

A PROGRAM SYSTEM FOR ANALYSIS AND SYNTHESIS OF CONTROL SYSTEMS

By

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Introduction

In recent years digital and analog computers are employed in a wide range of applications, among others in the research work and in the undergraduate education. Nowadays most of colleges, high schools and institutes own analog or digital (or hybrid) computers. In day-to-day education the computers are used regularly for demonstrations, simulations etc. Their role is very important in areas where the equations describing the problem or process are very complex and difficult to solve analytically.

Nowadays modern teaching of control theory is unimaginable without applying analog or digital computers. In the last decade a lot of program packages and program systems have been elaborated for digital computers. By means of these programs, the operation of the control systems is easy to illustrate and test. Let us mention, however, that these programs may also be very useful for solving problems in biology, sociology, economy etc.

Our purpose is to give a brief survey of the research work done at the Department of Automation, Technical University, Budapest (Hungary). In recent years the most important research work has been to elaborate a program system for computer-aided design of control systems. The aim was to develop a user- and procedure-oriented program package for the analysis and synthesis of control systems. The control system may be linear or it may contain some typical nonlinear elements (e.g. relay, backlash, hysteresis, dead band etc.), it may be either a single-loop or a multiloop one, it may also contain dead time. The control system is assumed to be of lumped parameters, time-invariant and deterministic.

The simplified flow chart of a SI-SO (single input – single output) control system is seen in Fig. 1.

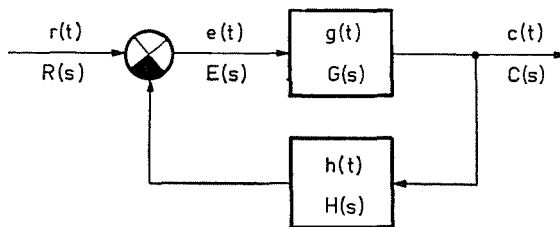


Fig. 1. Flow chart of a simplified SI-SO control system

1. Program system

In elaborating the program system, unified input data and simple application was a primary aim. The procedure package forms a direct connection between the user (scientists, students etc.) and the computer. The man-machine relation is shown in Fig. 2.

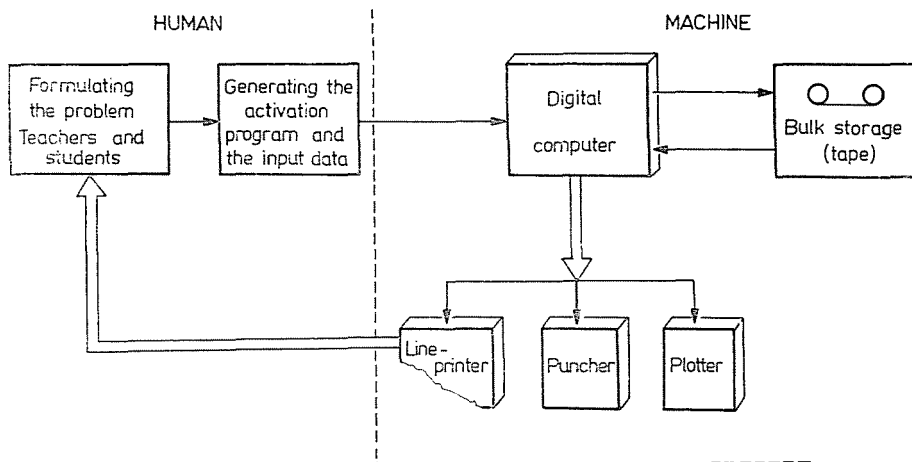


Fig. 2. The man-machine relation

The programs are stored in backing memory (disc or magnetic tape). Activation means that the total program must be assembled from the procedures (or a standard activation program may be used), it must be processed and the results printed or plotted. Depending on the results, a new input data set can be generated.

The elaborated program system consists of two main parts:

- I. Programs for analysis tasks.
- II. Programs for synthesis tasks.

The hierarchic system of the procedures is shown in Fig. 3.

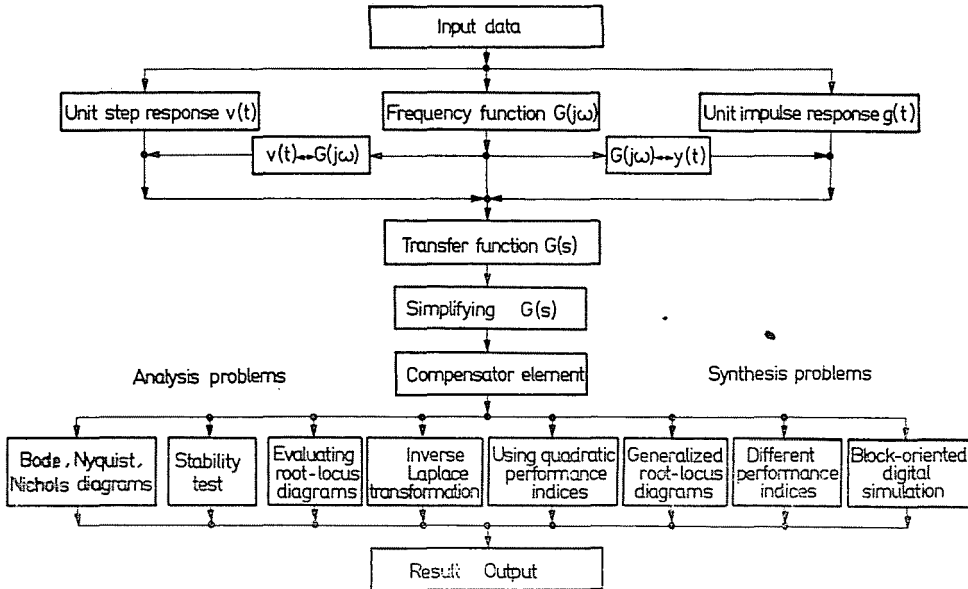


Fig. 3. Hierarchy of the program system

The analysis and synthesis of the control systems may be performed in the time domain, in the frequency domain or in the operator domain. In these presentations the control system may be described by the unit step response ($c(t)$) in the time domain, the closed-loop frequency response ($M(j\omega)$) in the frequency domain and the closed-loop transfer function ($M(s)$) in the operator domain. Let us note that for a single-loop feedback system, the closed-loop transfer function can be written as:

$$M(s) = \frac{G(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad (1)$$

where $G(s)H(s)$ will be termed as the open-loop transfer function of the control system. Naturally, when the control system is a multivariable one, instead of Eq. (1) the closed-loop transfer function matrix is defined as

$$M(s) = [I + G(s)H(s)]^{-1} G(s) \quad (2)$$

which is a $q \times p$ matrix.

In this paper only the single-variable control system will be considered.

To characterize the control system, the closed-loop (or the open-loop) transfer function will be given in polynomial form (i.e. in a rational fractional function), as well as in a factorized form:

$$M(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^m n_i s^i}{\sum_{j=0}^n d_j s^j} = K \frac{\prod_{i=1}^Z (s + z_i)^{v_i}}{\prod_{j=1}^P (s + p_j)^{\mu_j}} \quad (3)$$

$$\sum_{j=0}^P \mu_j = n$$

where m and n are the degrees of the numerator $N(s)$ of $M(s)$, and of the denominator $D(s)$ of $M(s)$, resp., Z is the number of distinct zeros z_i with multiplicity v_i of $M(s)$, P is the number of distinct poles p_j with multiplicity μ_j .

In the next section the first part of the program system, for the analysis of a linear control system, will be surveyed.

2. ALCOS — Analysis of Linear Control Systems

The main programs in the program package ALCOS (Analysis of Linear Control Systems) are listed as follows:

1. FSAF procedure computes the unit step response (or the unit impulse response) of the open-loop (or closed-loop) control system from the frequency function obtained by measurements:

$$\{M(\omega_i), \varphi(\omega_i)\}_{i=1}^N \Rightarrow \{c(k\Delta T)\}_{k=0}^N \quad (4)$$

where K is the number of measurements, ΔT is the interval between the k -th and $(k - 1)$ -th values of time response.

2. MAYS procedure computes the unknown parameters of the transfer function (in a rational function form $M(s)$) on the basis of the magnitude ($M(\omega) = |M(j\omega)|$) and phase ($\varphi(\omega) = \angle M(j\omega)$) of the closed-loop frequency function $M(j\omega)$ obtained by measurements:

$$\{M(\omega_i), \varphi(\omega_i)\}_{i=1}^N \Rightarrow M(s) = \frac{N(s)}{D(s)} \quad (5)$$

3. PRONY procedures determine the unknown parameters of the closed-loop transfer function $M(s)$ (in a rational function form) using the sampled unit step response $c(t)$ (or unit impulse response) given at N distinct points versus time:

$$\{c(k\Delta T)\}_{k=0}^N \Rightarrow M(s) = \frac{N(s)}{D(s)} \quad (6)$$

4. BONN procedure computes the Bode diagram, the Nyquist plot and the Nichols chart of the open-loop transfer function given in rational fractional function of the form in Eq. (3). Before activating the procedure BONN, one has to determine the frequency points where the frequency characteristics have to be computed by means of the procedure OMEG. To compute these frequency points, procedure OMEG determines the corner frequencies of the transfer function.

5. ROUTH procedure indicates whether the control system investigated is stable or not by means of the Routh-scheme.

6. STAB procedure computes the phase margin, the gain margin, the phase-crossover frequency and the gain-crossover frequency of the control system characterized by its open-loop transfer function.

7. SIGF procedure simplifies the flow-chart of a multi-loop control system. More precisely, it reduces the signal flow graph of the system to a branch or to a simple signal flow graph with four nodes.

8. LANC procedure computes the different reduced expressions of a transfer function expressed by a rational fractional function using the continued fraction representation. The result must be checked for stability of the system.

9. ROLO procedure computes the root-locus points of the control system (which may also have dead time) versus the loop-gain in the s -plane. Let us note that this procedure may determine the root-locus with regard to any parameter of the system, as time constant, damping ratio etc.

10. INVLAP procedure computes the inverse Laplace transform of a rational fraction function, i.e. the time response of a system. Two kinds of algorithms have been used; the first is valid only for systems with single poles, the second may be used in any control system without dead time and with lumped parameters.

11. DIFF procedure computes the time response of a control system with dead time and with unity feedback on the basis of the open-loop transfer function.

The procedure determines also the phase variables of the system.

Beside the above listed procedures with activation programs, a lot of other procedures were elaborated for basic mathematical operations. Only the most useful ones are listed below:

LINEQ procedure solves the linear matrix equation $\mathbf{Ax} = \mathbf{b}$ for vector \mathbf{x} .

INVERT procedure makes matrix inversion of an $n \times n$ square matrix.

ROOT procedure determines all real and complex roots of a polynomial with real coefficients. This procedure may be very useful in the case where the poles have multiplicity.

MULTI, ADIT procedures perform multiplication and addition of the real polynomials.

HORN procedure determines the coefficients in the Taylor expansion of a real polynomial.

To characterize the control system in the frequency domain (or in the operator domain), two kinds of data presentation were used: in the first case only the pole-zero configuration of the transfer function was known and it was given the computer as two matrices:

$$\text{POLE-ZERO} \Rightarrow \text{PZ} = \left. \begin{array}{cc} \text{real} & \text{imaginary} \\ \left[\begin{array}{cc} \text{Re}(z_1) & \text{Im}(z_1) \\ \text{Re}(z_1) & -\text{Im}(z_1) \\ \cdot & \cdot \\ \cdot & \cdot \\ \text{Re}(p_1) & \text{Im}(p_1) \\ \text{Re}(p_1) & -\text{Im}(p_1) \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{array} \right] & \left. \begin{array}{l} \text{zeros} \\ \\ \\ \text{poles} \end{array} \right\} \text{MUL} = \left[\begin{array}{c} \nu_1 \\ \nu_1 \\ \cdot \\ \cdot \\ \mu_1 \\ \mu_1 \\ \cdot \\ \cdot \\ \cdot \end{array} \right] \quad (7)$$

where z_i denotes the zeros of the transfer function and p_j denotes its poles.

The second kind of data presentation is that where the polynomials of the transfer function are known, i.e. the numerator and the denominator:

$$\begin{array}{ll} \text{numerator} & N = [n_0, n_1, \dots, n_m]^T \\ \text{denominator} & D = [d_0, d_1, \dots, d_n]^T \end{array} \quad (8)$$

In some cases both data presentations were used to accelerate results.

Naturally, there are two procedures which transfer the data from the first presentation into the second one and vice versa.

This program package was tested and used in the research work, as well as in teaching control system theory. By means of this program, the students can study more complex control systems than earlier. The computer prints or plots the results to illustrate the time domain (frequency domain, operator domain) performance of the control system.

In the next section a lot of examples are given to illustrate the effectiveness of the ALCOS program package.

3. Examples

3.1. As a first example let us consider the control system with the open-loop transfer function

$$G(s) = \frac{1}{(s + 0.5)(s + 1)} \quad (9)$$

and with unity feedback. The problem is to compute the unit step response (or the unit impulse response) of the open-loop system. Only the magnitude and phase curves of $G(j\omega)$ can be measured. The measured points were exactly generated as follows:

$$G(\omega) = |G(j\omega)| = \frac{2}{\sqrt{(2 - \omega^2)^2 + 9\omega^2}} \quad (10)$$

$$\varphi(\omega) = \angle G(j\omega) = -\arccos\left(\frac{2 - \omega^2}{\sqrt{(2 - \omega^2)^2 + 9\omega^2}}\right) \quad (11)$$

The simulated points are given in Table I and plotted in Fig. 4.

Table I

Simulated absolute values of $G(j\omega)$ and phase curve of $G(j\omega)$ in Ex. 3.1

ω	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$G(\omega)$	1	0.9757	0.9104	0.8213	0.7250	0.6324	0.5489	0.4761
$\varphi(\omega)$	0	-0.2970	-0.5779	-0.8318	-1.0552	-1.4164	-1.4164	-1.5612
ω	1.6	1.8	2.0	2.5	3.0	3.5	4.0	
$G(\omega)$	0.4138	0.3609	0.3162	0.2320	0.1754	0.1363	0.1084	
$\varphi(\omega)$	-1.6869	-1.7965	-1.8925	-2.0863	-2.2318	-2.3441	-2.4329	
ω	5.0	6.0	7.0	8.0	9.0	10.0	15.0	
$G(\omega)$	0.0728	0.0519	0.0388	0.0300	0.0239	0.0195	0.0087	
$\varphi(\omega)$	-2.5636	-2.6546	-2.7213	-2.7725	-2.8122	-2.8445	-2.9424	
ω	20.0	25.0	50.0					
$G(\omega)$	0.0049	0.00318	0.00079					
$\varphi(\omega)$	-2.9919	-3.0217	-3.0816					

The theoretical unit step response of the open-loop system (without noise) is:

$$c(t) = 1 + e^{-2t} - 2e^{-t} \quad (12)$$

and the unit impulse response is:

$$\omega(t) = 2e^{-t} - 2e^{-2t} \quad (13)$$

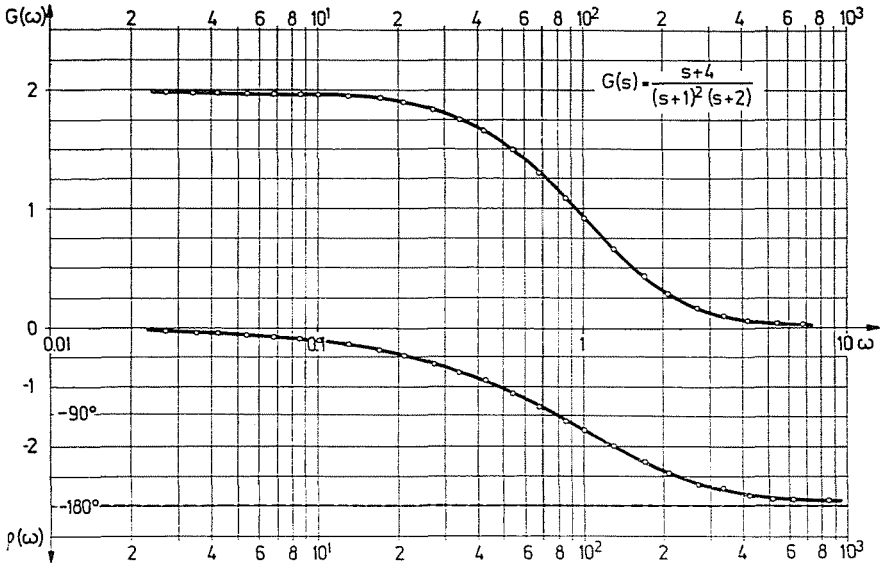


Fig. 4. Simulated magnitude-versus-frequency and phase-versus-frequency curves of $G(j\omega)$ in Ex. 3.1

The time response is also determined by means of procedure FSAF. The results of the computer program, as well as the exact $c(t)$ and $w(t)$ are seen in Fig. 5. Note that the relative error between the two curves is less than 0.1 %, so the computed and the theoretical time responses of the open-loop system are practically the same.

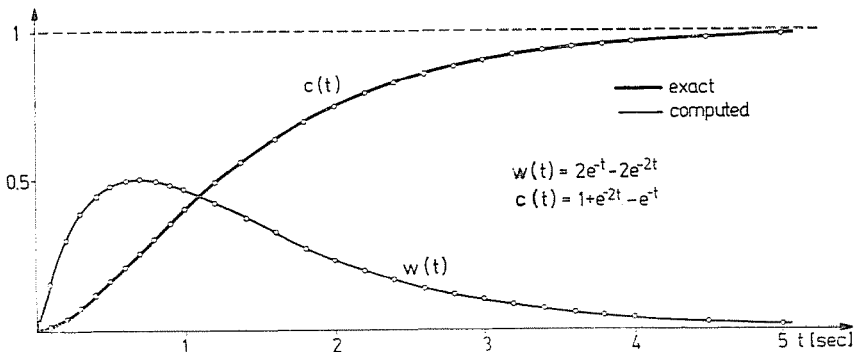


Fig. 5. The exact and the computed unit step responses ($c(t)$) and unit impulse responses ($w(t)$) of the system with $G(s) = 1/(s + 0.5)(s + 1)$

3.2. Let the open-loop transfer function of the control system with unity feedback be:

$$G(s) = \frac{3}{18 + 6s + s^2} = \frac{3}{(s + 3 + j3)(s + 3 - j3)} \tag{14}$$

Since the parameters of $G(s)$ are unknown, they have to be determined from the frequency function $G(j\omega)$ obtained by measurements:

$$G(j\omega_i) = G(\omega_i) \mid \varphi(\omega_i) \mid \quad i = 1, 2, \dots, N \quad (15)$$

In this case the magnitude and phase of $G(j\omega)$ were measured at 50 points in the dominant frequency region.

Procedure MAYS determined the unknown parameters using $\{G(\omega_i), \varphi(\omega_i)\}_{i=1}^{50}$ values. The results are for 5 digits:

$$\begin{aligned} n_0 &= 3.00000 & d_0 &= 18.00000 \\ & & d_1 &= 6.00000 \\ & & d_2 &= 1.00000 \end{aligned} \quad (16)$$

The results are seen to be exactly the same as the parameters of Eq. (14). Let us note that it is necessary to exactly estimate the order of the system to get good results.

The procedure MAYS also computes the following relative errors of the results:

$$\varepsilon_M = \frac{1}{N} \sum_{i=1}^N (G(\omega_i) - \hat{G}(\omega_i))^2 \quad (17)$$

and

$$\varepsilon_\varphi = \frac{1}{N} \sum_{i=1}^N (\varphi(\omega_i) - \hat{\varphi}(\omega_i))^2, \quad (18)$$

where N is the number of measuring points, $G(\omega_i)$ is the measured magnitude value at angular frequency, ω_i ; $\hat{G}(\omega_i)$ is the value estimated by the procedure MAYS.

In this example the values are:

$$\begin{aligned} \varepsilon_M &= 6.2769 \cdot 10^{-11} \\ \varepsilon_\varphi &= 3.0545 \cdot 10^{-10} \end{aligned} \quad (19)$$

3.3. Consider the control system with the open-loop transfer function

$$G(s) = \frac{s + 4}{(s + 1)^2 (s + 2)} = \frac{4 + s}{2 + 5s + 4s^2 + s^3}. \quad (20)$$

Now $G(s)$ has a multiple pole at $s = -1$. The problem is to find the unknown parameters of $G(s)$. The measured magnitude and phase of $G(j\omega)$ are plotted in Fig. 6. $G(\omega_i)$ and $\varphi(\omega_i)$ were measured at 50 frequency points.

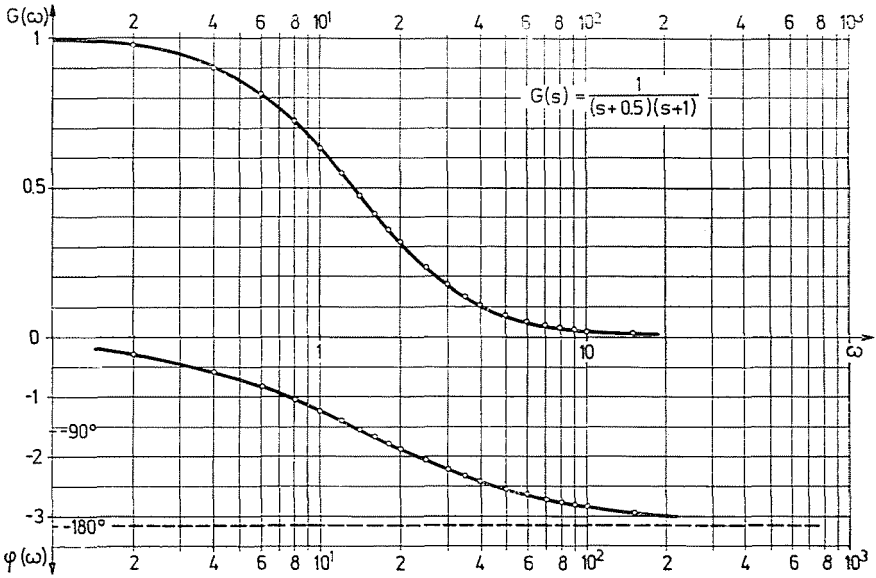


Fig. 6. Simulated $G(\omega)$ and $\varphi(\omega)$ curves in Ex. 3.3

The results of the procedure MAYS are:

$$\begin{aligned}
 n_0 &= 4.000001 & d_0 &= 2.0000007 \\
 n_1 &= 1.000000 & d_1 &= 5.0000015 \\
 & & d_2 &= 4.0000012 \\
 & & d_3 &= 1.0000000 .
 \end{aligned}
 \tag{21}$$

The relative error values in this case are:

$$\begin{aligned}
 \varepsilon_M &= 9.3367 \cdot 10^{-8} \\
 \varepsilon_\varphi &= 8.8863 \cdot 10^{-8} .
 \end{aligned}
 \tag{22}$$

3.4. Consider the open-loop transfer function of a control system with unity feedback:

$$G(s) = \frac{s^2 + 2s + 3}{s^4 + 18s^3 + 118s^2 + 340s + 357} .
 \tag{23}$$

The closed-loop transfer function is:

$$M(s) = \frac{s^2 + 2s + 3}{s^4 + 18s^3 + 119s^2 + 342s + 360} = \frac{(s + 1)(s + 2)}{(s + 3)(s + 4)(s + 5)(s + 6)}
 \tag{24}$$

On the basis of the measured $M(\omega)$ and $\varphi(\omega)$ values, these functions are shown in Fig. 7. $M(\omega)$ and $\varphi(\omega)$ were simulated at 50 frequency points. Using the simulated values of $M(\omega)$ and $\varphi(\omega)$, procedure MAYS was activated. The result gives the unknown parameters of the closed-loop transfer function $M(s)$ of Eq. (24).

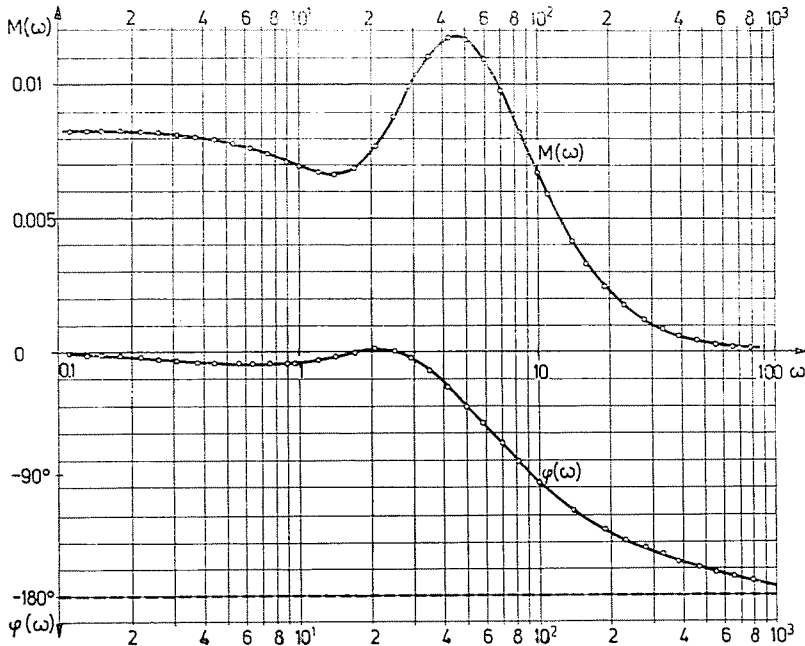


Fig. 7. Simulated $G(\omega)$ and $\varphi(\omega)$ curves in Ex. 3.4

The results:

numerator	denominator	
$n_0 = 3.0234$	$d_0 = 363.082$	
$n_1 = 2.0065$	$d_1 = 342.886$	
$n_2 = 1.0000$	$d_2 = 119.128$	(25)
	$d_3 = 18.006$	
	$d_4 = 1.000$	

and the relative error values:

$$\begin{aligned} \varepsilon_M &= 2.794 \cdot 10^{-5} \\ \varphi_M &= 3.351 \cdot 10^{-3} \end{aligned} \quad (26)$$

After determining the parameters of the closed-loop transfer function, the unit impulse response of the system was also computed by means of Laplace transformation. Using the INVLAP procedure, the unit impulse response was computed and plotted, as seen in Fig. 8.

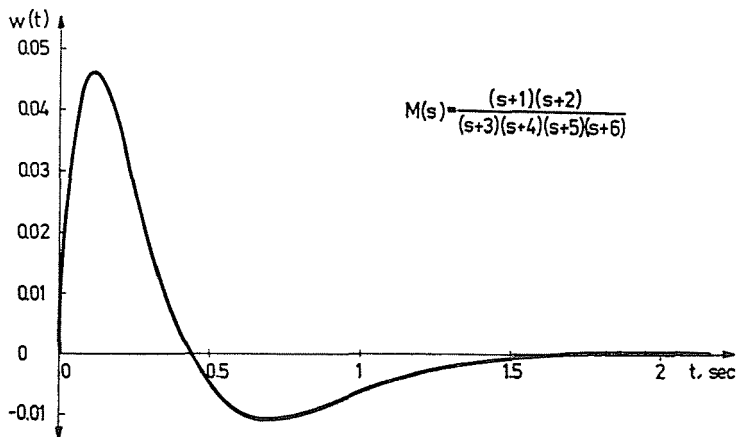


Fig. 8. Unit impulse response of the system in Ex. 3.4

3.5. In many practical cases the parameters of the open-loop (or closed-loop) transfer function are unknown, but the input-output time signal can be measured: if the input signal is the unit impulse function, then the output signal will be the unit impulse response. On the basis of the sampled values of the unit impulse response (or the unit step response) the unknown parameters of the open-loop (or closed-loop) transfer function can be determined by means of PRONY's method [5].

Let the exact unit impulse function be:

$$w(t) = 20e^{-0,02t} - 15e^{-0,25t} + 10e^{-0,05t} - 15e^{-0,1t} . \quad (27)$$

The values of $w(i\Delta T)$ were generated at 300 points with $\Delta T = 5$ sec. If the coefficients and the exponents of the function $w(t)$ are known, then the transfer function can easily be determined [14]:

$$M(s) = L[\omega(t)] = L\left[\sum_{i=0}^m A_i e^{p_i t}\right] = \sum_{i=1}^m \frac{A_i}{s + p_i} . \quad (28)$$

Using the simulated unit impulse response, procedure PRONY computes the unknown parameters. The results computed for distinct order of the control system (here the order of the control system means the number of the exponent terms in Eq. (28)) are tabulated in Table II.

Table II

Parameters of the exact unit impulse function and the parameters identified by procedure PRONY in Ex. 3.5

Number of sampling points	Sampling time [sec]	Degree of approximation	Parameters of the unit impulse function				Mean square error
			Re A_i	Im A_i	Re P_i	Im P_i	
300	5	4	20	0	-0.02	0	—
			-15	0	-0.25	0	
			10	0	0.05	0	
			-15	0	-0.1	0	

results of procedure PRONY

300	5	4	19.9101	0	-0.01996	0	10^{-8}
			-14.9618	0	-0.25000	0	
			10.1748	0	-0.04998	0	
			-15.1225	0	-0.10000	0	
300	5	3	24.4630	0	-0.02201	0	10^{-5}
			-10.4060	0	-0.27366	0	
			-14.0568	0	-0.14593	0	
300	5	2	23.0254	0	-0.02079	0	10^{-3}
			-23.0255	0	-0.20495	0	

By means of the tabulated results, procedure KOZNEV determines the transfer function in a rational fraction form. The exact transfer function is:

$$\begin{aligned}
 M(s) &= 4.35 \frac{s^2 + 0.1776s + 0.0056896}{s^4 + 0.42s^3 + 0.0505s^2 + 0.0021s + 0.000025} = \\
 &= 4.35 \frac{(s + 0.042)(s + 0.136)}{(s + 0.05)(s + 0.1)(s + 0.02)(s + 0.25)} \quad (29)
 \end{aligned}$$

The third-order and second-order transfer functions, approximating the exact $M(s)$, are:

$$\hat{M}_3(s) = 4.36 \frac{s + 0.197}{s^3 + 0.4416s^2 + 0.04917s + 0.000879} \quad (30)$$

$$\hat{M}_2(s) = 4.24 \frac{1}{s^2 + 0.22574s + 0.00426} \quad (31)$$

The fourth-order approximation $\hat{M}_4(s)$ practically equals $M(s)$.

Fig. 9 shows the exact and computed unit impulse responses. The three curves agree very well. There is only about 1 % deviation between $w(t)$, $\hat{w}_3(t)$ and $\hat{w}_2(t)$.

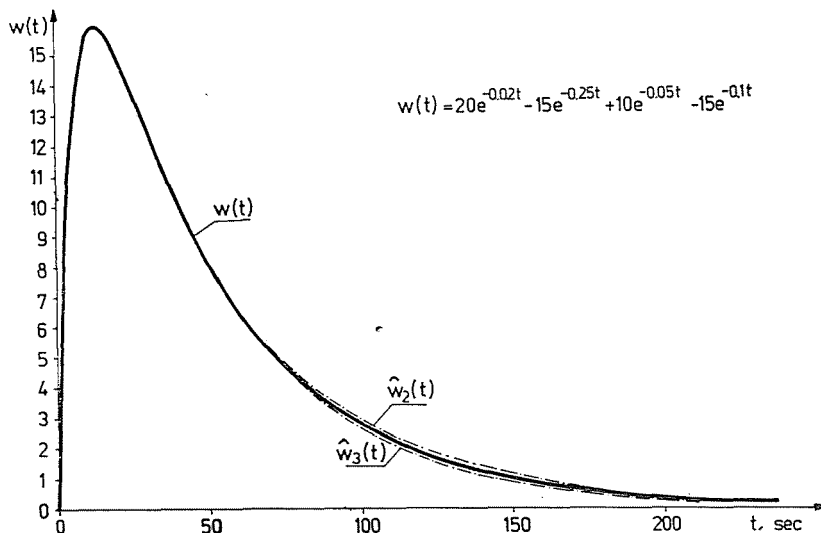


Fig. 9. The exact and the approximated unit impulse responses in Ex. 3.5. $\hat{w}_2(t)$ is the second-order, $\hat{w}_3(t)$ the third-order approximation

3.6. Let us consider the compensated servo control system in Fig. 10.

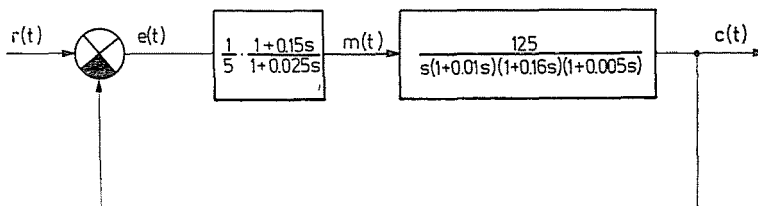


Fig. 10. Flow chart of a servo system with unity feedback

The closed loop-transfer function is:

$$M(s) = \frac{1.5628 \cdot 10^7(s + 10)}{s^5 + 356.25s^4 + 37187.5s^3 + 1.21875 \cdot 10^6 \cdot s^2 + 2.1878 \cdot 10^7 \cdot s + 1.5610 \cdot 10^8} \tag{32}$$

Since this transfer function is of high order, it is advantageous to reduce it into a transfer function of lower order. This can be done using the main program LANC. The resulting transfer functions are to be checked with respect to stability, because the main program LANC does not check it.

Among the reduced transfer functions the fourth-order transfer function is unstable and the first-order transfer function is very inaccurate. The acceptable transfer functions are the following:

$$\widehat{M}_3(s) = 5.3138 \frac{-s^2 + 101.285s + 1103.3}{s^3 + 38.486s^2 + 772.71s + 5863} \quad (33)$$

$$\widehat{M}_2(s) = 7.059 \frac{s + 26.76}{s^2 + 14.614s + 188.9} \quad (34)$$

The unit step responses generated by the distinct-order transfer functions, such as $M(s)$, $\widehat{M}_3(s)$, $\widehat{M}_2(s)$, are given in Fig. 11. Observing them, the third-order approximation is to be seen very good, but even the second-order one is acceptable.

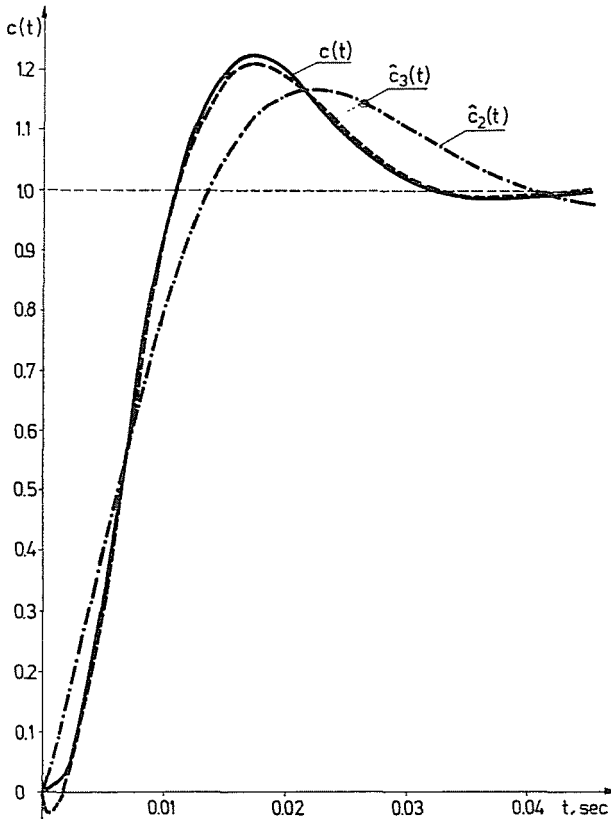


Fig. 11. The exact and the approximated unit step responses in Ex. 3.6. $\hat{c}_2(t)$ is the second-order, $\hat{c}_3(t)$ the third-order approximation

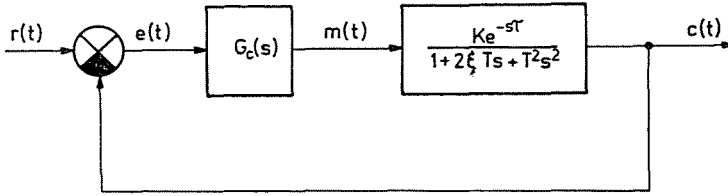


Fig. 12. Flow chart of a second-order control system with dead time

3.7. Let us consider the control system in Fig. 12. The transfer function of the plant is:

$$G_p(s) = \frac{Ke^{-s\tau}}{1 + 2\zeta Ts + T^2 s^2} \quad (35)$$

where $K = 2$; $\tau = 1$ sec; $\zeta = 0.5$; $T = 0.1$ sec; so Eq. (35) may also be written as:

$$G_p(s) = \frac{200e^{-s}}{100 + 10s + s^2} = \frac{200e^{-s}}{(s + 5 + j5\sqrt{3})(s + 5 - j5\sqrt{3})} \quad (36)$$

The characteristic equation of the control system without compensation is:

$$s^2 + 10s + 100 + 200e^{-s} = 0. \quad (37)$$

Since this control systems has one dead time, procedure ROUTH cannot be applied. Procedure STABH shows, however the uncompensated system to be unstable. The critical value of the loop gain is $K_c = 0.9621$. So a lower value $K_1 = 0.6$ is chosen to study the performance of the uncompensated system.

First of all, the frequency characteristics were computed by procedure BONN. Fig. 13 shows the Nyquist plot of the system. In this case the system is seen to be stable (since $K_1 < K_c$). The Nyquist plot indicates, however, that the step response of the closed-loop system will be quite oscillatory. Figs 14 and 15 show the Bode diagram and the Nichols-chart of the uncompensated system, respectively.

The transient performance of the control system is usually characterized by the use of a unit step response. Therefore, the unit step response with $K_1 = 0.6$ was computed by the procedure DIFF. The result is seen in Fig. 16. The peak overshoot was determined as 85 %. The rise time was very small (after the dead time) and the setting time was about 0.5 sec. This performance is unacceptable. Naturally, if the loop-gain is much smaller, the system can meet even the performance indices. This is illustrated in Fig. 17, where the unit step response is given with $K = 0.0081$. In this case, a too low loop-gain makes the performance of the system much better, than of the case with $K = 0.6$. However, the amplitude of the loop-gain is too small, so there is a strong attenuation in the system.

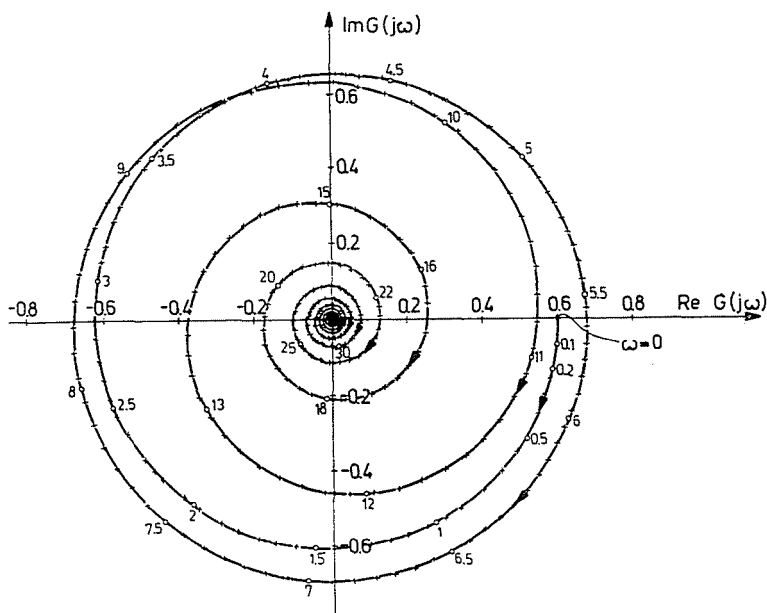


Fig. 13. Nyquist-plot of the uncompensated control system in Ex. 3.7 with $K = 0.6$

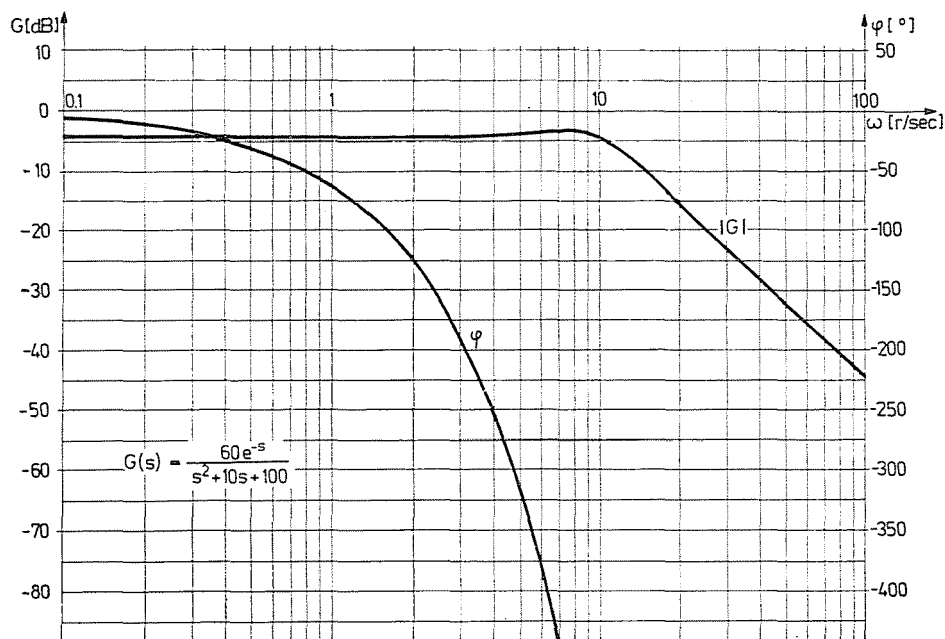


Fig. 14. Bode-diagram of the uncompensated control system in Ex. 3.7 with $K = 0.6$

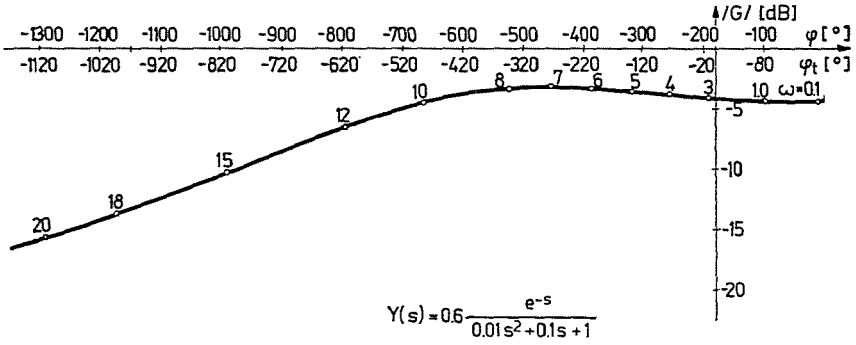


Fig. 15. Nichols-chart of the uncompensated control system in Ex. 3.7 with $K = 0.6$

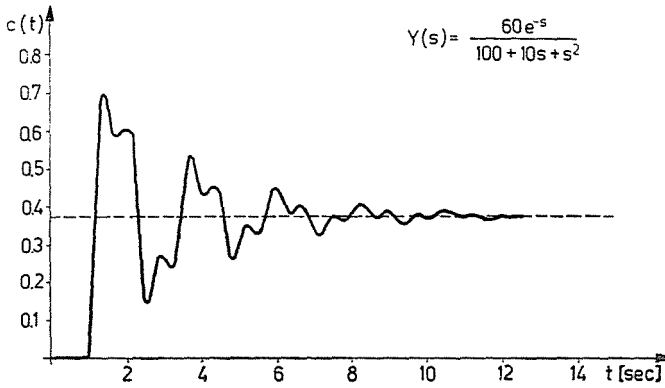


Fig. 16. Unit step response of the uncompensated control system in Ex. 3.7 with $K = 0.6$

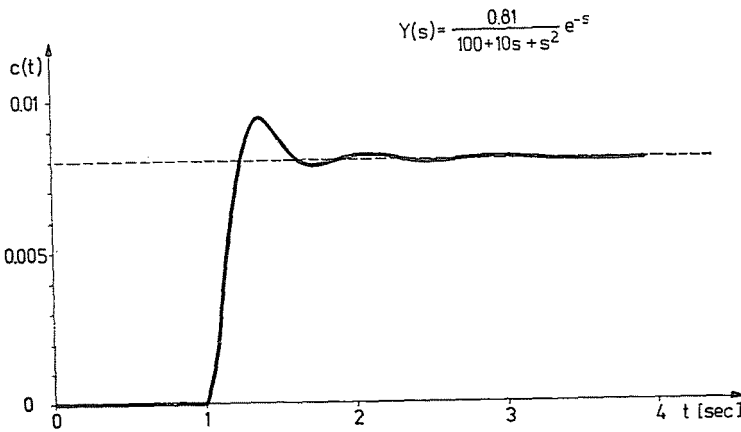


Fig. 17. Unit step response of the uncompensated control system in Ex. 3.7 with $K = 0.0081$

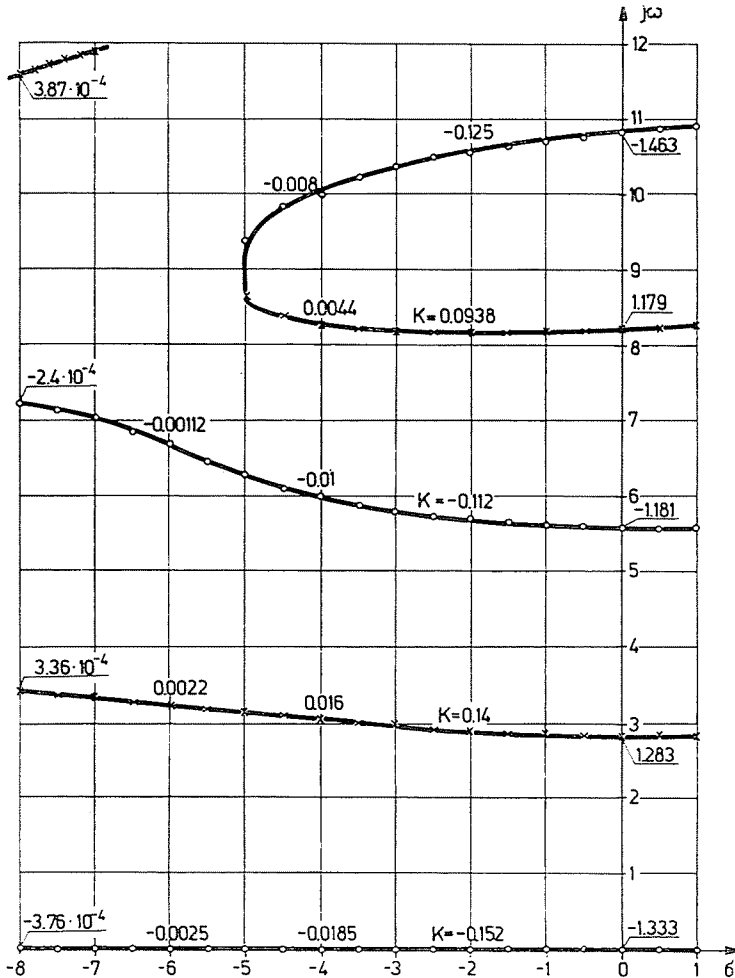


Fig. 18. Root-locus of the uncompensated control system, \times — points with positiv loop-gain, \circ — points with negative loop-gain

To illustrate the effect of the dead time on the root-locus diagram, Fig. 18 shows the root-locus of the uncompensated system. Let us note that root-loci for the negative value of the loop-gain K are also plotted.

As the performance of the uncompensated system is not acceptable, one has to choose a compensator element. Let the compensator be a PID type with the following parameters:

$$G_c(s) = K_{c0} \left(1 + sT_D + \frac{1}{sT_i} \right) = K_{c0} \left(1 + s + \frac{7}{s} \right) \quad (38)$$

i.e. $T_D = 1$ sec and $T_i = 1/7$ sec.

These parameters were chosen on the basis of [6, 7].

Now the open-loop transfer function of the compensated control system is:

$$\begin{aligned} G(s) &= G_c(s) G_p(s) = 200K_{c0}e^{-s} \frac{s^2 + s + 7}{s^3 + 10s^2 + 100s} = \\ &= 200 K_{c0}e^{-s} \frac{(s + 0.5 + j2.598)(s + 0.5 - j2.598)}{s(s + 5 + j5\sqrt{3})(s + 5 - j5\sqrt{3})} \end{aligned} \quad (39)$$

The open-loop system has both a conjugate complex zero and a pole.

In this case the critical gain is $K_c = 0.4625$ determined by procedure STABH. So K_{c0} was chosen to be 0.07. The root-locus of the compensated system was computed by the procedure ROLO, and plotted in Fig. 19. Fig. 20 presents the unit step response of the compensated system. Now the peak over-

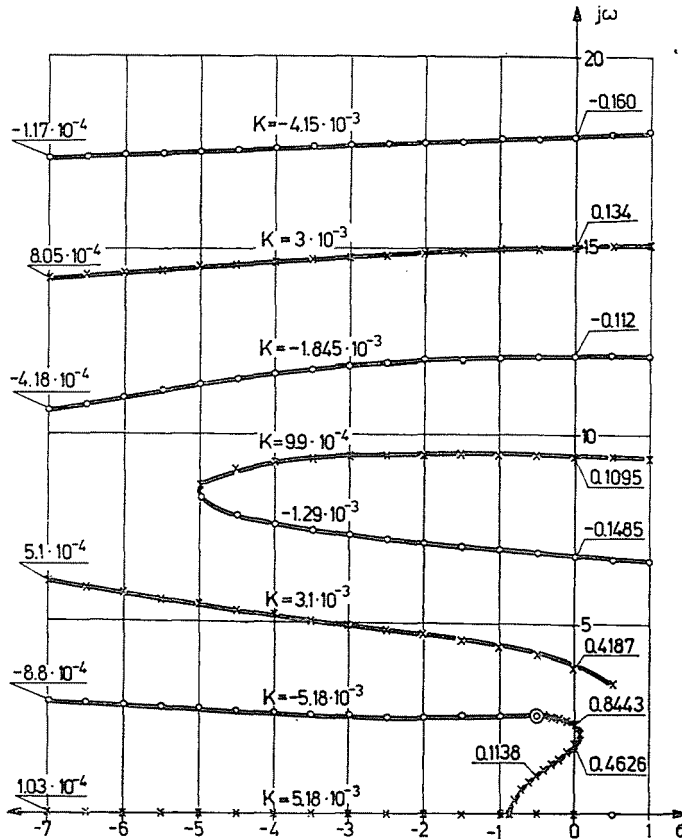


Fig. 19. Root-locus of the PID compensated control system, \times - points with positive loop-gain, \circ - points with negative loop-gain

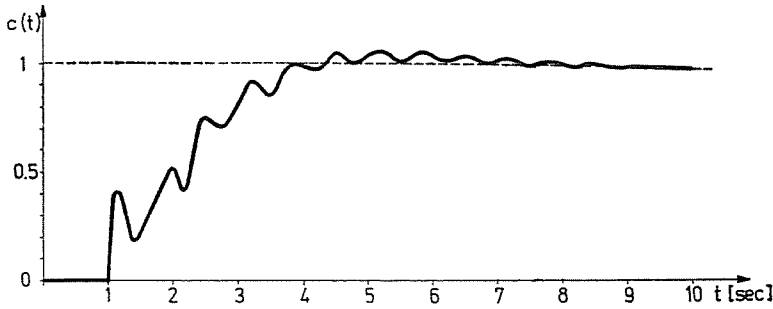


Fig. 20. Unit step response of the PID compensated control system in Ex. 3.7

shoot is only 5 %, the rise time is a little longer but the setting time is about 4 sec. Comparing the unit step responses of the uncompensated and compensated control systems in Figs 16 and 20, respectively, according to expectations, performance indices of the compensated system are seen to be much better than those of the uncompensated one.

3.8. Let us consider the multiloop control system in Fig. 21. The open-loop transfer function of the inner-loop is:

$$G_{i1}(s) = \frac{300s}{(s+2)^2(s+5)(s+10)} = \frac{300s}{s^4 + 19s^3 + 114s^2 + 260s + 200} \quad (40)$$

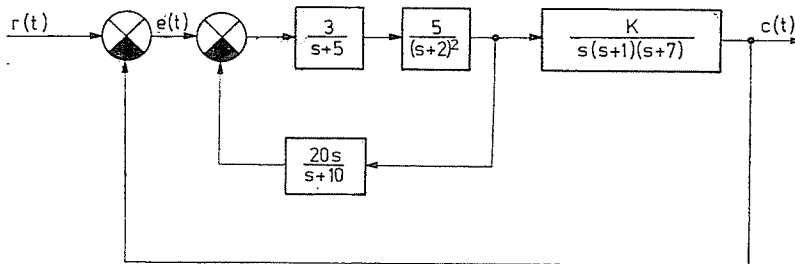


Fig. 21. Flow chart of a multiloop control system

The inner-loop has a double pole at $s = -2$ and the inner feedback element is a derivative type. The open-loop transfer function of the system is given by

$$G(s) = \frac{15K(s+10)}{s^7 + 27s^6 + 273s^5 + 1605s^4 + 5478s^3 + 5520s^2 + 1400s} \quad (41)$$

The pole-zero configuration computed by procedure ROOT of the open-loop system is as follows:

$$\begin{aligned}
 z_1 &= -10 \\
 p_1 &= 0 \\
 p_2 &= -0.38549 \\
 p_3 &= -1 \\
 p_4 &= -2.5283 - j5.6458 \\
 p_5 &= -2.5283 + j5.6458 \\
 p_6 &= -7 \\
 p_7 &= -13.5579.
 \end{aligned} \tag{42}$$

The open-loop transfer function is seen to have one real zero, two conjugate complex poles and five real poles.

The problem is to find the critical value of the loop-gain K . Procedure STABH computes the critical loop-gain K_c and the gain-crossover frequency ω_c . The results are given in Table III. The loop gains corresponding to the prescribed phase-margin φ_{pm} are also given. Note that the control system cannot satisfy the performance specification $\varphi_{pm} = -45^\circ$.

Table III

Phase margin and gain margin of the uncompensated control system in Ex. 3.8

$\varphi_{pm} [^\circ]$	ω_c	$\varphi [^\circ]$	K
0	0.53487	-180	9.6563
30	0.30713	-150	3.8307
45	0.21858	-135	2.3995
60	0.14037	-120	1.4077
75		unsatisfiable	

φ_{pm} — phase margin,

ω_c — gain crossover frequency,

φ — phase angle, K — loop-gain

Fig. 22 shows the Nyquist-plot of the open-loop transfer function with $K = 2.4$. In this case the phase-margin is 45° .

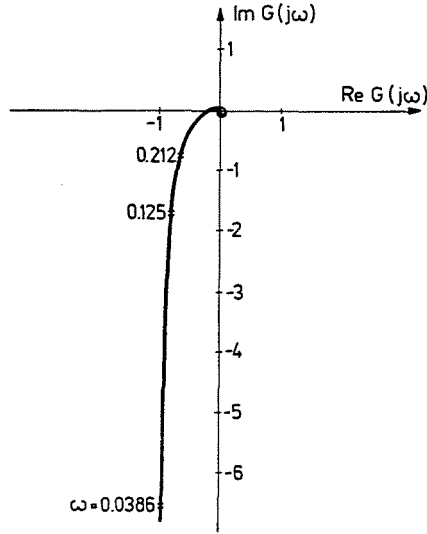


Fig. 22. Nyquist-plot of the uncompensated multiloop control system in Ex. 3.8 with $K = 2.4$

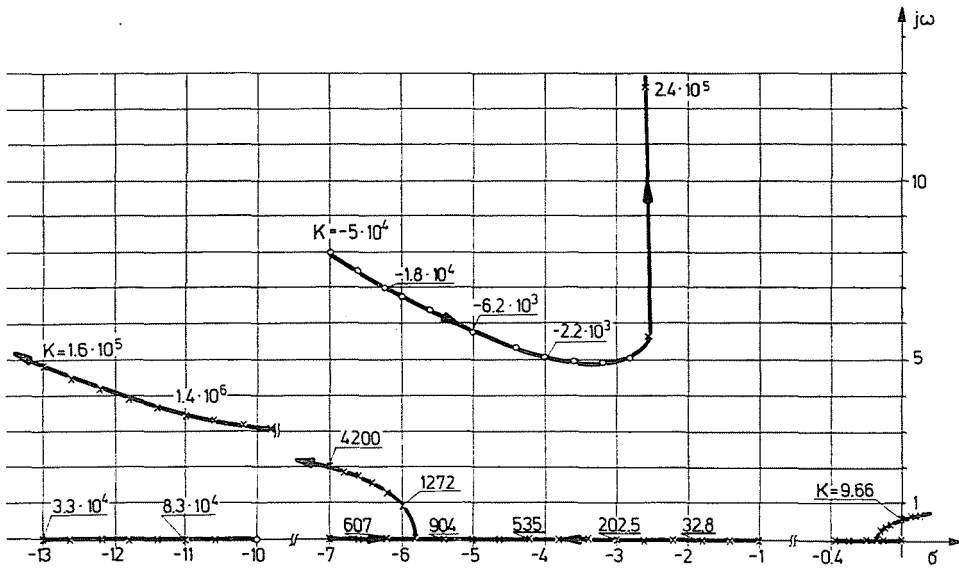


Fig. 23. Root-locus of the uncompensated multiloop system in Ex. 3.8

The root-locus diagram of the system has also been computed by procedure ROLO and it is given in Fig. 23. The critical loop-gain is seen to be low. The poles of the closed-loop system corresponding to phase-margin 45° are:

$$\begin{aligned}
 p_1 &= -0.1195 + j0.26837 \\
 p_2 &= -0.1195 - j0.26837 \\
 p_3 &= -1.1483 \\
 p_4 &= -2.5280 + j5.6473 \\
 p_5 &= -2.5280 - j5.6473 \\
 p_6 &= -6.9989 \\
 p_7 &= -13.5578
 \end{aligned} \tag{43}$$

The smaller conjugate complex pole may be considered as the dominant pole of the closed-loop system. Since this pole is small, the unit step response will be an oscillatory type and the settling time of the system may be too high.

Therefore it may be useful to apply a PI compensator element in the feedforward path. The PI compensator may be approximated by a passive phase-lead network [6]. The following phase-lead transfer function was chosen:

$$G_c(s) = \frac{1}{5} \frac{1 + 5s}{1 + s} = \frac{s + 0.2}{s + 1} \tag{44}$$

So the open-loop transfer function of the compensated control system is

$$G(s) = \frac{15K(s^2 + 10.2s + 2)}{s^8 + 28s^7 + 300s^6 + 1878s^5 + 7083s^4 + 10998s^3 + 6920s^2 + 1400s} \tag{45}$$

The critical loop-gain and the gain-crossover frequency were computed by procedure STABH and the results are given in Table IV. Now the critical gain is 45.29 at the gain-crossover frequency 0.944 rad. This value is much higher than the critical gain of the uncompensated system.

Table IV

Phase margin and gain margin of the compensated control system in Ex. 3.8

φ_{pm} [°]	ω_c	φ [°]	K
0	0.94403	-180	45.291
15	0.77835	-165	32.520
30	0.63381	-150	23.918
45	0.50640	-135	17.889
60	0.38483	-120	13.416
75	0.26490	-105	9.665
90	0.06952	-90	3.129

The dominant parts of the root-locus curve both for the uncompensated and the compensated system are given in Fig. 24, clearly showing the effect of the phase-lead compensation.

To compare the performance of the uncompensated and the compensated systems in the time domain, the unit step responses are given for both cases in Fig. 25. The unit step responses were computed by procedure INV LAP and

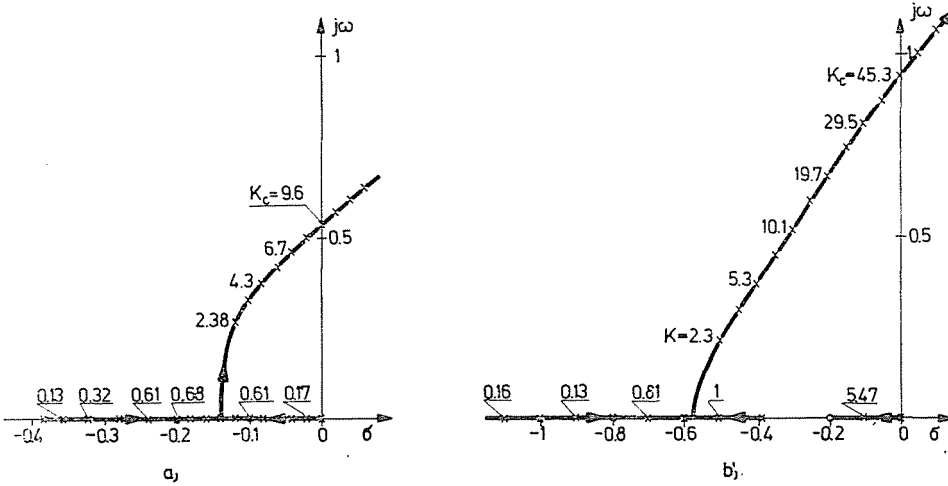


Fig. 24. Dominant part of root-loci of the uncompensated (a) and the compensated (b) multi-loop control systems

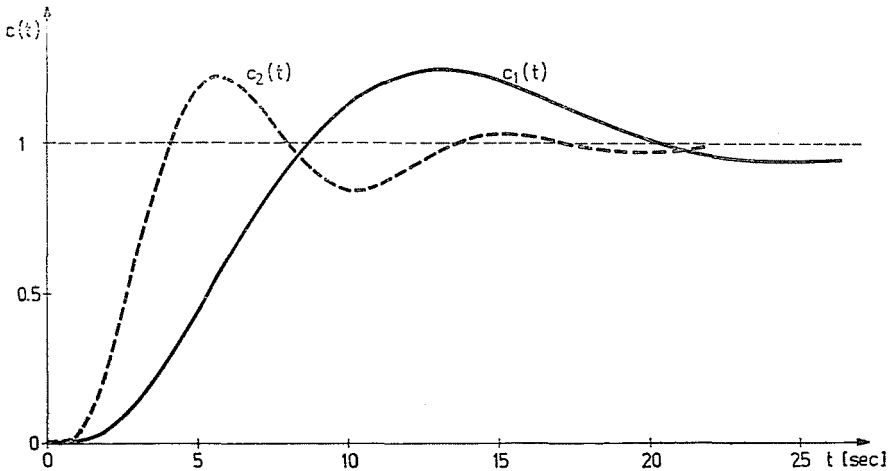


Fig. 25. Unit step response of the uncompensated ($c_1(t)$) and the compensated ($c_2(t)$) control systems in Ex. 3.8

may be written in analytical form. The unit step response of the uncompensated system is

$$c_1(t) = 1 + 1.2186e^{-0.1195t} \cos(0.268t - 3.8818) - 0.1e^{-1.148t} + 1.6358 \cdot 10^{-4}e^{-6.9989t} + 4.9406 \cdot 10^{-4}e^{-2.528t} \cos(5.647t - 10.049) + 4.1841 \cdot 10^{-6}e^{-13.557t} \quad (46)$$

and that of the compensated system

$$c_2(t) = 1 - 0.20358e^{-0.1673t} + 1.4126 \cdot 10^{-3}e^{-6.99t} + 0.4779e^{-0.199t} \cos(0.667t - 4.032) - 0.1929e^{-1.832t} + 3.339 \cdot 10^{-5}e^{-13.557t} + 1.906 \cdot 10^{-3}e^{-2.527t} \cos(5.658t - 9.29) . \quad (47)$$

$c_2(t)$ is seen to be much faster than $c_1(t)$. The performance criteria of both systems have been confronted in Table V.

Table V

Comparison of the performance indices of the uncompensated and compensated control systems in Ex. 3.8

	Critical gain	Maximum overshoot [%]	Delay time [sec]	Rise time [sec]	Settle time [sec]	Time at maximum overshoot [sec]
Uncompensated system	9.6	23	5.3	5.3	30	13
Compensated system	45.3	22	2.6	2.3	12.5	5.7

Several examples were given to illustrate the program system. All examples were for analysis problems. Let us note that the parameters of the compensator elements were not optimally chosen. No optimization is possible but by means of program package for synthesis tasks. In the second part of this paper the synthesis programs will be surveyed.

Summary

A short survey is given of the program system elaborated by the staff of the Department of Automation, Technical University, Budapest, facilitating the analysis of linear, time invariant control systems with lumped parameters and dead time. Most of the programs have been written in ALGOL-60. Several examples are presented to illustrate how to use this program package.

References

1. CHU, Y.: Digital Simulation of Continuous Systems. McGraw-Hill Book Company, New York, 1973.
2. CSÁKI, F.: Dynamics of Control Systems,* Publishing House of the Hung. Acad. of Sciences, Budapest (1966).
3. CSÁKI, F.: Modern Control Theories (Nonlinear, Optimal and Adaptive Systems), Publishing House of the Hung. Acad. of Sciences, Budapest (1972).
4. CSÁKI, F.—BARS, R.: Automation,* Tankönyvkiadó, Budapest (1972).
5. CSÁKI, F.—GÁL, T.—HABER, R.—HABERMAYER, M.—IVÁNCZY, SZ.—KEVICZKY, L.—KOVÁCS, T.—LAKATOS, L.—VAJTA, M. JR.: Procedure Oriented Program System for Computer-Aided Teaching of Automatic Control Theory in High Level Education. 2nd World Conference IFEP on "Computers in Education", Marseille, France, 1—5, Sept. 1975.
6. HABERMAYER, M.: Stability Degree Analysis of Linear Control Systems with Dead Time by a Digital Computer. Periodica Polytechnica El. Eng. **16** (1972), 271—284.
7. HABERMAYER, M.: Analysis of Linear Control Systems with Dead Time in the Time Domain. Periodica Polytechnica El. Eng., **18** (1974), 271—284.
8. KEVICZKY, L.—CSÁKI, F.: Design of Control Systems with Dead Time in the Time Domain. Acta Technica, **74** (1973), 63—84.
9. KEVICZKY, L.—HABERMAYER, M.—BÁNYÁSZ, Cs.: TAPSO — Transient Analysator for the Purpose of Simulation and Optimization, AICA Symposium on Hybrid Computation in Dynamic Systems Design, Rome (1974).
10. KOVÁCS, T.: An ALGOL program for Generating Root Locus Diagrams. Periodica Polytechnica El. Eng., **13** (1969), 99—118.
11. KOVÁCS, T.—LAKATOS, L.—HABER, R.: Root-Locus Method for Compensating and Performance Testing Multiloop Linear Control Systems. Periodica Polytechnica El. Eng., **18** (1974), 27—36.
12. KUO, F.—KAISER, J. F.: System Analysis by Digital Computer. John Wiley, New York (1966).
13. KUO, B. C.: Automatic Control Systems. Prentice Hall, New York (1975).
14. Analysis of Linear Control Systems,* (Ed. Prof. F. Csáki), Műszaki Könyvkiadó, Budapest (1976).
15. VAJTA, M. JR.—KOVÁCS, T.: Inverse Laplace Transformation for Arbitrary Pole-Zero Configuration Using a Digital Computer. Periodica Polytechnica El. Eng., **18** (1974), 37—38.

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