FIELD DISTRIBUTION OF A ROTATIONALLY SYMMETRIC PERIODIC ELECTROSTATIC FOCUSING SYSTEM*

By

S. Gallai and M. Szilágyi

Department of Physical Sciences, Kandó Kálmán Institute of Electrical Engineering

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List of symbols

$x = \frac{2\pi z}{p}$ and r	variables of the cylindrical co-ordinate system
p	period of the focusing system
r_0	radius of the focusing cylinder
$\alpha = \frac{2\pi c}{p}$	size of the gap between the focusing electrodes
U_F and U_f	electrode voltages
$\varphi_{(x,r)} = \frac{2U(x,r)}{U_F - U_f}$	normalized potential function
$\Phi_{(x)} = \varphi_{(x,0)}$	normalized axial potential
$\varphi_0 = \frac{U_F + U_f}{U_F - U_f}$	normalized average potential
φ_{2k+1}	coefficient of the $(2k+1)$-order member of the normalized potential function
$I_0(x)$	first kind zero order modified Bessel function
C_n	coefficient of the n-th member of the Fourier series
$F(x)$	potential function in the gap
$f(x)$	approximated potential function in the gap

Periodic electrostatic focusing is a well-known method of focusing long cylindrical electron beams widely used in practice. Some difficulties arise in connection with the design of focusing systems since the boundary condition of Laplace's equation describing the field in the gap between the electrodes is not known. The published solutions [2, 3, 4, 5] all substitute some simple function — "comfortable" in computation — for the field in the gap. These approximations are mostly rough and the arising errors will tamper with the potential distribution of the field.

For the theoretical determination of focusing parameters, electron trajectories have to be known for which the potential distribution of the focusing field must be given. In the case of thick beams it is indispensable to know the field far from the axis. The potential function in the gap between the electrodes must be such as to satisfy Laplace's equation both inside the

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electrodes ($r < r_0$) and in the filed outside the electrodes ($r > r_0$). As the solution of the differential equation is searched for in form of an infinite series, meeting the former condition leads to lengthy mathematical computations. The problem can be solved by determining the value of the potential function in he gap in an experimental way by using a resistance network or electrolytic tank analogue. Then the obtained function is approximated.

The aim of our work is to determine the field distribution of the focusing system assuming the potential function in the gap to be of order $(2K + 1)$, approaching reality rather well. (By an appropriate adoption of the co-ordinate system the second-order symmetry can be made a good use of.)

In Fig. 1 the scheme and the potential function of a simple cylindrical

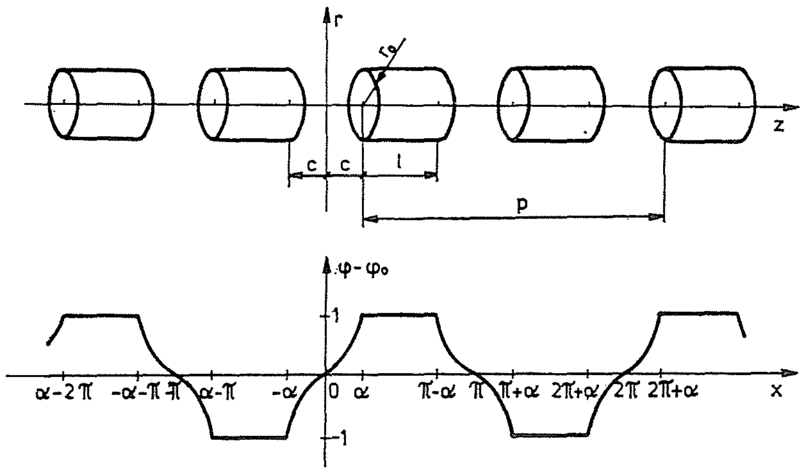

Fig. 1.

periodic electrostatic focusing system are presented, permitting to write the normalized potential function of the boundary value:

$$\varphi(x, r_0) - \varphi_0 = \begin{cases} \sum_{k=0}^{K} \frac{\varphi_{2k+1}}{\alpha^{2k+1}} x^{2k+1}, & \text{if } 0 \leqq x \leqq \alpha \quad \text{(1a)} \\ 1, & \text{if } \alpha \leqq x \leqq \frac{\pi}{2} . \quad \text{(1b)} \end{cases}$$

The condition (1b) is the consequence of the normalization of the potential function. For $x = \alpha$ the conditions (1a) and (1b) simultaneously hold. It follows that

$$\sum_{k=0}^{K} \varphi_{2k+1} = 1. \tag{2}$$

Let the coefficients φ_{2k+1} now be regarded as known constants. Their definition will follow later.

To determine the maximum order of approximation, let us deal with the number of the points of measurement in the gap. In case of the geometrical arrangement shown in Fig. 1 (second and third-order symmetry) the order of the approximating function $f(x)$ is limited by the number of measurement points available in the gap width c. If the number of measurement points is M in the interval c, the following relationship holds:

$$M \geqq K + 1, \tag{3}$$

where K is the superscript in (1a).

If the boundary value is known the solution of Laplace's equation in cylindrical co-ordinates is:

$$\varphi(x, r) - \varphi_0 = \sum_{n=1}^{\infty} C_n \sin(nx)\, I_0(nr)\,. \tag{4}$$

The coefficients C_n are determined by equating the substitutive value of (4) at $r = r_0$ and the relations (1a, b) and considering the orthogonality of trigonometric functions, as well. Defining the C_n calues the potential distribution in the gap will be

$$\varphi_{(x, r_0)} = \varphi_0 + \frac{4}{\pi} \sum_{n=0}^{\infty} A_{(n,K,\alpha)} \sin(2n+1)\, x, \tag{5}$$

where:

$$A_{(n,K,\alpha)} = \frac{\cos(2n+1)\alpha}{2n+1} - \sum_{k=0}^{K} \varphi_{2k+1}(2k+1)! \sum_{l=0}^{k} \frac{(-1)^l}{[2(k-l)+1]!\,[(2n+1)\alpha]^{2l+1}} \times$$
$$\times \left[\frac{2(k-l)+1}{2n+1} \sin(2n+1)\alpha - \alpha\cos(2n+1)\,\alpha\right]. \tag{5a}$$

When defining the normalized potential function $\varphi(x, r)$, getting its approximate value of a finite number of members is sufficient.

After (5) is known the solution of the differential equation becomes:

$$\varphi(x, r) = \varphi_0 + \frac{4}{\pi} \sum_{n=0}^{N} A_{(n,K,\alpha)} \frac{I_0\left[(2n+1)\frac{2\pi r}{p}\right]}{I_0\left[(2n+1)\frac{2\pi r_0}{p}\right]} \sin(2n+1)\, x. \tag{6}$$

From expression (6) the normalized axial potential $\Phi(x) = \varphi(x, 0)$ can be given considering $I_0(0) = 1$.

Expressions derived for a third-order approximation ($K = 1$) with $N = 20$ have been numerically investigated on a computer. The results have been plotted, for example in Fig. 2, showing a 1/4 period of the focusing system. The curves in full line are measurement results determined by the resistance network analogue model [11]. The results of linear approximation are presented in dash line [2], while the dotted line shows the results of the third-order approximation calculated by us.

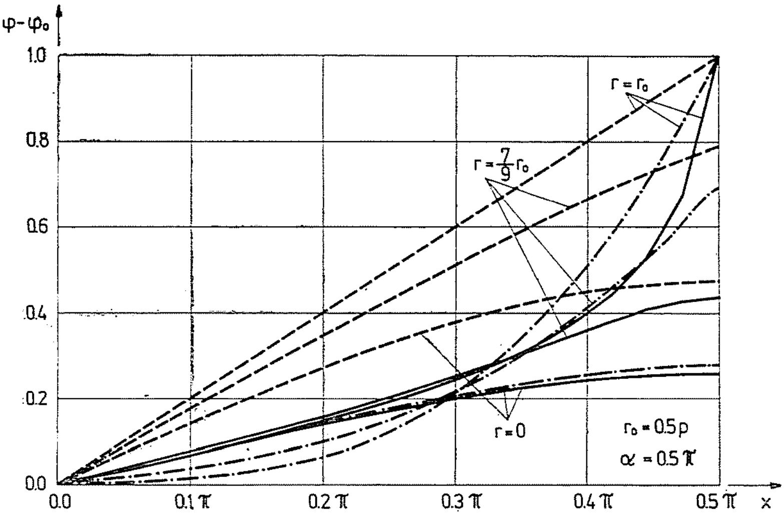

Fig. 2.

Let us now consider the determination of φ_{2k+1}. Let $f(x)$ approximate $F(x)$ to meet the condition

$$\int_0^\alpha [f(x) - F(x)]\, dx = 0. \tag{7}$$

(It is to be noted that $f(x)$ is equivalent to expression (1a).) Introducing the variable transformation $\frac{x}{\alpha}$:

$$\varphi_1 = -1 + 4 \int_0^1 F\left(\frac{x}{\alpha}\right) d\left(\frac{x}{\alpha}\right), \tag{8}$$

φ_3 can be determined by using (2).

Function $F\left(\frac{x}{\alpha}\right)$ has been integrated by Simpson's method.

Discussion of the results

The potential distribution of a rotationally symmetric periodic electrostatic focusing system has been determined assuming an approximation of order $(2K + 1)$ in the gap. The results of our computations for a given geometrical arrangement have been presented through an example. The approximation discussed is of a special importance if potential distribution is to be determined in a field far from the axis.

The coefficients φ_{2k+1} necessary for the approximation can be determined from measurement results using (2) and (8).

Our computation results show that in the case of small electrode diameter and large gap-size an approximation of higher order improves the computation accuracy.

It should be pointed out that in the case where in expression (3) the equality holds, the substitution value of the approximate polynomial (1a) adopted at the points of measurement equals the measured function value (M-point Lagrange approximation). It is advisable to densify measurement points near the interval boundary ($x \to \alpha$).

Summary

The theoretical investigation of periodic electrostatic focusing systems has been called into existence by the development of microwave valves. The design of open focusing systems is hampered by lengthy unpractical mathematical computations needed for the exact solution of Laplace's equation describing the potential field. This problem may be eased by the combination of numerical and analytical methods as follows: first the potential distribution in the gap between the focusing electrodes is determined in an experimental way — by measurements — and then it is approximated by a "well-fitting" n-order function using some approximation method. Finally, Laplace's equation is solved by the help of the resulting boundary condition. This method permits to determine the focusing conditions of thick electron beams, as it well describes the field even near the electrodes.

References

1. PRIESTLAND, P. B.—HARTNAGEL, H. L.: IEEE Trans. *ED*-**15**, 11. p. 915 (1968)
2. TIEN, P. K.: J. Appl. Phys. **25**, 10. p. 1281 (1954)
3. CLOGSTON, A. M.—HEFFNER, H.: J. Appl. Phys. **25**, 4. p. 435 (1954)
4. Игрицкий, А. Л.: Радиотехника и электроника *6*, 4. p. 613 (1961)
5. Игрицкий, А. Л.: Радиотехника и электроника *6*, 6. p. 964 (1961)
6. MEMBER, T. S.: Electronics and Communications **53**-*B*, 8. p. 76 (1970)
7. NAGY, Gy.—SZILÁGYI, M.: Bevezetés a tértöltésoptika elméletébe (Introduction to the theory of space-charge optics), Akadémiai Kiadó, Budapest (1967).
8. Молоковский, С. И.—Сушков, А. Д.—Трегубов, В. Ф.: Радиотехника и электроника *15*, 1. p. 217 (1970)
9. Волков, В. И.—Свешников, А. Т.—Семашко, Н. Н.: Журнал технической физики *41*, 8. p. 1602 (1971)
10. Литвиненко, Н. Х.—Мороз, Е. Е.—Сальникова, Л. П.—Шестопалов, В. П.: Журнал технической физики *42*, 12. p. 2529 (1972)
11. SZILÁGYI, M.: Image Processing and Computer-Aided Design in Electron Optics (Edited by P. W. Hawkes) Academic Press London and New York (1973).
12. Градштейн, И. Ш.—Рыжик, И. М.: Таблицы интегралов, сумм, рядов и произведений. Изд. «Наука» Москва (1971)

Sándor GALLAI
Dr. Miklós SZILÁGYI

Kandó Kálmán Villamosipari Műszaki Főiskola
Természettudományi Tanszék
1084 Budapest, Tavaszmező u. 17. Hungary