is given by

$$u(t)=\frac{C(z^{-1})y_r-G(z^{-1})y(t-d_2)}{B(z^{-1})F(z^{-1})}=\frac{C(z^{-1})y_r-G(z^{-1})y_m(t)}{B(z^{-1})F(z^{-1})}.$$

This control law is seen to be a strategy just like a minimum variance strategy of a process with time delay $d=d_1+d_2$. The increase of the loss function comparing the cases determined by $d_2\equiv 0$ and $d_2\neq 0$ is

$$\Delta V=\lambda^2(1+f_1^2+\ldots+f_{d_1+d_2-1}^2)-\lambda^2(1+f_1^2+\ldots+f_{d_1}^2-1)=$$

$$=\lambda^2(f_{d_1}^2+f_{d_1+1}^2+\ldots+f_{d_1+d_2-1}^2).$$

4. Limitation of the control signal

The various limitations of the control signal can be interpreted by an additional term in the loss function:

$$V=E\{[y(t+d)-y_r]^2+\alpha u^2(t)\},$$

where $\alpha=$const. is a factor to weigh the loss due to the control action. Using Eq. (4), a second-order term of $u(t)$ should be minimized:

$$V=E\left\{\left[\frac{B(z^{-1})F(z^{-1})}{C(z^{-1})}u(t)+\frac{G(z^{-1})}{C(z^{-1})}y(t)+\lambda F(z^{-1})e(t+d)-y_r\right]^2+\alpha u^2(t)\right\}. \qquad (5)$$

Let's have the following notations:

$$B(z^{-1})Fz^{-1})=b_0+Q(z^{-1})z^{-1}$$
$$C(z^{-1})=1+P(z^{-1})z^{-1}.$$

Thus to achieve the minimum of the loss function the condition

$$b_0[b_0u(t)+Q(z^{-1})u(t-1)+G(z^{-1})y(t)-C(z^{-1})y_r]+\alpha u(t)+\alpha P(z^{-1})u(t-1)=0$$

has to be met. From this condition the optimal control is given by

$$u(t)=\frac{b_0}{b_0^2+\alpha}\{C(z^{-1})y_r-G(z^{-1})y(t)-[Q(z^{-1})+\alpha P(z^{-1})]u(t-1)\}$$

or introducing

$$S(z^{-1})=b_0^2+\alpha+z^{-1}[Q(z^{-1})+\alpha P(z^{-1})]$$

by

$$u(t)=\frac{C(z^{-1})y_r-G(z^{-1})y(t)}{S(z^{-1})}\cdot b_0\,. \tag{6}$$

Naturally, this strategy turns into Eq. (3) for $\alpha=0$. Fig. 3 shows the optimal strategy for $\alpha\neq 0$.

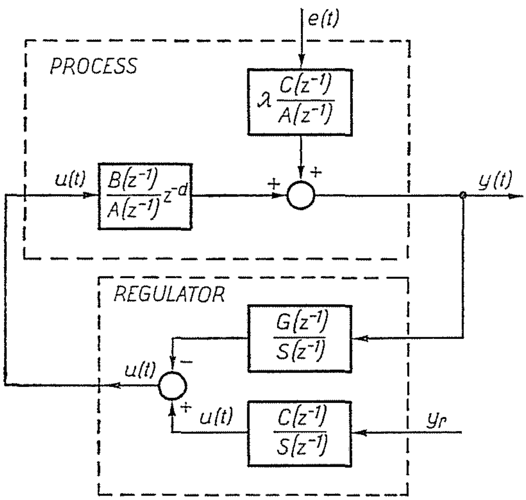

Fig. 3. Optimal control in case of limitations of the control signal

In many industrial control systems the control signal can be limited by saturation, that is, it has to satisfy the inequality

$$-U_0<u(t)<U_0\,, \tag{7}$$

where U_0 is the limit of the range of admissible control signals. It means that

$$\alpha=0, \quad \text{if} \quad -U_0<u(t)<U_0$$

and

$$\alpha=\infty, \quad \text{if} \quad |u(t)|>U_0.$$

If a self-tuning algorithm by Eq. (3) is working and Eq. (7) holds in every moment, then no influence is exerted by the limitation. If the control signal produced by Eq. (3) is

$$|u(t)|>U_0 \tag{8}$$

then the real value of the process input is

$$u_r(t)=U_0 \operatorname{sign}\{u(t)\}.$$

It is clear that the more the number of the events increases if Eq. (8) is satisfied, the greater the loss function increases. To investigate the simplest case, if $y_r=0$ and Eq. (8) holds for every t, the input of the process is a binary sequence of $+U_0$ and $-U_0$ and the controlled signal has the form:

$$y(t+d)=U_0\frac{B(z^{-1})}{A(z^{-1})}\operatorname{sign}\left\{\frac{-G(z^{-1})}{B(z^{-1})F(z^{-1})}y(t)\right\}+$$
$$+\lambda\frac{G(z^{-1})}{A(z^{-1})}e(t)+\lambda F(z^{-1})e(t+d).$$

Writing $y(t)$ according to the equation above and eliminating $e(t)$ we obtain

$$y(t+d)=\frac{G(z^{-1})}{C(z^{-1})}y(t)+U_0\frac{B(z^{-1})}{A(z^{-1})}\left[\operatorname{sign}\left\{\frac{-G(z^{-1})}{B(z^{-1})F(z^{-1})}y(t)\right\}-\right.$$
$$\left.-\frac{G(z^{-1})}{C(z^{-1})}\operatorname{sign}\left\{\frac{-G(z^{-1})}{B(z^{-1})F(z^{-1})}y(t-d)\right\}\right]+$$
$$+\lambda F(z^{-1})\,e\,(t+d). \tag{9}$$

Eq. (9) shows that even in this simple case analytical determination of the loss function involves great difficulties because of nonlinearity. Thus the simulations have great importance even from this point of view. The simulation results show that a self-tuning regulator can considerably reduce the influence of the disturbance, even if strong limitations of the control signal exist [*Wittenmark* (1973)].

5. Extension of the minimum variance control strategy for the case of observable noise

In its original form the minimum variance control stands for reducing stochastic disturbances represented by filtered white noise. In addition to this type of noise, another type of additive noise to be derived from an observable input signal exists in practical industrial loops. For example such a process is the control loop of the temperature of the fresh steam in a traditional power plant, where the temperature

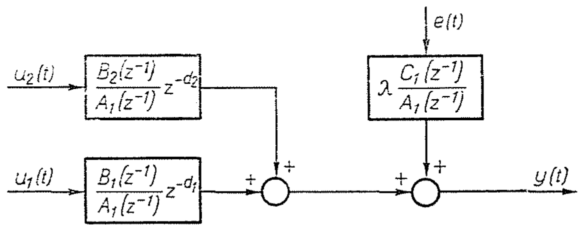

Fig. 4. Flow chart of the modified ÅSTRÖM-model in the case of observable noise

after the superheater is influenced by the load outside the random noises. Another typical application of the use of this structure may be the thickness control of cold or hot rolled sheets by means of the compressive force, where the observable inputs are the sheet thickness and quality previous to rolling. Further applications will be mentioned later. Assuming the system parameters to be known or previously identified MISO ÅSTRÖM model has the form (see Fig. 4):

$$y(t)=\frac{B_1(z^{-1})}{A_1(z^{-1})}u_1(t-d_1)+\frac{B_2(z^{-1})}{A_1(z^{-1})}u_2(t-d_2)+\lambda\frac{C_1(z^{-1})}{A_1(z^{-1})}e(t), \quad (10)$$

where $z^{-d_1}B_1(z^{-1})/A_1(z^{-1})$ is the pulse transfer function of the process, $z^{-d_2}B_2(z^{-1})/A_1(z^{-1})$ is the pulse transfer function for the observable input signal $u_2(t)$. In this structure

$u_2(1), u_2(2), \ldots, u_2(t)$ are the observable inputs,
$y(1), y(2), \ldots, y(t)$ are the observable outputs,
$u_1(1), u_1(2), \ldots, u_1(t-1)$ are known and $u_1(t)$ is to be determined in such a way that the loss function

$$V=E\{y^2(t+d_1)\} \quad (11)$$

is minimum. For simplicity's sake now $y_r=0$ is considered, but the results will be generalized for $y_r\neq 0$. Using the separation

$$C_1(z^{-1})=A_1(z^{-1})F_1(z^{-1})+z^{-d_1}G_1(z^{-1})$$

the optimal control law is easy to derive from Eq. (10) (*Hetthéssy* 1974):

$$u_1(t)=\frac{-G_1(z^{-1})}{B_1(z^{-1})F_1(z^{-1})}y(t)-\frac{B_2(z^{-1})}{B_1(z^{-1})}u_2(t-d_2+d_1). \quad (12)$$

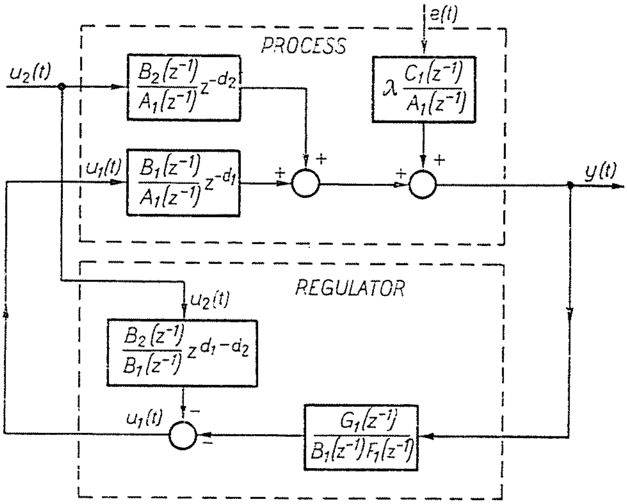

Fig. 5. Optimal control for $d_2 \geqq d_1$

The flow chart of the optimal control is shown in Fig. 5. However, this strategy can only be physically realized if the condition

$$d_2 \geqq d_1$$

holds, otherwise $u_1(t)$ ought to be determined from an observation after t, which is impossible. For $d_1 > d_2$ let

$$d = d_1 - d_2 > 0,$$

and the constant and stochastic components of $u_2(t)$ have to be separated by

$$u_2(t) = U_2 + \mu \frac{C_2(z^{-1})}{A_2(z^{-1})} v(t),$$

where $v(t)$ is white noise. Using the polynomial separation

$$B_2(z^{-1})F_1(z^{-1})C_2(z^{-1}) = C_1(z^{-1})A_2(z^{-1})F_2(z^{-1}) + G_2(z^{-1})z^{-d},$$

where

$$F_2(z^{-1}) = f_{20} + f_{21}z^{-1} + \ldots + f_{2,(d-1)}z^{1-d}$$

$$G_2(z^{-1}) = g_{20} + g_{21}z^{-1} + \ldots + g_{2,(n_1+n_2+d-1)}z^{1-n_1-n_2-d}$$

the optimal control law is given by

$$u_1(t) = \frac{-G_1(z^{-1})}{B_1(z^{-1})F_1(z^{-1})} y(t) - \frac{G_2(z^{-1})}{C_2(z^{-1})B_2(z^{-1})F_1(z^{-1})} u_2(t) +$$

$$+ \left[\frac{G_2(z^{-1})}{C_2(z^{-1})B_2(z^{-1})F_1(z^{-1})} - \frac{B_2(z^{-1})}{B_1(z^{-1})} \right] U_2. \qquad (13)$$

The minimum value of the loss function is

$$V_{\min}=\lambda^2(1+f_{11}^2+\ldots+f_{1\cdot d_1}^2)+\mu^2(f_{20}^2+f_{21}^2+\ldots+f_{2,(d-1)}^2).$$

If minimization of the loss function

$$V=E\{[y(t+d_1)-y_r]^2\} \quad (y_r\neq 0)$$

is required, then the optimal control signal is determined by the relationship:

$$\begin{aligned} u_1(t)=&\frac{C_1(z^{-1})}{B_1(z^{-1})F_1(z^{-1})}y_r-\frac{G_1(z^{-1})}{B_1(z^{-1})F_1(z^{-1})}y(t)-\\ &-\frac{G_2(z^{-1})}{C_2(z^{-1})B_2(z^{-1})F_1(z^{-1})}u_2(t)+\\ &+\left[\frac{G_2(z^{-1})}{C_2(z^{-1})B_2(z^{-1})F_1(z^{-1})}-\frac{B_2(z^{-1})}{B_1(z^{-1})}\right]U_2. \end{aligned} \qquad (14$$

Fig. 6 shows the optimal control.

Eqs. (12), (13) and (14) demonstrate that if the structure in Eq. (11) exists, the minimum variance control can be realized by using simultaneous feedback and feedforward.

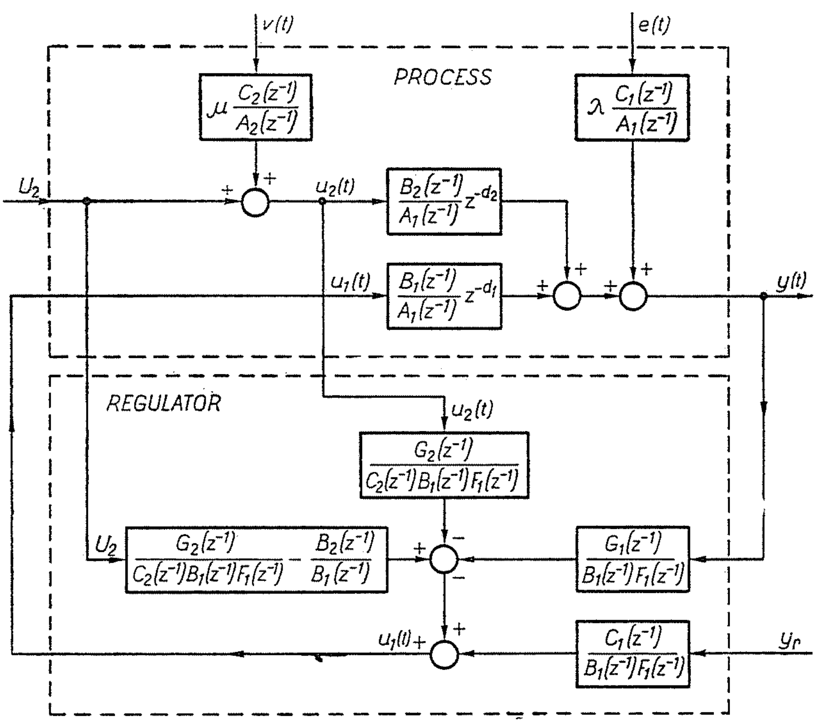

Fig. 6. Optimal control for $d_1>d_2$

6. Minimum variance control of MISO systems

First consider a system of two inputs (Hoffmann 1971) (see Fig. 7):

$$y(t+d)=\frac{B_1(z^{-1})}{A(z^{-1})}u_1(t)+\frac{B_2(z^{-1})}{A(z^{-1})}u_2(t)+$$

$$+\lambda\frac{C(z^{-1})}{A(z^{-1})}e(t+d). \tag{15}$$

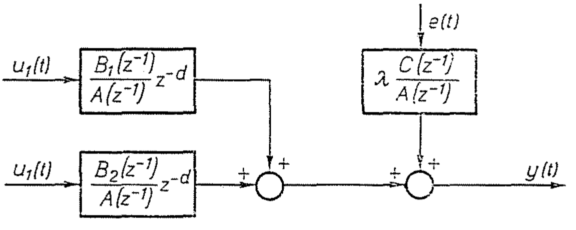

Fig. 7. The MISO extension of the ÅSTRÖM-model

Let the purpose of the control be to determine $u_1(t)$ and $u_2(t)$ in such a way that the loss function

$$V=E\{y^2(t+d)\} \tag{16}$$

is minimum. Using the separation by Eq. (4) we have

$$y(t+d)=\frac{B_1(z^{-1})F(z^{-1})}{C(z^{-1})}u_1(t)+\frac{B_2(z^{-1})F(z^{-1})}{C(z^{-1})}u_2(t)+$$

$$+\frac{G(z^{-1})}{C(z^{-1})}y(t)+\lambda F(z^{-1})\,e\,(t+d),$$

thus the optimal control law has to meet the condition:

$$B_1(z^{-1})F(z^{-1})u_1(t)+B_2(z^{-1})F(z^{-1})u_2(t)+G(z^{-1})y(t)=0. \tag{17}$$

Eq. (17) shows that the problem is undetermined, because $u_1(t)$ or $u_2(t)$ can be arbitrarily chosen. To complete the problem it is necessary to modify the loss function by weighting the control signals. There are two possibilities.

Either $u_1(t)$ and $u_2(t)$ can be determined by Eq. (17) if the loss function

$$V_u=E\{\alpha_1 u_1^2(t)+\alpha_2 u_2^2(t)\} \tag{18}$$

is also taken into consideration. It is easy to show that V_u is minimum if

$$u_2(t)=\frac{\alpha_1}{\alpha_2}\cdot\frac{b_{20}}{b_{10}}u_1(t). \tag{19}$$

Then using the notation

$$B(z^{-1})=B_1(z^{-1})+\frac{\alpha_1}{\alpha_2}\cdot\frac{b_{20}}{b_{10}}B_2(z^{-1}) \tag{20}$$

by means of Eq. (17) we obtain

$$u_1(t) = \frac{-G(z^{-1})}{B(z^{-1})F(z^{-1})} y(t).$$

Or as the second possibility, an extended loss function

$$V_e = E\{y^2(t+d) + \alpha_1 u_1^2(t) + \alpha_2 u_2^2(t)\} \tag{21}$$

is minimized. Since Eq. (19) holds also in this case, using the notations by Eq. (20) and

$$\alpha = \alpha_1 + \frac{\alpha_1^2}{\alpha_2} \cdot \frac{b_{20}^2}{b_{10}^2}, \tag{22}$$

the extended loss function has the following form:

$$V_e = E\left\{\left[\frac{F(z^{-1})B(z^{-1})}{C(z^{-1})} u_1(t) + \frac{G(z^{-1})}{C(z^{-1})} y(t)\right]^2 + \right.$$
$$\left. + \alpha u_1^2(t) + \lambda^2 [F(z^{-1})\, e\,(t+d)]^2 \right\}. \tag{23}$$

Thus the problem is traced back to a previous SISO problem. Because of Eq. (5) the optimal control is

$$u_1(t) = \frac{-b_0 G(z^{-1})}{S(z^{-1})} y(t). \tag{24}$$

In connection with both methods some remarks arise.

If the minimization of loss function

$$V = E\{[y(t+d) - y_r]^2\}$$

is required ($y_r \neq 0$), then the expressions of the optimal control signals contain an additional, constant term.

Both methods can easily be extended for the case of three or more (r) control signals ($r > 1$) introducing the notations:

$$\alpha = \alpha_1 + \frac{\alpha_1^2}{b_{10}^2} \sum_{j=2}^{r} \frac{b_{jo}^2}{\alpha_j};$$

$$B(z^{-1}) = B_1(z^{-1}) + \frac{\alpha_1}{b_{10}} \sum_{j=2}^{r} \frac{b_{jo}}{\alpha_j} B_j(z^{-1});$$

$$u_j(t) = \frac{\alpha_1}{\alpha_j} \cdot \frac{b_{jo}}{b_{10}} u_1(t), \qquad (j = 2, 3, \ldots, r).$$

Finally, if the system is described by

$$A(z^{-1})y(t)=B_1(z^{-1})u_1(t-d_1)+B_2(z^{-1})u_2(t-d_2)+\ldots+B_r(z^{-1})u_2(t-d_r)+ \\ +\lambda C(z^{-1})e(t),$$

where

$$d_1\neq d_2\neq\ldots\neq d_r,$$

then let

$$d=\min\{d_1, d_2, \ldots, d_r\}.$$

Then extending the polynomials $B_i(z^{-1})$ $(i=1, 2, 3, \ldots, r)$ by a proper number of zero coefficients, it is possible to use the original system model given by Eq. (15).

7. Minimum variance control of MIMO systems

Consider the multivariable extension of the ÅSTRÖM model (see Fig. 8):

$$\mathbf{A}(z^{-1})\mathbf{y}(t)=\mathbf{B}(z^{-1})\mathbf{u}(t-d)+\mathbf{C}(z^{-1})\mathbf{\Lambda e}(t), \tag{25}$$

where $\mathbf{y}$ is the q-dimensional output vector, $\mathbf{u}$ is the q-dimensional control vector, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are polynomial matrices of z^{-1}:

$$\mathbf{A}(z^{-1})=\mathbf{I}_q+\sum_{j=1}^{n} z^{-j}\mathbf{A}_j$$

$$\mathbf{B}(z^{-1})=\sum_{j=0}^{m} z^{-j}\mathbf{B}_j$$

$$\mathbf{C}(z^{-1})=\mathbf{I}_q+\sum_{j=1}^{k} z^{-j}\mathbf{C}_j,$$

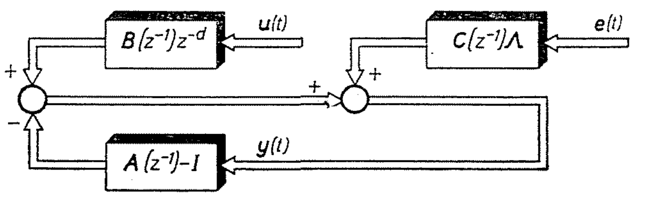

Fig. 8. The MIMO extension of the ÅSTRÖM-model

further $\mathbf{e}$ is a sequence of normally distributed independent vector variables with zero mean value and covariance matrix $\mathbf{I}_q$. Here $\mathbf{I}_q$ is a $q\times q$ unit matrix. The loss function to be minimized is

$$V=[\mathbf{y}(t+d)-\mathbf{y}_r]^T[\mathbf{y}(t+d)-\mathbf{y}_r]. \tag{26}$$

Using the separation

$$\mathbf{C}(z^{-1})=\mathbf{A}(z^{-1})\mathbf{F}(z^{-1})+z^{-d}\mathbf{G}(z^{-1})$$

Eq. (25) has the following form:

$$\mathbf{y}(t+d)=[\mathbf{F}(z^{-1})\mathbf{C}^{-1}(z^{-1})\mathbf{B}(z^{-1})\mathbf{u}(t)+\mathbf{A}^{-1}(z^{-1})\mathbf{G}(z^{-1})\mathbf{C}^{-1}(z^{-1})\mathbf{A}(z^{-1})\mathbf{y}(t)]+ \\ +\mathbf{F}(z^{-1})\Lambda\mathbf{e}(t+d), \quad (27)$$

thus the optimal control strategy is:

$$\mathbf{u}(t)=\mathbf{B}^{-1}(z^{-1})[\mathbf{C}(z^{-1})\mathbf{F}^{-1}(z^{-1})\mathbf{y}_r-\mathbf{G}(z^{-1})\mathbf{F}^{-1}(z^{-1})\mathbf{y}(t)]. \quad (28)$$

To avoid producing $\mathbf{B}^{-1}(z^{-1})$ and $\mathbf{F}^{-1}(z^{-1})$ let us have

$$\mathbf{B}(z^{-1})=\mathbf{B}_0+\tilde{\mathbf{B}}(z^{-1});$$
$$\mathbf{F}(z^{-1})=\mathbf{I}_q+\tilde{\mathbf{F}}(z^{-1});$$

and

$$\mathbf{v}(t)=\mathbf{F}^{-1}(z^{-1})\mathbf{y}(t)=\mathbf{y}(t)-\tilde{\mathbf{F}}(z^{-1})\mathbf{v}(t);$$
$$\mathbf{v}_r=\mathbf{F}^{-1}(z^{-1})\mathbf{y}_r=\mathbf{y}_r-\tilde{\mathbf{F}}(z^{-1})\mathbf{v}_r.$$

Rewriting Eq. (28) by means of these notations we get

$$\mathbf{u}(t)=\mathbf{B}_0^{-1}[\mathbf{C}(z^{-1})\mathbf{v}_r-\mathbf{G}(z^{-1})\mathbf{v}(t)-\tilde{\mathbf{B}}(z^{-1})\mathbf{u}(t)].$$

The optimal strategy is shown in Fig. 9.

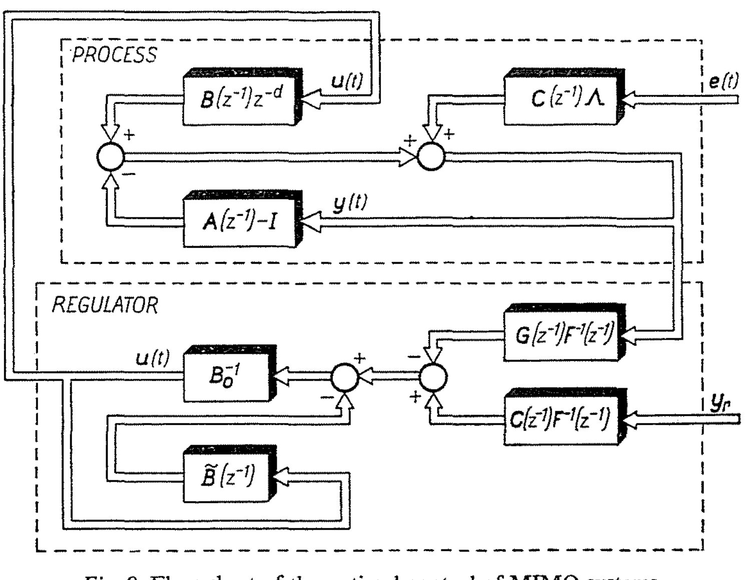

Fig. 9. Flow chart of the optimal control of MIMO systems

8. Relations between the classical design and the minimum variance control

The latest results of the stochastic control theory have clearly shown that the classical (*PI*, *PID*, ... etc.) regulators are not sufficient to achieve the theoretically best control, if the system is not influenced by deterministic disturbances (x_{det}) alone. In the presence of stochastic disturbances (x_{stoc}) a more complicated signal formation is needed than by the classical controllers, but this complicated signal formation is simple to produce by a digital computer. Although a minimum variance controller yields good results even for transient processes (start up, shut down, change of the set point), that sort of processes often require optimum control from some other aspects (minimal overshot, minimal reset time, ... etc.). In other words, a combination of the minimum variance and classical controllers has to be constructed in such a way that the stochastic disturbances are handled by an internal minimum variance regulator and the deterministic disturbances by an external classical controller. To do this, the separation of the disturbances is needed, possible by a filter cutting a range of frequencies due to the stochastic disturbances (Fig. 10). Thus the stochastic component is given by

$$y_s = y - y_f,$$

where y_f is the filtered output. The stochastic component is connected to the minimum variance controller via a sampling unit, while y_f is the input of the classical controller (Fig. 10). The control signal is derived from two sources, the classical controller, and the minimum variance controller via a holding unit. In connection with Fig. 10 it has to be remarked that y_s slightly differs from the real stochastic component, due to the characteristic of the filter.

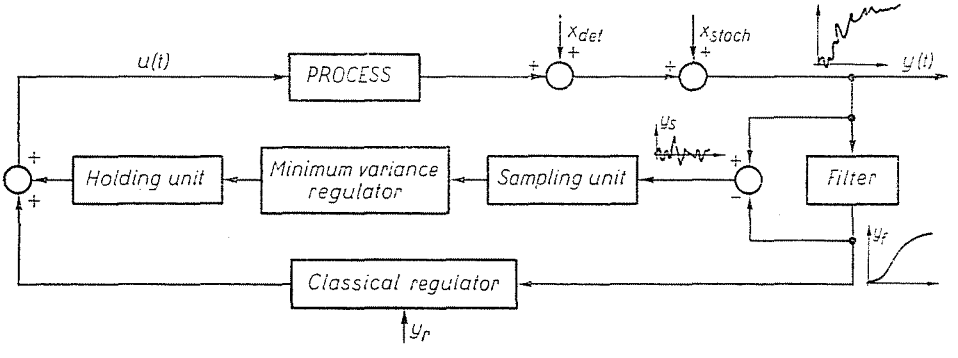

Fig. 10. A possible combination of the MV and classical controllers

9. Conclusions

In our paper the minimum variance control is considered. Besides the well-known classical control cases some problems encountered for practical control systems are discussed. These regulators are unlike to be more complicated than the basic case but lend themselves for much more complex control tasks. We hope our experience will give suggestions to other researchers and engineers for further investigation.

Summary

The optimum system design according to the classical, quadratic — by the sense of Wiener — criterion is a little bit difficult.

The minimum variance (MV) control introduced by ÅSTRÖM makes considerably easier to solve this task by parametrizing the system and designing directly in the time domain, and it promotes its practical applications, too. Nowadays, when the research work is mostly directed towards the adaptive, self-tuning solution of the minimum variance control, it is necessary to make a review and outlook about the above subject. Besides the classical MV regulators such control cases are considered in this paper which have been developed in accordance with the industrial demands during the practical applications, e.g. the relationships of the MV regulator for delayed observations are also given. The different restrictions regarding the control signals are also discussed. The MV regulator is extended to the case of observable disturbances, too. Basic establishments are formulated for MV control of MISO systems. Finally the equation of MV regulator for MIMO systems is given and a possible solution of effective cooperation of classical and MV regulators is treated.

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