ELECTROMAGNETIC FIELD EXCITED BY POWER LINES WITH NONSINUSOIDAL CURRENTS

By

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Introduction

In the past two decades the application of equipment consuming or generating nonsinusoidal current has become general, due to the rapid development of power electronic devices. Currents of such equipment flowing in the supply lines may cause dangerous conducted or radiated signals likely to lead to faulty operation or breakdown of the telecommunication, measuring, control, etc. equipment and systems adjacent to the power equipment and lines. On the other hand, most power devices and systems normally consuming or generating sinusoidal currents produce nonsinusoidal ones if a breakdown, e.g. short circuiting happens. Therefore for designing and using small signal equipment as mentioned above, calculation or at least estimation of the interferencing electromagnetic field seems to be very useful.

This paper presents the derivation of some general formulae easier to apply for practical problems than the other ones known from the literature. These formulae yield useful guidelines such as in example 1 simplifying to decide whether the simple static approximate calculation is sufficient or the difficulter general method is necessary. Two other examples are given too, demonstrating use of the formulae.

Derivation of the formulae

If the changes of the currents and charges exciting the electromagnetic field are "fast", the calculation of the electromagnetic field can only be made on the basis of the Maxwell equations. Solving these equations is very difficult and in most cases only numerical results are possible.

The well known methods [1], [2], [3] are restricted to the sinusoidal changes of the currents and charges. Though the problem of nonsinusoidal excitement can be solved on the basis of the sinusoidal changes — using the Fourier integral or series — this method has some disadvantages:

— the time domain solution can only be obtained by quite cumbersome procedure using the appropriate Fourier integrals or series;

— the analysis made in the frequency domain blurs most of the physical insight;

— since in most cases the shape of the current exciting the electromagnetic field is only known approximately, the inherent error of the direct method of the Fourier integral is difficult or impossible to estimate, etc.

Formulae derived in the following give the solution of the problem in the time domain. These formulae are similar to the Coulomb and Biot-Savart laws and can be regarded as their generalized forms.

The time-dependent Maxwell equations are of the form:

$$\text{rot}\,\mathbf{H}=\mathbf{J}+\frac{\partial}{\partial t}\mathbf{D} \quad \text{(I)} \qquad \text{div}\,\mathbf{B}=0 \quad \text{(III)}$$

$$\text{rot}\,\mathbf{E}=-\frac{\partial}{\partial t}\mathbf{B} \quad \text{(II)} \qquad \text{div}\,\mathbf{D}=\varrho \quad \text{(IV)}$$

If the current or charge is known, the solution is given by the scalar and vector potentials or by the Hertz vector

$$\mathbf{H}=rot\mathbf{A} \tag{1}$$

$$\mathbf{E}=-\mu\frac{\partial}{\partial t}\mathbf{A}-\text{grad}\,\varphi \tag{2}$$

or

$$\mathbf{H}=\varepsilon\frac{\partial}{\partial t}\text{rot}\,\mathbf{\Pi} \tag{3}$$

$$\mathbf{E}=-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{\Pi}+\text{grad div}\,\mathbf{\Pi} \tag{4}$$

where

$$\mathbf{A}(\mathbf{r},t)=\frac{1}{4\pi}\int_{V'}\frac{1}{R}\mathbf{I}\left(\mathbf{r}',t-\frac{R}{c}\right)\mathrm{d}V' \tag{5}$$

$$\varphi(\mathbf{r},t)=\frac{1}{4\pi\varepsilon}\int_{V'}\frac{1}{R}\varrho\left(\mathbf{r}',t-\frac{R}{c}\right)\mathrm{d}V' \tag{6}$$

$$\mathbf{\Pi}(\mathbf{r},t)=\frac{1}{\varepsilon}\int_0^t \mathbf{A}(\mathbf{r},t)\,\mathrm{d}t \tag{7}$$

The distances in above formulae are as defined in Fig. 1. On the other hand, the gradient, divergence and curl operations here and in all the following formulae have to be calculated with the variable $\mathbf{r}\equiv(x, y, z)$, e.g. in terms of right-handed rectangular Cartesian "components" such as:

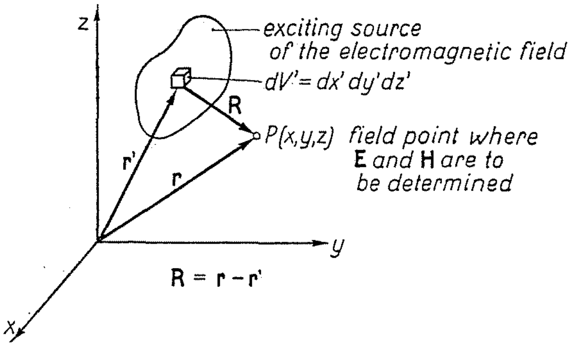

Fig. 1

$$\operatorname{grad} \varrho(\mathbf{r}', \mathbf{r}, t) \equiv \frac{\partial}{\partial x} \varrho(\mathbf{r}', \mathbf{r}, t)\mathbf{i} + \frac{\partial}{\partial y} \varrho(\mathbf{r}', \mathbf{r}, t)\mathbf{j} + \frac{\partial}{\partial z} \varrho(\mathbf{r}', \mathbf{r}, t)\mathbf{k}$$

$$\operatorname{div} \mathbf{J}(\mathbf{r}', \mathbf{r}, t) \equiv \frac{\partial}{\partial x} J_x(\mathbf{r}', \mathbf{r}, t) + \frac{\partial}{\partial y} J_y(\mathbf{r}', \mathbf{r}, t) + \frac{\partial}{\partial z} J_z(\mathbf{r}', \mathbf{r}, t)$$

$$\operatorname{rot} \mathbf{J}(\mathbf{r}', \mathbf{r}, t) \equiv \left(\frac{\partial}{\partial y} J_z(\mathbf{r}', \mathbf{r}, t) - \frac{\partial}{\partial z} J_y(\mathbf{r}', \mathbf{r}, t) \right) \mathbf{i} +$$

$$+ \left(\frac{\partial}{\partial z} J_x(\mathbf{r}', \mathbf{r}, t) - \frac{\partial}{\partial x} J_z(\mathbf{r}', \mathbf{r}, t) \right) \mathbf{j} + \left(\frac{\partial}{\partial x} J_y(\mathbf{r}', \mathbf{r}, t) - \frac{\partial}{\partial y} J_x(\mathbf{r}', \mathbf{r}, t) \right) \mathbf{k}$$

It is also to be noted that in our case the vector and scalar point functions **J** and ϱ depend on **r** via $\left(t - \frac{R}{c} \right)$ where

$$\left(t - \frac{R}{c} \right) = \left(t - \frac{1}{c} |\mathbf{r} - \mathbf{r}'| \right).$$

Derivation of formula **H**

The magnetic field at point **r** excited by an infinitesimal part of the source at point **r'** can be expressed using (1) and (5):

$$\mathrm{d}\mathbf{H} = \operatorname{rot} \mathrm{d}\mathbf{A} = \operatorname{rot} \left\{ \frac{1}{4\pi} \frac{1}{R} \mathbf{J} \left(\mathbf{r}', t - \frac{R}{c} \right) \mathrm{d}V' \right\}.$$

Thus we obtain

$$\mathrm{d}\mathbf{H} = \frac{1}{4\pi} \left[\operatorname{grad} \left(\frac{1}{R} \right) \times \mathbf{J} + \frac{1}{R} \operatorname{rot} \mathbf{J} \right] \mathrm{d}V'.$$

Now, we make use of the special form of vector $\mathbf{J}\left(\mathbf{r}', t - \frac{R}{c} \right)$ and replace its curl by its time derivative:

$$\operatorname{rot} \mathbf{J} \left(\mathbf{r}', t - \frac{R}{c} \right) = -\frac{1}{c} \mathbf{R}^\circ \times \frac{\partial}{\partial t} \mathbf{J} \left(\mathbf{r}', t - \frac{R}{c} \right)$$

where $\mathbf{R}^\circ = \frac{1}{R}\mathbf{R}$ a unit vector pointing in the same direction as **R**.

Since $\operatorname{grad}\left(\frac{1}{R}\right) = -\frac{1}{R^2}\mathbf{R}^\circ$ we obtain:

$$\mathrm{d}\mathbf{H} = \frac{1}{4\pi}\left(\frac{1}{R^2}\mathbf{J} + \frac{1}{cR}\frac{\partial}{\partial t}\mathbf{J}\right)\times\mathbf{R}^\circ\,\mathrm{d}V'. \tag{8}$$

The formula for a finite source

$$\mathbf{H}(\mathbf{r}, t) = \frac{1}{4\pi}\int_{V'}\left[\frac{1}{R^2}\mathbf{J}\left(\mathbf{r}', t-\frac{R}{c}\right) + \frac{1}{cR}\frac{\partial}{\partial t}\mathbf{J}\left(\mathbf{r}', t-\frac{R}{c}\right)\right]\times\mathbf{R}^\circ\,\mathrm{d}V' \tag{9}$$

or if the source is a current along a filamentary conductor:

$$\mathbf{H}(\mathbf{r}, t) = \frac{1}{4\pi}\oint_{l(\xi)}\left[\frac{1}{R^2}I\left(\xi, t-\frac{R}{c}\right) + \frac{1}{cR}\frac{\partial}{\partial t}I\left(\xi, t-\frac{R}{c}\right)\right](\mathbf{l}^\circ\times\mathbf{R}^\circ)\,\mathrm{d}\xi \tag{10}$$

where $\mathbf{l}^\circ = \mathbf{l}^\circ(\xi)$ is the tangent unit vector of the curve (filamentary conductor) and $R = R(\xi) = |\mathbf{r}'(\xi) - \mathbf{r}|$.

Formulae (9) or (10) give the exact solution of the magnetic field if the current density $\mathbf{J}(\mathbf{r}; t)$ is given or known. The first term in (9) or (10) gives the induction field, the second one the radiated field. Omitting the second terms, the formulae are equal to the Biot-Savart law. Rearranging (9) as:

$$\mathbf{H}(\mathbf{r}, t) = \frac{1}{4\pi}\int_{V'}\frac{1}{R^2}\left[\mathbf{J}\left(\mathbf{r}', t-\frac{R}{c}\right) + \frac{R}{c}\frac{\partial}{\partial t}\mathbf{J}\left(\mathbf{r}', t-\frac{R}{c}\right)\right]\times\mathbf{R}^\circ\,\mathrm{d}V'$$

it has the same form as the Biot-Savart law only then **J** has to be replaced by $\left(\mathbf{J} + \frac{R}{c}\frac{\partial}{\partial t}\mathbf{J}\right)$ and $\mathbf{J}$, $\frac{\partial}{\partial t}\mathbf{J}$ have to be retarded. Therefore (9) and (10) can be considered generalized Biot-Savart laws both by form and by substance.

Derivation of formula **E** *using* **J** *alone*

Using (4), (5) and (7) the electric field at point **r** excited by an infinitesimal source at point **r′** is

$$\mathrm{d}\mathbf{E} = \left[-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathrm{d}\mathbf{\Pi} + \operatorname{grad}\operatorname{div}\mathrm{d}\mathbf{\Pi}\right] =$$

$$= \frac{1}{4\pi\varepsilon}\left[-\frac{1}{c^2 R}\frac{\partial}{\partial t}\mathbf{J}\left(\mathbf{r}', t-\frac{R}{c}\right) + \operatorname{grad}\operatorname{div}\int_0^t\frac{1}{R}\mathbf{J}\left(\mathbf{r}', \tau-\frac{R}{c}\right)\mathrm{d}\tau\right]\mathrm{d}V'. \tag{11}$$

Evaluating the divergence in (11) the fact will be made use of again that the spatial derivative of **J** can be replaced by its time derivative, that is:

$$\operatorname{div}\mathbf{J}\left(\mathbf{r}', t-\frac{R}{c}\right) = -\frac{1}{c}\mathbf{R}^\circ\frac{\partial}{\partial t}\mathbf{J}\left(\mathbf{r}', t-\frac{R}{c}\right). \tag{12}$$

Thus

$$\operatorname{div} \int_0^t \frac{1}{R} \mathbf{J}\, d\tau = -\frac{1}{cR} \mathbf{R}^\circ \mathbf{J} - \frac{1}{R^2} \mathbf{R}^\circ \int_0^t \mathbf{J}\, d\tau.$$

Now, in evaluating the gradient we use the following vector identity

$$\nabla (\mathbf{F}\mathbf{G}) = (\mathbf{F}\nabla)\mathbf{G} + (\mathbf{G}\nabla)\mathbf{F} + \mathbf{F}\times(\nabla\times\mathbf{G}) + \mathbf{G}\times(\nabla\times\mathbf{F}) \tag{13}$$

where

$$(\mathbf{F}\nabla)\mathbf{G} = (\mathbf{F}\nabla G_x)i + (\mathbf{F}\triangle G_y)\mathbf{j} + (\mathbf{F}\triangle G_z)\mathbf{k}. \tag{14}$$

By (13) and (14)

$$\begin{aligned}\operatorname{grad}\left(\frac{1}{Rc}\mathbf{R}^\circ\mathbf{J}\right) &= -\frac{1}{R^2c}\mathbf{R}^\circ(\mathbf{R}^\circ\mathbf{J}) + \frac{1}{Rc}\operatorname{grad}(\mathbf{R}^\circ\mathbf{J}) = \\ &= -\frac{2}{R^2c}\mathbf{R}^\circ(\mathbf{R}^\circ\mathbf{J}) + \frac{1}{R^2c}\mathbf{J} - \frac{1}{Rc^2}\mathbf{R}^\circ\left(\mathbf{R}^\circ\frac{\partial}{\partial t}\mathbf{J}\right)\end{aligned} \tag{15}$$

and

$$\begin{aligned}\operatorname{grad}\left(\frac{1}{R^2}\mathbf{R}^\circ\int_0^t \mathbf{J}\, d\tau\right) &= -\frac{2}{R^3}\mathbf{R}^\circ\left(\mathbf{R}^\circ\int_0^t \mathbf{J}\, d\tau\right) + \frac{1}{R^2}\operatorname{grad}\left(\mathbf{R}^\circ\int_0^t \mathbf{J}\, d\tau\right) = \\ &= -\frac{3}{R^3}\mathbf{R}^\circ\left(\mathbf{R}^\circ\int_0^t \mathbf{J}\, d\tau\right) + \frac{1}{R^3}\int_0^t \mathbf{J}\, d\tau - \frac{1}{R^2c}\mathbf{R}^\circ(\mathbf{R}^\circ\mathbf{J})\end{aligned} \tag{16}$$

Using (15) and (16) the final formula for a finite source

$$\begin{aligned}\mathbf{E}(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon}\int_{V'} \Bigg\{ &\frac{1}{R^3}\left[-\int_0^t \mathbf{J}\, d\tau + 3\mathbf{R}^\circ\left(\mathbf{R}^\circ\int_0^t \mathbf{J}\, d\tau\right]\right. + \frac{1}{R^2c}[-\mathbf{J} + 3\mathbf{R}^\circ(\mathbf{R}^\circ\mathbf{J})] + \\ &+ \frac{1}{Rc^2}\left[-\frac{\partial}{\partial t}\mathbf{J} + \mathbf{R}^\circ\left(\mathbf{R}^\circ\frac{\partial}{\partial t}\mathbf{J}\right)\right]\Bigg\}\, dV'\end{aligned} \tag{17}$$

where

$$\mathbf{J} = \mathbf{J}\left(\mathbf{r}', t - \frac{R}{c}\right).$$

If the finite source is a current along a filamentary conductor formula, (17) looks like

$$\begin{aligned}\mathbf{E}(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon}\oint_{l(\xi)} \Bigg\{ &\frac{1}{R^3}\int_0^t I\left(\xi, \tau - \frac{R}{c}\right) d\tau[-\mathbf{l}^\circ + 3\mathbf{R}^\circ(\mathbf{R}^\circ\mathbf{l}^\circ)] + \frac{1}{R^2c} I\left(\xi, t - \frac{R}{c}\right)[-\mathbf{l}^\circ + \\ &+ 3\mathbf{R}^\circ(\mathbf{R}^\circ\mathbf{l}^\circ)] + \frac{1}{Rc^2}\frac{\partial}{\partial t} I\left(\xi, t - \frac{R}{c}\right)[-\mathbf{l}^\circ + \mathbf{R}^\circ(\mathbf{R}^\circ\mathbf{l}^\circ)]\Bigg\}\, d\xi\end{aligned} \tag{18}$$

where $\mathbf{l}^\circ = \mathbf{l}^\circ(\xi)$ is the tangent unit vector of the curve $l(\xi)$ and $R = R(\xi) = |\mathbf{r}(\xi) - \mathbf{r}|$.

Formule (17) and (18) give the exact solution of the electric field if the current density $\mathbf{J}(\mathbf{r}'\ t)$ is given or known. The first term in (17) or (18) gives the static field, the second one the induction field and the third one the radiated field. In some respects they can be regarded the generalized form of the Coulomb law.

Derivation of formula **E** *using* **J** *and* ϱ

With (2), (5) and (6) the electric field at point **r** excited by a finite source can be obtained with the same procedure as above

$$\mathbf{E}(\mathbf{r}, t)=\frac{1}{4\pi\varepsilon}\int\limits_{V'}\left[\frac{1}{R^2}\varrho\left(\mathbf{r}', t-\frac{R}{c}\right)\mathbf{R}^\circ+\frac{1}{Rc}\frac{\partial}{\partial t}\varrho\left(\mathbf{r}', t-\frac{R}{c}\right)\mathbf{R}^\circ-\right.$$
$$\left.-\frac{1}{Rc^2}\frac{\partial}{\partial t}\mathbf{J}\left(\mathbf{r}', t-\frac{R}{c}\right)\right]\mathrm{d}V' \tag{19}$$

and for a filamentary conductor

$$\mathbf{E}(\mathbf{r}, t)=\frac{1}{4\pi\varepsilon}\oint\left[\frac{1}{R^2}q\left(\xi, t-\frac{R}{c}\right)\mathbf{R}^\circ+\frac{1}{Rc}\frac{\partial}{\partial t}q\left(\xi, t-\frac{R}{c}\right)\mathbf{R}^\circ-\right.$$
$$\left.-\frac{1}{Rc^2}\frac{\partial}{\partial t}I\left(\xi, t-\frac{R}{c}\right)\mathbf{l}^\circ\right]\mathrm{d}\xi \tag{20}$$

where $\mathbf{l}^\circ=\mathbf{l}^\circ(\xi)$ is the tangent unit vector of the curve $l(\xi)$ and $R=R(\xi)=|\mathbf{r}(\xi)-\mathbf{r}|$.

Formulae (19) and (20) give the exact solution of the electric field if both the current density $\mathbf{J}(\mathbf{r}'\ t)$ and the charge density $\varrho(\mathbf{r}'\ t)$ are given or known. Here the first term produces the static field, the third one the radiated field. The second term produces the induction field, it contributes, however, to the radiated field, too. This last fact is obvious by comparing (19) and (17) or (20) and (18).

Regarding the application of the formulae derived, some remarks have to be made.

a) For the exact calculation of the electromagnetic field, exact knowledge of the exciting sources is necessary. In most cases, however, the exact dependance of the sources **J** and ϱ from $\mathbf{r}'$ is not known. These functions are, however, known at a good approximation in practical cases, permitting a good approximation of the solution.

b) Since using (17) or (18) for calculating the electric field we have to know the value of $\int\limits_0^t \mathbf{J}\,\mathrm{d}\tau$, the application of these formulae for stationary cases is disadvantageous. In these cases (19) or (20) are easier to use (see examples 1 and 2). Calculating the transient field, however, (17) or (18) seem to be more advantageous (see example 3).

Applications

Example 1

In practice, the question frequently arises in connection with the calculation of the electromagnetic field whether the well-known static, stationary formulae give a good approximation or the Maxwell equations have to be recoursed to (or e.g. the generalized formulae presented in this paper).

To answer this question let us consider a wire of length dl conducting a current $i(t)$.

The maximum value of the stationary magnetic field excited by this current element is:

$$\left| \frac{1}{R^2} i_{\max}(t) \mathbf{R}^\circ \times \mathbf{dl} \right|$$

and the radiated field

$$\left| \frac{1}{Rc} \left[\frac{\partial}{\partial t} i(t) \right]_{\max} \mathbf{R}^0 \times \mathbf{dl} \right|$$

Comparing the two formulae the stationary field appears to be stronger than the radiated one if

$$\frac{1}{R^2} i_{\max}(t) > \frac{1}{Rc} \left(\frac{\partial}{\partial t} i(t) \right)_{\max}$$

that is

$$R < c i_{\max} \Big/ \left(\frac{\partial}{\partial t} i \right)_{\max} \tag{21}$$

In most cases systems can be characterised by a dominant time constant T thus the current has the form:

$$I(t) = I_0(1 - e^{-t/T}).$$

Here time constant T is of the same order as the rise time τ that is commonly used and known. Since we want to determine the maximum value of the field (worst case), the time constant can be replaced by the rise time. Thus (21) can be written as:

$$R < c\tau. \tag{22}$$

Considering a numerical example, if the rise time is $1\,\mu$ sec, the stationary field is stronger than the radiated one in the range $R < 300$ m.

Now, let us consider the electric field with the formula (19). Comparing the first two terms, the static field is stonger if

$$\frac{1}{R^2}\varrho_{\max} > \frac{1}{Rc}\left(\frac{\partial}{\partial t}\varrho\right)_{\max}$$

This is true in the range $R < c\varrho_{\max} \Big/ \left(\frac{\partial}{\partial t}\varrho\right)_{\max}$ or $R < c\tau$. Comparing the effects of **J** and ϱ, the second one has a greater effect if $\varrho > \frac{1}{c}\mathbf{J}$.

This last relation can only be evaluated if the arrangement is known. For example the charge density on two infinite parallel wires of radius r and spaced at a distance d between centers is, by static approximation

$$\mathbf{q} \approx \frac{\pi\varepsilon}{\ln\frac{d}{r}} U.$$

Accordingly, the effect of the charge (generated by the voltage U connected to the wires) is greater if

$$I < c\frac{\pi\varepsilon}{\ln\frac{d}{r}} U.$$

Considering a numerical example where $d = 10$ cm, $r = 2.5$ mm and $U = 220$ V the effect of the voltage is greater if $I < 0.48$ A.

Now, assume current and voltage are not uniform along the conductor and the current and voltage waves travel approximately at light velocity c along the conductor.

A current of amplitude I_0 and rise time τ along a conductor of length L can be expressed as:

$$I(x,t) = \frac{I_0}{\tau}\left[1\left(t-\frac{x}{c}\right)\left(t-\frac{x}{c}\right) - 1\left(t-\tau-\frac{x}{c}\right)\left(t-\tau-\frac{x}{c}\right)\right]$$

and its time derivative

$$\frac{\partial}{\partial t} I(x,t) = \frac{I_0}{\tau}\left[1\left(t-\frac{x}{c}\right) - 1\left(t-\tau-\frac{x}{c}\right)\right].$$

If $\tau > \frac{L}{c}$ and $\tau > t > \frac{L}{c}$ then $\frac{\partial}{\partial t} I(x,t) = \frac{I_0}{\tau}$ along the whole conductor, thus the radiated magnetic field is approximately

$$\sim \frac{1}{Rc}\frac{I_0}{\tau} L$$

and the induction field is

$$\sim \frac{1}{R^2} I_0 L$$

wich gives the result obtained above, that is, the induction field is stronger in the range $R<c\tau$. But for $\tau<\frac{L}{c}$ the current is changing only along a part of length τc thus the radiated field is expressed by

$$\sim\frac{1}{Rc}\,\frac{I_0}{\tau}\,\tau c.$$

Comparing this value with that of the induction field we obtain that the induction field is greater than the radiated one in the range:

$$R<L.$$

Example 2

Let us consider two parallel wires shown in Fig. 2 excited by a voltage of amplitude U_0 and conducting a current $I(t)=I_0(1-e^{-t/T})$. Assume the current to be constant along the wire and $d\ll L$.

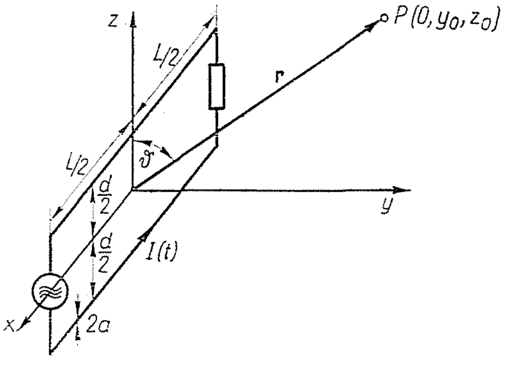

Fig. 2

We calculate the magnetic and electric fields in the yz plane using (10) and (20). The stationary magnetic field is:

$$\mathbf{H}_{\text{ind}}=\frac{1}{4\pi}\oint_L\frac{I}{R^2}\,\mathrm{d}\mathbf{l}\times\mathbf{R}^0\cong\left(\frac{I}{4\pi}\int_{-L/2}^{L/2}\left[r\cos\vartheta\left(\frac{1}{R_2^3}-\frac{1}{R_1^3}\right)+\frac{d}{2}\left(\frac{1}{R_1^3}+\frac{1}{R_2^3}\right)\right]\mathrm{d}x\right)\mathbf{j}+$$

$$+\left(\frac{I}{4\pi}\int_{-L/2}^{L/2}r\sin\vartheta\left(\frac{1}{R_1^3}-\frac{1}{R_2^3}\right)\mathrm{d}x\right)\mathbf{k}$$

where

$$R_1=\sqrt{x^2+(r\sin\vartheta)^2+(r\cos\vartheta-d/2)^2},$$

$$R_2=\sqrt{x^2+(r\sin\vartheta)^2+(r\cos\vartheta+d/2)^2}.$$

Calculating the integrals we obtain:

$$\mathbf{H}_{\mathrm{ind}}=\left(\frac{IL}{4\pi}\left(r\cos\vartheta(C_2-C_1)+\frac{d}{2}(C_1+C_2)\right)\mathbf{j}+\left(\frac{IL}{4\pi}r\sin\vartheta\,(C_1-C_2)\right)\mathbf{k}\right.$$

where

$$C_1=\frac{1}{[(r\sin\vartheta)^2+(r\cos\vartheta-d/2)^2]\sqrt{(L/2)^2+(r\sin\vartheta)^2+(r\cos\vartheta-d/2)^2}}$$

$$C_2=\frac{1}{[(r\sin\vartheta)^2+(r\cos\vartheta+d/2)^2]\sqrt{(L/2)^2+(r\sin\vartheta)^2+(r\cos\vartheta+d/2)^2}}$$

In most cases it is enough to determine the field at $\vartheta=0$ and $\vartheta=\pi/2$ thus the formula presented above is simplified.

The radiated magnetic field:

$$\mathbf{H}_{\mathrm{rad}}=\frac{1}{4\pi}\oint_L\frac{1}{Rc}\frac{\partial}{\partial t}I\left(t-\frac{R}{c}\right)\mathrm{d}\mathbf{l}\times\mathbf{R}^\circ\approx\left(\frac{1}{4\pi c}\int_{-L/2}^{L/2}\left[r\cos\vartheta\left(\frac{1}{R_2^2}\frac{\partial}{\partial t}I\left(t-\frac{R_1}{c}\right)-\right.\right.\right.$$

$$\left.\left.\left.-\frac{1}{R_2^2}\frac{\partial}{\partial t}I\left(t-\frac{R_2}{c}\right)\right)+\frac{d}{2}\left(\frac{1}{R_1^2}\frac{\partial}{\partial t}I\left(t-\frac{R_1}{c}\right)+\frac{1}{R_2^2}\frac{\partial}{\partial t}I\left(t-\frac{R_2}{c}\right)\right)\right]\mathrm{d}x\right)\mathbf{j}+$$

$$+\left(\frac{1}{4\pi c}\int_{-L/2}^{L/2}r\sin\vartheta\left(\frac{1}{R_1^2}\frac{\partial}{\partial t}I\left(t-\frac{R_1}{c}\right)-\frac{1}{R_2^2}\frac{\partial}{\partial t}I\left(t-\frac{R_2}{c}\right)\right)\mathrm{d}x\right)\mathbf{k}$$

The absolute value of the radiated magnetic field is simpler in form:

$$H_{\mathrm{rad}}=\frac{1}{4\pi c}\int_{-L/2}^{L/2}\left[\frac{1}{R_1}\frac{\partial}{\partial t}I\left(t-\frac{R_1}{c}\right)\sin\varphi_1-\frac{1}{R_2}\frac{\partial}{\partial t}I\left(t-\frac{R_2}{c}\right)\sin\varphi_2\right]\mathrm{d}x$$

where

$$\sin\varphi_1=\frac{r}{R_1}\sqrt{\sin^2\vartheta+(\cos\vartheta-d/2r)^2}$$

$$\sin\varphi_2=\frac{r}{R_2}\sqrt{\sin^2\vartheta+(\cos\vartheta+d/2r)^2}$$

The denominator can be simplified by approximation:

$$R_1\approx R_2\approx r$$

then

$$\sin\varphi_1\approx\sin\varphi_2\approx 1$$

thus, we obtain

$$H_{rad}=\frac{1}{4\pi}\int_{-L/2}^{L/2}\frac{1}{rc}\left[\frac{\partial}{\partial t}I\left(t-\frac{R_1}{c}\right)-\frac{\partial}{\partial t}I\left(t-\frac{R_2}{c}\right)\right]dx\approx$$

$$=\frac{dL\cos\vartheta}{4\pi c^2 r}\frac{\partial^2}{\partial t^2}I\left(t-\frac{r}{c}\right)$$

Considering that for the far field

$$E_{rad}/H_{rad}\approx\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

we obtain

$$E_{rad}\approx\frac{dL\cos\vartheta}{4\pi c^3\varepsilon r}\frac{\partial^2}{\partial t^2}I\left(t-\frac{r}{c}\right).$$

Since $I=I_0(1-e^{-t/T})$ the final formulae are

$$H_{rad}=\frac{dL\cos\vartheta}{4\pi c^2 r}\frac{I_0}{T^2}e^{-(t-r/c)/T}$$

$$E_{rad}=\frac{dL\cos\vartheta}{4\pi c^3\varepsilon r}\frac{I_0}{T^2}e^{-(t-r/c)/T}.$$

With the same procedure we obtain for the near electric field:

$$\mathbf{E}_{static}=\frac{\mathbf{q}L}{4\pi\varepsilon}\{(r(C_2-C_1)\sin\vartheta)\mathbf{j}+((r\cos\vartheta+d/2)C_2-(r\cos\vartheta-d/2)C_1)\mathbf{k}\}$$

where

$$q=\frac{U_0\varepsilon\pi}{\ln\frac{d}{a}}.$$

Example 3

Finally, let us consider a straight conductor conducting a current pulse at velocity v as it would be a transmission line.

It can be proven analytically that in a certain time interval radiated magnetic and electric field (far from the transmission line) have the same shape as the current pulse.

Since the conductor of length L is in the x axis, the current has the form:

$$I(x,t)=I\left(t-\frac{x}{v}\right)$$

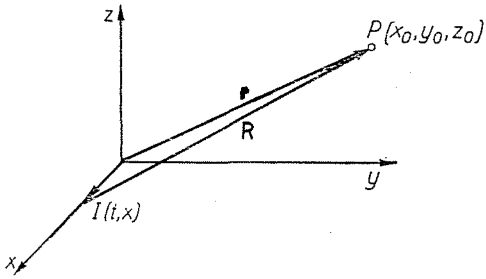

Fig. 3

Calculating the field at a great distance, only the radiation field terms dominate (10) and (18), thus

$$\mathbf{H}\approx\frac{1}{4\pi c}\int_0^T \frac{1}{R}\frac{\partial}{\partial t}I\left(t-\frac{x}{v}-\frac{R}{c}\right)\mathbf{R}^\circ\times\mathbf{i}\,\mathrm{d}x$$

$$\mathbf{E}\approx\frac{1}{4\pi\varepsilon c^2}\int_0^L \frac{1}{R}\left[-\frac{\partial}{\partial t}I\left(t-\frac{x}{v}-\frac{R}{c}\right)\mathbf{i}+\mathbf{R}^\circ\left(\mathbf{R}^\circ\mathbf{i}\frac{\partial}{\partial t}I\left(t-\frac{x}{v}-\frac{R}{c}\right)\right)\right]\mathrm{d}x.$$

If $L\ll R$ then $R\approx$constant thus

$$|\mathbf{H}|\approx\frac{1}{4\pi cR}\sin\varphi\int_0^L \frac{\partial}{\partial t}I\left(t-\frac{x}{v}-\frac{R}{c}\right)\mathrm{d}x$$

$$|\mathbf{E}|\approx\frac{1}{4\pi\varepsilon c^2 R}\sin\varphi\int_0^L \frac{\partial}{\partial t}I\left(t-\frac{x}{v}-\frac{R}{c}\right)\mathrm{d}x$$

where φ is the angle between the x axis and the vector pointing to P.

Since

$$\frac{\partial}{\partial t}I\left(t-\frac{x}{v}-\frac{R}{c}\right)=-v\frac{\partial}{\partial x}I\left(t-\frac{x}{v}-\frac{R}{c}\right)$$

the equations above can be written as:

$$H\approx\frac{\sin\varphi}{4\pi cR}v\left[I\left(t-\frac{R}{v}\right)-I\left(t-\frac{L}{v}-\frac{R}{c}\right)\right]$$

$$E\approx\frac{\sin\varphi}{4\pi c^2\varepsilon R}v\left[I\left(t-\frac{R}{v}\right)-I\left(t-\frac{L}{v}-\frac{R}{c}\right)\right].$$

For $R/v < t < L/v + R/c$ that is, the pulse did not reach the end of the transmission line;

$$H \approx \frac{v}{4\pi c R} \sin \varphi \, I\left(t - \frac{R}{c}\right)$$

$$E \approx \frac{v}{4\pi \varepsilon c^2 R} \sin \varphi I\left(t - \frac{R}{c}\right).$$

This result may be useful for calculating the electromagnetic field far from a power line when a short-cut happens or when a switching transient wave developes. On the other hand, by measuring the far field, parameters of a travelling wave on a power line can be determined.

Summary

General formulae simplifying the calculation of electromagnetic fields excited by currents with arbitrary time dependence are derived.

These may be very useful for calculating or estimating the interferencing electromagnetic fields generated by nonsinusoidal currents of power lines and equipment. Use of the formulae is shown on some practical examples likely of importance.

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