MULTI ELEMENT FAULT ISOLATION IN ELECTRONIC CIRCUITS*

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1. Introduction

A method of fault isolation in the linear circuits for a single faulty element has been suggested by Martens and Dyck[1] and others [2]–[3] using bilinear transformation. The method is graphical in nature, producing frequency domain loci which depend upon the value of the assumed faulty element. The faulty element is identified by the location of the test measurements of the equipment on a particular locus. Besides having the limitation of being applicable to a single element fault only, the method is cumbersome to run on an automatic computer. In this paper we present analytical expression for the identification of one or two faulty elements and derive a general algorithm for the multi elements fault case. The symbolic transfer function (or different transfer functions between different sets of terminals) is assumed to be known [4]–[6] The method of fault identification is explained by means of examples.

2. Problem statement and assumptions

The problem consists in isolating faulty elements (one or more) in a lumped, linear time invariant, active electronic network. The topology of the network and the nominal value of the components are assumed to be known, so that the symbolic transfer function can be obtained. It is further assumed that failures are not catastrophic and enough test terminals are available to obtain frequency response which is dependent on the value of elements which are faulty.

3. Method of solution

3.1. Single faulty element

Let the various elements such as transistors, resistors etc. of the network be symbolized by $p_1, p_2, \ldots, p_n$, n being the total number of elements. The nominal value of these elements be given by $p_{10}, p_{20}, \ldots, p_{no}$.

* The paper is written after Prof. Puri's lecture held with the same title at the Technical University of Budapest in 16th April of 1976.

Let

$$0<\frac{p_i}{p_{i0}}=\chi_i, \qquad i=1, \ldots, n \qquad 3.1.1$$

when all χ_i, $(i=1, \ldots, n)$ are unity, the equipment is not faulty.

Let us consider the situation when a particular k^{th} component is faulty, such that $\chi_k \neq 1$. Problem reduces to determining all different χ's and thus the faulty element and its value. The system transfer function $T(s)$ can be written in n different forms[7] as

$$T(s)=T(s, \chi_k)=\frac{A_k(s)\chi_k+B_k(s)}{C_k(s)\chi_k+D_k(s)}, \qquad (k=1, \ldots, n) \qquad 3.1.2$$

Let us obtain the frequency of this equipment at one particular frequency ω, involving phase and amplitude measurement and hence the real $R(\omega)$ and imaginary part $x(\omega)$, yielding

$$T(j\omega)=R(\omega)+j\chi(\omega)=\frac{A_k(j\omega)\chi_k+B_k(j\omega)}{C_k(j\omega)\chi_k+D_k(j\omega)}, \qquad (k=1, \ldots, n). \qquad 3.1.2$$

The test frequency is chosen with regard to the nominal locations of poles and zeros, as discussed by Seshu and Waxman[8].

Let

$$\begin{aligned} A_k(j\omega)&=A_{k_1}(\omega)+jA_{k_2}(\omega)\\ B_k(j\omega)&=B_{k_1}(\omega)+jB_{k_2}(\omega)\\ C_k(j\omega)&=C_{k_1}(\omega)+jC_{k_2}(\omega)\\ D_k(j\omega)&=D_{k_1}(\omega)+jD_{k_2}(\omega). \end{aligned} \qquad 3.1.4$$

Substituting 3.1.4 into 3.1.3 and equating real and imaginary part,

$$\begin{aligned} &[R(\omega)C_{k_1}(\omega)-\chi(\omega)C_{k_2}(\omega)]\chi_k+[R(\omega)D_{k_1}(\omega)-\chi(\omega)D_{k_2}(\omega)]=A_{k_1}(\omega)\chi_k+B_{k_1}(\omega)\\ &[R(\omega)C_{k_2}(\omega)+\chi(\omega)C_{k_1}(\omega)]\chi_k+[R(\omega)D_{k_2}(\omega)+\chi(\omega)D_{k_1}(\omega)]=A_{k_2}(\omega)\chi_k+B_{k_2}(\omega). \end{aligned} \qquad 3.1.5$$

Relationships 3.1.5 are only true if the kth element symbolized by χ_k is the faulty element. The quantities $A_{k_1}(\omega)$ etc. are computed for the nominal values of other elements (except of course the kth element). Since both equations 3.1.5 should give the same value of χ_k, the condition that kth element is indeed the faulty one is given by eliminating χ_k from equations 3.1.5 to yield

$$\begin{aligned} &[\chi^2(\omega)+R^2(\omega)][C_{k_1}(\omega)D_{k_2}(\omega)-C_{k_2}(\omega)D_{k_1}(\omega)]+\\ &+R(\omega)[D_{k_1}(\omega)A_{k_2}(\omega)+C_{k_2}(\omega)B_{k_1}(\omega)-D_{k_2}(\omega)A_{k_1}(\omega)-C_{k_1}(\omega)B_{k_2}(\omega)]+\\ &+\chi(\omega)[C_{k_1}(\omega)B_{k_1}(\omega)+C_{k_2}(\omega)B_{k_2}(\omega)-D_{k_1}(\omega)A_{k_1}(\omega)-D_{k_2}(\omega)A_{k_2}(\omega)]+\\ &+(A_{k_1}B_{k_2}-A_{k_2}B_{k_1})=\Delta_k(\omega)=0 \end{aligned} \qquad 3.1.6$$

Theoretically only frequency measurement at one frequency is necessary, but due to noise consideration, the condition 3.1.6 may be verified at different frequencies. If

$$\sqrt{\frac{1}{m}\sum_{i=1}^{m}\Delta_k^2(\omega_i)}$$

is less than some precomputed number ϵ, then the condition 3.1.6 is considered fulfilled. Furthermore, value of χ_k has to be positive.

3.2. Example

Consider simple circuit given in Figure 3.2.1. Let the nominal values of these various elements be

$$\begin{aligned} R_1 &= R_{10} = 1000\ \text{k}\Omega \\ C &= C_0 = 1\ \mu\text{F} = 10^{-6}\text{F} \\ R_2 &= R_{20} = 3000\ \text{k}\Omega \end{aligned} \qquad 3.2.1$$

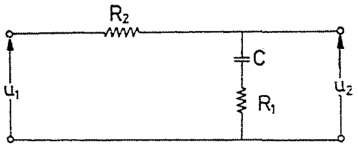

Fig. 1

The actual values of these elements are such that the frequency response $\left[\frac{u_2(s)}{u_1(s)}\right]$ at $\omega=1$ is

$$R(\omega)=\frac{11}{26}$$
$$X(\omega)=-\frac{3}{26}\,. \qquad 3.2.2$$

Only one element is considered off the nominal value. We are required to find this element and its value.

Let

$$\frac{R_1}{R_{10}}=\chi_1, \qquad \frac{R_2}{R_{20}}=\chi_2, \qquad \frac{C}{C_0}=\chi_3 \qquad 3.2.3$$

$$T(s)=\frac{u_2(s)}{u_1(s)}=\frac{1+ScR_1}{1+Sc(R_1+R_2)}=\frac{A(s)\chi+B(s)}{C(s)\chi+D(s)}\,. \qquad 3.2.4$$

Thus

$$T(s,\chi_1)=T_1(s)=\frac{1+SC_0R_{10}\chi_1}{1+SC_0(R_{10}\chi_1+R_{20})} \qquad 3.2.5$$

$$T(s,\chi_2)=T_2(s)=\frac{1+SC_0R_{10}}{1+SC_0(R_{10}+R_{20}\chi_2)} \qquad 3.2.6$$

$$T(s,\chi_3)=T_3(s)=\frac{1+SC_0R_{10}\chi_3}{1+SC_0(R_{10}+R_{20})\chi_3} \qquad 3.2.7$$

$$\begin{array}{lll}
A_{11}(\omega)=0 & A_{21}(\omega)=0 & A_{31}(\omega)=0 \\
A_{12}(\omega)=\omega C_0 R_{10}=1 & A_{22}(\omega)=0 & A_{32}(\omega)=\omega C_0 R_{10}=1 \\
B_{11}(\omega)=1 & B_{21}(\omega)=1 & B_{31}(\omega)=1 \\
B_{12}(\omega)=0 & B_{22}(\omega)=\omega C_0 R_{10}=1 & B_{32}(\omega)=0 \\
C_{11}(\omega)=0 & C_{21}(\omega)=0 & C_{31}(\omega)=0 \\
C_{12}(\omega)=\omega C_0 R_{10}=1, & C_{22}(\omega)=\omega C_0 R_{20}=3, & C_{32}(\omega)=\omega C_0 R_{10}+R_{20}=4 \\
D_{11}(\omega)=1 & D_{21}(\omega)=1 & D_{31}(\omega)=1 \\
D_{12}(\omega)=\omega C_0 R_{20}=3 & D_{22}(\omega)=\omega C_0 R_{10}=1 & D_{32}(\omega)=0.
\end{array}$$

Substituting these values in 3.1.6, we obtain

$$\Delta_1=\frac{130}{(26)^2}(-1)+\frac{11}{26}(2)-\frac{3}{26}(-3)-1=0 \qquad 3.2.8$$

$$\Delta_2=\frac{130}{(26)^2}(-3)+\frac{11}{26}(3)-\frac{3}{26}(3)=\frac{4}{13} \qquad 3.2.9$$

$$\Delta_3=\frac{130}{(26)^2}(-4)+\frac{11}{26}(5)-\frac{3}{26}(0)-1=\frac{9}{26} \qquad 3.2.10$$

Thus element R_1 is faulty and its value is

$$\chi_1=\chi_k=\frac{B_{k_1}(\omega)-R(\omega)D_{k_1}(\omega)+\chi(\omega)D_{k_2}(\omega)}{R(\omega)C_{k_1}(\omega)-\chi(\omega)C_{k_2}(\omega)-A_{k_1}(\omega)}=\left(\frac{26-20}{26}\right)\left(\frac{26}{3}\right)=2 \qquad 3.2.11$$

Hence actual value of R_1 is 2000 K.
In case none of the determinants $\Delta_k(\omega)$ are zero, then there is more than one element which is faulty. In case of degeneracy such function such as input impedance is used for identification[9].

3.3. *Fault with two faulty elements*

Let k_1 and k_2 be two faulty elements symbolized by χ_{k_1} and χ_{k_2} respectively. The transfer function can be written as

$$T(s)=T(s,k_1,k_2)=\frac{A_{k_1,k_2}(s)\chi_{k_1}\chi_{k_2}+B_{k_1,k_2}(s)\chi_{k_1}+C_{k_1,k_2}(s)\chi_{k_2}+D_{k_1,k_2}(s)}{E_{k_1,k_2}(s)\chi_{k_1}\chi_{k_2}+F_{k_1,k_2}(s)\chi_{k_1}+G_{k_1,k_2}(s)\chi_{k_2}+H_{k_1,k_2}(s)} \qquad 3.3.1$$

where $A_{k_1,k_2}(s)$ etc. are polynomials in s.
Let
$A_{k_1,k_2}(j\omega)=A_1(\omega)+jA_2(\omega)$ etc. dropping the indices k_1, k_2 for convenience

$$T(j)=R(\omega)+j\chi(\omega)$$

The real and imaginary part of 3.3.1 can yield the following two equations.

$$[R(\omega)E_1(\omega)-\chi(\omega)E_2(\omega)-A_1(\omega)]\chi_{k_1}\chi_{k_2}+[(R(\omega)F_1(\omega)-\chi(\omega)F_2(\omega)-B_1(\omega)]\chi_k+$$
$$+[R(\omega)G_1(\omega)-\chi(\omega)G_2(\omega)-C_1(\omega)]_{k_2}+[R(\omega)H_1(\omega)-\chi(\omega)H_2(\omega)-D_1(\omega)]=0 \qquad 3.3.2$$

$$[R(\omega)E_2(\omega)+\chi(\omega)E_1(\omega)-A_2(\omega)]\chi_{k_1}\chi_{k_2}+[R(\omega)F_2(\omega)+\chi(\omega)F_1(\omega)-B_2(\omega)]\chi_{k_1}+$$
$$+[R(\omega)G_2(\omega)+\chi(\omega)G_1(\omega)-C_2(\omega)]\chi_{k_2}+[R(\omega)H_2(\omega)+\chi(\omega)H_1(\omega)-D_2(\omega)]=0 \quad 3.3.3$$

These equations can be rewritten as

$$P(\omega)\chi_{k_1}\chi_{k_2}+Q(\omega)\chi_{k_1}+U(\omega)\chi_{k_2}+V(\omega)=0 \quad 3.3.4$$

$$K(\omega)\chi_{k_1}\chi_{k_2}+L(\omega)_{k_1}+M(\omega)_{k_2}+N(\omega)=0 \quad 3.3.5$$

where

$$\begin{aligned}
P(\omega)&=R(\omega)E_1(\omega)-\chi(\omega)E_2(\omega)-A_1(\omega)\\
Q(\omega)&=R(\omega)F_1(\omega)-\chi(\omega)F_2(\omega)-B_1(\omega)\\
U(\omega)&=R(\omega)G_1(\omega)-\chi(\omega)G_2(\omega)-C_1(\omega)\\
V(\omega)&=R(\omega)H_1(\omega)-\chi(\omega)H_2(\omega)-D_1(\omega)\\
K(\omega)&=R(\omega)E_2(\omega)+\chi(\omega)E_1(\omega)-A_2(\omega)\\
L(\omega)&=R(\omega)F_2(\omega)+\chi(\omega)F_1(\omega)-B_2(\omega)\\
M(\omega)&=R(\omega)G_2(\omega)+\chi(\omega)G_1(\omega)-C_2(\omega)\\
N(\omega)&=R(\omega)H_2(\omega)+\chi(\omega)H_1(\omega)-D_2(\omega).
\end{aligned} \quad 3.3.6$$

Let us now perform the frequency response test at two different frequencies ω_1 and ω_2 yielding $R(\omega_1)+j\chi(\omega_1)$ and $R(\omega_2)+j\chi(\omega_2)$. Substituting these in 3.3.6

$$P(\omega)|_{\omega=1}=p_1 \quad \text{etc., we obtain}$$
$$P(\omega)|_{\omega=2}=p_2$$

$$\begin{aligned}
&p_1\chi_{k_1}\chi_{k_2}+q_1\chi_{k_1}+u_1\chi_{k_2}+v_1=0\\
&k_1\chi_{k_1}\chi_{k_2}+l_1\chi_{k_1}+m_1\chi_{k_2}+n_1=0\\
&p_2\chi_{k_1}\chi_{k_2}+q_2\chi_{k_1}+u_2\chi_{k_2}+v_2=0\\
&k_2\chi_{k_1}\chi_{k_2}+l_2\chi_{k_1}+m_2\chi_{k_2}+n_1=0
\end{aligned} \quad 3.3.7$$

eliminating the product terms, these equations can be rewritten as

$$\begin{aligned}
&a_{11}\chi_{k_1}+a_{12}\chi_{k_2}+a_{13}=0\\
&a_{21}\chi_{k_1}+a_{22}\chi_{k_2}+a_{23}=0\\
&a_{31}\chi_{k_1}+a_{32}\chi_{k_2}+a_{33}=0
\end{aligned} \quad 3.3.8$$

where

$$a_{11}=\frac{q_1}{p_1}-\frac{l_1}{k_1}, \qquad a_{12}=\frac{u_1}{p_1}-\frac{m_1}{k_1}, \qquad a_{13}=\frac{v_1}{p_1}-\frac{n_1}{k_1}$$
$$a_{21}=\frac{q_1}{p_1}-\frac{q_2}{p_2}, \qquad a_{22}=\frac{u_1}{p_1}-\frac{u_2}{p_2}, \qquad a_{23}=\frac{v_1}{p_1}-\frac{v_2}{p_2}$$
$$a_{31}=\frac{q_1}{p_1}-\frac{l_2}{k_2}, \qquad a_{32}=\frac{u_1}{p_1}-\frac{m_2}{k_2}, \qquad a_{33}=\frac{v_1}{p_1}-\frac{n_2}{k_2}$$

The condition that the elements k_1 and k_2 are faulty is given as

$$\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33} \end{vmatrix}=\Delta_A=0, \qquad \chi_1\geqq 0, \qquad \chi_2\geqq 0 \quad 3.3.9$$

In case 3.3.9 is satisfied, the variables χ_{k_1} and χ_{k_2} can be solved from the first two equations 3.3.8. There are circuits where degeneracy occurs and one transfer function is not sufficient in itself to obtain the faulty elements. In such a situation more than one function is necessary to identify the faulty elements. Thus in case of example shown in Fig. 4.3.1, the first two equations of 3.3.7 are obtained by one function and the other two by another independent function.

Example for two faulty elements:

Consider the example in Fig. 4.3.1. Two elements are assumed faulty. At a frequency $\omega=1$, the transfer function frequency response and input admittance frequency response are given as

$$T(I)=\frac{13}{37}-j\frac{4}{37}, \qquad Y(I)=\left(\frac{7}{37}-j\frac{5}{37}\right)10^{-6}V, \qquad 3.3.10$$

We are required to identify the faulty component. This is a degenerate network where different combinations of R_1, R_2 and C may yield the same transfer function. Hence two different functions are used for identification. Three different possible sets are:

$$\left.\begin{aligned} T_{12}(s)&=\frac{1+SC_{10}R_{10}\chi_1}{1+SC_{10}R_{10}\chi_1+SC_{10}R_{20}\chi_2}=\frac{1+S\chi_1}{1+S\chi_1+3S\chi_2} \\ Y_{12}(s)&=\frac{1+SC_0}{1+SC_0R_{10}\chi_1+SC_0R_{20}\chi_2}=\frac{(1+S)}{1+S\chi_1+3S\chi_2} \end{aligned}\right] R_1 \text{ and } R_2 \text{ are faulty} \qquad 3.3.11$$

$$\left.\begin{aligned} T_{23}(s)&=\frac{1+SR_{10}C_0\chi_3}{1+SR_{10}C_0\chi_3+SR_{20}C_0\chi_2\chi_3}=\frac{1+S\chi_3}{1+S\chi_3+3S\chi_2\chi_3} \\ Y_{23}(s)&=\frac{1+SC_0\chi_3}{1+SR_{10}C_0\chi_3+SR_{20}C_0\chi_2\chi_3}=\frac{1+S\chi_3}{1+S\chi_3+3S\chi_2\chi_3} \end{aligned}\right] R_2 \text{ and } C \text{ are faulty} \qquad 3.3.12$$

$$\left.\begin{aligned} T_{31}(s)&=\frac{1+SR_{10}C_0\chi_1\chi_3}{1+SR_{10}C_0\chi_1\chi_3+SR_{20}C_0\chi_3}=\frac{1+S\chi_1\chi_3}{1+S\chi_1\chi_3+3S\chi_3} \\ Y_{31}(s)&=\frac{1+SC_0\chi_3}{1+SR_{10}C_0\chi_1\chi_3+SR_{20}C_0\chi_3}=\frac{1+S\chi_3}{1+S\chi_1\chi_3+3S\chi_3} \end{aligned}\right] R_1 \text{ and } C \text{ are faulty} \qquad 3.3.13$$

Case 1. R_1 and R_2 are considered faulty.

From $T_{12}(s)$, $A=E=C=0$, $D=H=1$, $B=F=j$, $G=3j$

$$P=K=0, \quad Q=\frac{4}{37}, \quad U=\frac{12}{37}, \quad V=\frac{24}{37}$$

$$L=\frac{24}{37}, \quad M=\frac{39}{37}, \quad N=\frac{4}{37}.$$

Equations for variables χ_1 and χ_2, from $T_{12}(s)$, are

$$\chi_1+3\chi_2=6$$
$$-24\chi_1+39\chi_2=4.$$

Yielding $\chi_1=2$, $\chi_2=4/3$.
From $Y_{12}(s)$

$$P=0,\quad Q=5/37,\quad U=15/37,\quad V=-30/37$$
$$K=0,\quad L=7/37,\quad M=21/37,\quad N=-212/37.$$

The two equations in variable χ_1 and χ_2 obtained from $Y_{12}(s)$ degenerate into one equation

$\chi_1+3\chi_2=6$, satisfied by the solutions

$\chi_1=2$ and $\chi_2=4/3$, confirming that indeed resistors R_1 and R_2 are the faulty components. Their true value is $R_1=2000$ KΩ, $R_2=4000$ KΩ.

Case 2. R_2 and C are considered faulty.

From $T_{23}(s)$

$$A=B=0,\quad C=j,\quad D=1,\quad E=3j,\quad F=0,\quad G=j,\quad H=1$$
$$P=\frac{12}{37},\quad Q=0,\quad U=\frac{4}{37},\quad V=-\frac{24}{37}$$
$$K=\frac{39}{37},\quad L=0,\quad M=-\frac{24}{37},\quad N=-\frac{4}{37}.$$

The equations in variables χ_2 and χ_3 are

$$12\chi_2\chi_3+4\chi_3=24$$
$$39\chi_2\chi_3-24\chi_3=4$$

The solution is

$$\chi_3=2,\quad \chi_2=2/3.$$

From $Y_{23}(s)$, which is the same as $T_{23}(s)$

$$P=\frac{15}{37},\quad Q=0,\quad U=5/37,\quad V=-\frac{30}{37}$$
$$K=\frac{21}{37},\quad L=0,\quad M=-\frac{30}{37},\quad N=-\frac{5}{37}.$$

The two equations for variables χ_2 and χ_3 are

$$15\chi_2\chi_3+5\chi_3=30$$
$$21\chi_2\chi_3-30\chi_3=5$$

yielding the solution $\chi_3=1$ $\chi_2=5/3$. Since this solution is not the same as obtained from $T_{23}(s)$, the elements R_2 and C are not the faulty elements as a pair.

Case 3. R_1 and C are considered faulty.

From $T_{31}(s)$

$$A=j\omega,\quad B=0,\quad C=0,\quad D=1$$
$$E=j\omega,\quad F=0,\quad G=j3\omega,\quad H=1$$
$$P=\frac{4}{37},\quad Q=0,\quad U=\frac{12}{37},\quad V=-\frac{24}{37},$$
$$K=-\frac{24}{37},\quad L=0,\quad M=\frac{39}{37},\quad N=-\frac{4}{37}$$

The equations in χ_1 and χ_3, from the function $T_{31}(s)$ are:

$$\chi_1\chi_3+3\chi_3=6$$
$$-24\chi_1\chi_3+39\chi_3=4$$

Thus

$$\chi_1=3/2,\qquad \chi_3=\frac{4}{3}.$$

From $Y_{31}(s)$

$$A=B=0,\quad C=j\omega,\quad D=1$$
$$E=j\omega,\quad F=0,\quad G=j3\omega,\quad H=1$$
$$P=\frac{5}{37},\quad Q=0,\quad U=\frac{15}{37},\quad V=-\frac{30}{37}$$
$$K=\frac{7}{37},\quad L=0,\quad M=-\frac{16}{37},\quad N=-5/37.$$

The equations in χ_1 and χ_3 from function $Y_{31}(s)$ are

$$\chi_1\chi_3+3\chi_3-6=0$$
$$7\chi_1\chi_3-16\chi_3-5=0$$

Yielding

$$\chi_1=3,\quad \chi_3=1.$$

Solutions from $Y_{13}(s)$ and $T_{13}(s)$ do not agree with each other, thus R_1 and C are not the faulty pair. The conclusion is that R_1 and R_2 are the faulty resistors having actual values of 2000 K and 4000 K, respectively.

3.4. Fault with three or more simultaneously faulty elements

With three faulty elements to be simultaneously identified either we need four different function to be measured at one frequency (real and imaginary part) or one function at four different frequencies (if no degeneracy occurs). In general, for a m-element fault 2^{m-1} measurements (real and imaginary part) of different functions, or one function test at 2^{m-1} different frequencies is performed. For three-element fault, the equations obtained are of the form

$$a^{(i)}_{k_1k_2k_3}\chi_{k_1}\chi_{k_2}\chi_{k_3}+a^{(i)}_{k_1k_2}\chi_{k_1}\chi_{k_2}+a^{(i)}_{k_2k_3}\chi_{k_2}\chi_{k_3}+a^{(i)}_{k_3k_1}\chi_{k_3}\chi_{k_1}$$
$$+a^{(i)}_{k_1}\chi_{k_1}+a^{(i)}_{k_2}\chi_{k_2}+a^{(i)}_{k_3}\chi_{k_3}+a_0^{(i)}=0, \qquad (i=1, \ldots, 8),$$
$$(k_1\neq k_2\neq k_3,\ k_1, k_2, k_3=1, \ldots, n). \qquad 3.4.1$$

The product terms are eliminated by successive subfractions to obtain equations of the form

$$\begin{aligned} a_{11}\chi_{k_1}+a_{12}\chi_{k_2}+a_{13}\chi_{k_3}+a_{14}&=0\\ a_{21}\chi_{k_1}+a_{22}\chi_{k_2}+a_{23}\chi_{k_3}+a_{24}&=0\\ a_{31}\chi_{k_1}+a_{32}\chi_{k_2}+a_{33}\chi_{k_3}+a_{34}&=0\\ a_{41}\chi_{k_1}+a_{42}\chi_{k_2}+a_{43}\chi_{k_3}+a_{44}&=0. \end{aligned} \qquad 3.4.2.$$

The condition that k_1, k_2 and k_3 are the faulty elements is given by

$$\Delta_A=\text{det of } A=0, \qquad A=\{a_{ij}\}, \quad i, j=1, \ldots, 4 \qquad 3.4.3$$
$$\chi_{k_1}\geqq 0, \qquad \chi_{k_2}\geqq 0, \qquad \chi_{k_3}\geqq 0 \qquad 3.4.4$$

The quantities χ_{k_1}, χ_{k_2}, χ_{k_3} are solved by the first three equations. The extension to an m-element fault is obvious.

4. Conclusion

A method has been presented for the isolation of a fault where m elements are simultaneously faulty. In general 2^{m-1} different measurements (real and imaginary part) are made to identify the faulty elements. In case the transfer function in itself cannot identify the faulty components, other functions such as input impedance or transfer function between different sets of terminals is used. The method is suitable for automatic fault identification via digital computer.

Summary

A method is developed for the identification of each of the faulty elements in a linear electronic circuit when one or more of the elements are simultaneously off from their nominal value. Exact analytical expressions are presented for the identification of one or two simultaneouly faulty elements, along with the algorithm for the multiple faulty elements case. In general for an m element fault, 2^{m-1} different measurements are made (real and imaginary part). These measurements may involve transfer function frequency response at 2^{m-1} different frequencies, or in case of degeneracy, different functions such as input impedances, etc, such that total different measurements are 2^{m-1}. Method is discussed by means of examples.

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