DIGITAL COMPUTER SIMULATION OF THREE-PHASE THYRISTOR BRIDGE CIRCUITS

By

L. LAKATOS

Department of Automation, Technical University, Budapest

Received September 14, 1976

Presented by Prof. Dr. F. CSÁKI

Introduction

The digital computer is a very good tool for solving any kind of modelling problems. With the development of the digital computers and computing methods, the area of simulation has been extended over a wide range of sciences. About 10 years, papers have been published on simulating power semiconductor circuits. Some of these papers deal with the simulation of three-phase thyristor bridge circuits. In this paper a very simple and systematic method using the state-space method for the concerned problem is shown. Some details of the program using this method are mentioned.

1. The circuit to be simulated

The circuit is seen in Fig. 1. The following assumptions are made: the supply voltages are three-phase symmetrical sinusoidal voltages. The linkage reactances and the resistances reduced on the secondary winding of the supplying transformer are the same in the three phases. The loading impedance consists of series inductance and resistance and parallel capacitance. The thyristors (or diodes) are regarded as ideal switches. The firing angle of the thyristors may be set arbitrarily (certainly under a reasonable limit). It is very important that the program only simulates the state of the continuous conduction, when the current is conducted by two or three semiconductors. On the other hand, the aim of the program is only to simulate the relatively slow power circuit transients, rather than the fast switching transients. Thus the integrating step size is constant.

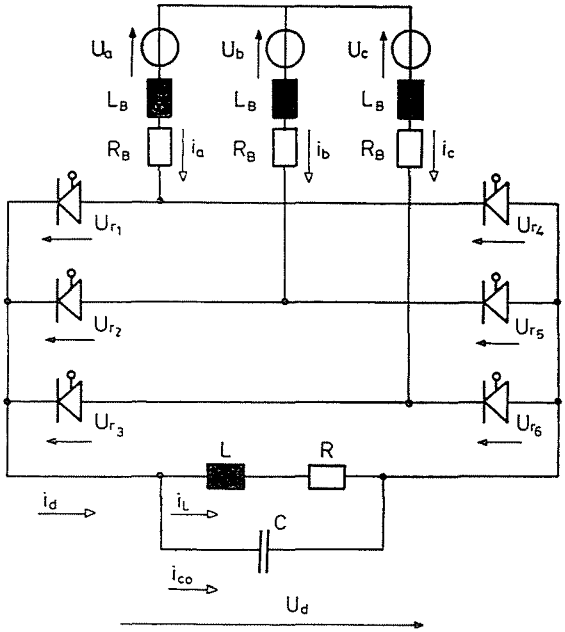

Fig. 1

2. The logic of the simulation and the state equations

Let us suppose that the semiconductors are thyristors with rather small firing angle and the loading inductance is infinite. Then in quasi stationary state the loading current is constant. If the transformer impedances are low, then the course of the direct voltage is as seen in Fig. 2, between the thick

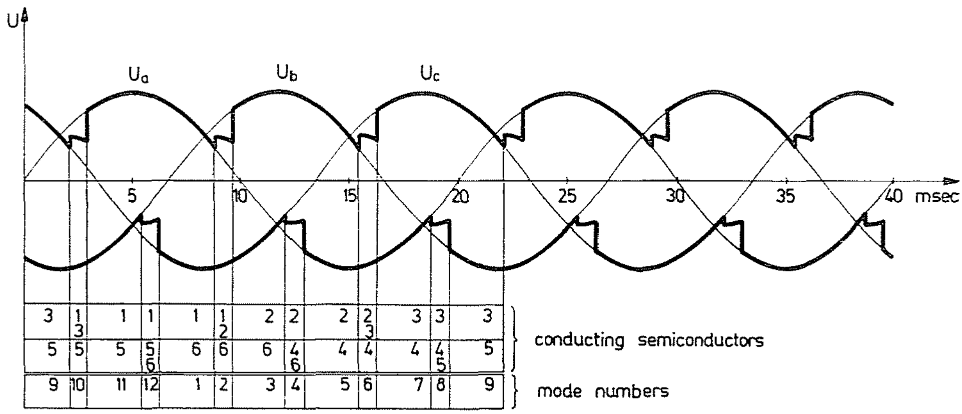

Fig. 2

lines. In the figure the conducting semiconductors are denoted. The (arbitrarily chosen) numbers of the 12 conducting modes are also denoted. Considering the conducting modes, it is seen that the state equations of only three circuits need to be constructed, because in modes 2, 6, 10, in modes 4, 8, 12 and in the modes numbered by odd numbers the configuration of the circuit is the same.

The state equations for the modes 2, 6 and 10 will be determined considering the circuit of mode 2 (see Fig. 3). The nonconducting thyristors are drawn for facilitating the computation of the reverse voltages.

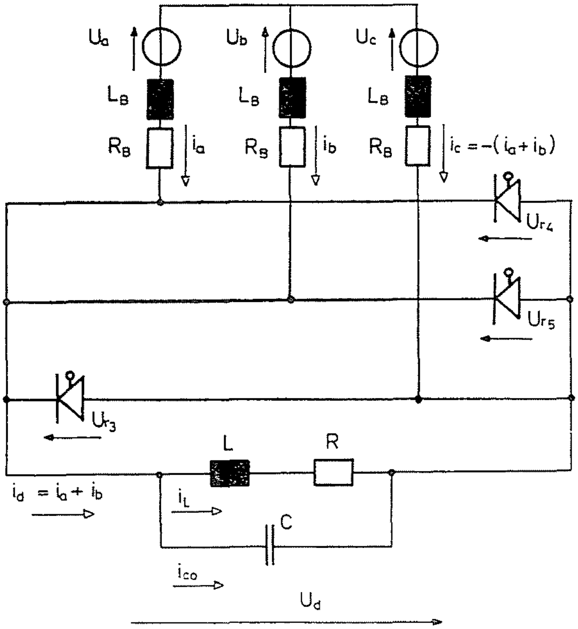

Fig. 3

The Kirchhoff voltage equations are the following:

$$u_a - u_b + R_B i_b + L_B \frac{di_b}{dt} - R_B i_a - L_B \frac{di_a}{dt} = 0 \tag{1}$$

$$u_a - u_c - R_B(i_a + i_b) - L_B\left(\frac{di_a}{dt} + \frac{di_b}{dt}\right) - u_d - R_B i_a - L_B \frac{di_a}{dt} = 0 \tag{2}$$

$$u_d = i_L R + L \frac{di_L}{dt} \,. \tag{3}$$

The Kirchhoff current equation is:

$$i_a + i_b - i_L = \frac{du_d}{dt}, \tag{4}$$

Disregarded the mathematical manipulations, the system of the state equations will be the following:

$$\dot{\mathbf{x}} = \mathbf{A}_1\mathbf{x} + \mathbf{B}_1\mathbf{u} \tag{5}$$

denoting

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} \tag{6}$$

and

$$\mathbf{x} = [i_a\, i_b\, i_L\, u_d]^T \tag{7}$$

$$\mathbf{u} = [u_a\, u_b\, u_c]^T \tag{8}$$

$$\mathbf{A}_1 = \begin{bmatrix} -\frac{R_B}{L_B} & 0 & 0 & -\frac{1}{3L_B} \\ 0 & -\frac{R_B}{L_B} & 0 & -\frac{1}{3L_B} \\ 0 & 0 & -\frac{R}{L} & \frac{1}{L} \\ \frac{1}{C} & \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \tag{9}$$

$$\mathbf{B}_1 = \begin{bmatrix} \frac{2}{3L_B} & -\frac{1}{3L_B} & -\frac{1}{3L_B} \\ -\frac{1}{3L_B} & \frac{2}{3L_B} & -\frac{1}{3L_B} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{10}$$

The superscript T denotes transposition.

By integrating numerically these state equations, the course of the voltages and currents in mode 2 can be evaluated. The other voltages and currents are expressed by the state variables as follows:

$$i_c = -(i_a + i_b) \tag{11}$$

$$i_d = i_a + i_b \tag{12}$$

$$i_{c\sigma} = i_a + i_b - i_L \tag{13}$$

$$u_{r1} = u_{r2} = u_{r6} = 0 \tag{14}$$

$$u_{r3} = u_{r4} = u_{r5} = -u_d. \tag{15}$$

During simulation, the sign of current i_a is to be checked, because in case of $i_a < 0$ a mode change appears. After that the simulation is to be performed using the state equations of the next mode (No. 3). In modes 6 and 10, the same state equation system is valid, but the currents and voltages are to be changed systematically. The assign of the different variables in modes 6 and 10 to the variables in mode 2 is seen in Table 1. The table allows to find out: which variables in modes 6 and 10 correspond to the variables of mode 2.

Table 1

Mode	Supply voltages			Currents			Reverse voltages						Current to be checked
2	u_a	u_b	u_c	i_a	i_b	i_c	u_{r1}	u_{r2}	u_{r3}	u_{r4}	u_{r5}	u_{r6}	i_a
6	u_b	u_c	u_a	i_b	i_c	i_a	u_{r2}	u_{r3}	u_{r1}	u_{r5}	u_{r6}	u_{r4}	i_b
10	u_c	u_a	u_b	i_c	i_a	i_b	u_{r3}	u_{r1}	u_{r2}	u_{r6}	u_{r4}	u_{r5}	i_c

The state equations for the modes 4, 6 and 12 will be determined considering the circuit of mode 12 (see Fig. 4).

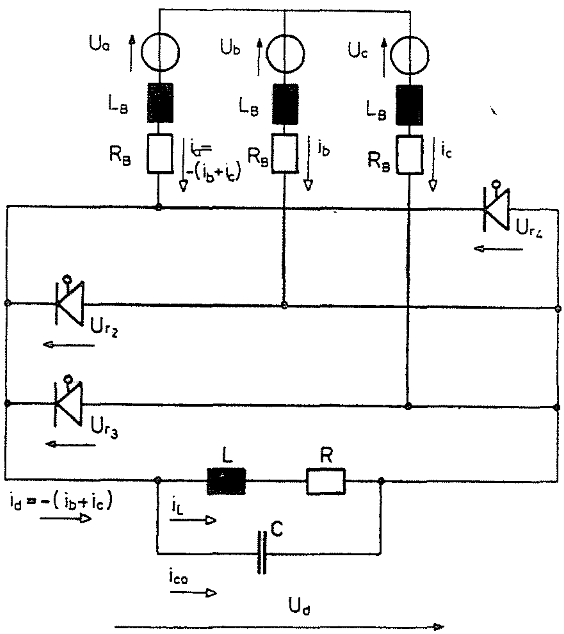

Fig. 4

The non-conducting thyristors are drawn, too. In this case the Kirchhoff equations need not to be written for constructing the state equations, since the state equations of mode 12 can be written from the state equations of mode 2 simply by changing signs and letters. The following changes are to be made

$$\begin{aligned} i_a &\to i_c \\ u_a &\to u_c \\ u_c &\to u_a \\ u_d &\to -u_d \\ i_L &\to -i_L \, , \end{aligned} \tag{16}$$

yielding the following state equations:

$$\dot{\mathbf{x}} = \mathbf{A}_3\mathbf{x} + \mathbf{B}_3\mathbf{u} \tag{17}$$

where

$$\mathbf{x} = [i_b \; i_c \; i_L \; u_d]^T \tag{18}$$

and

$$\mathbf{A}_3 = \begin{bmatrix} -\dfrac{R_B}{L_B} & 0 & 0 & \dfrac{1}{3L_B} \\ 0 & -\dfrac{R_B}{L_B} & 0 & \dfrac{1}{3L_B} \\ 0 & 0 & -\dfrac{R}{L} & \dfrac{1}{L} \\ -\dfrac{1}{C} & -\dfrac{1}{C} & -\dfrac{1}{C} & 0 \end{bmatrix} \tag{19}$$

$$\mathbf{B}_3 = \begin{bmatrix} -\dfrac{1}{3L_B} & \dfrac{2}{3L_B} & -\dfrac{1}{3L_B} \\ -\dfrac{1}{3L_B} & -\dfrac{1}{3L_B} & \dfrac{2}{3L_B} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{20}$$

The expressions of the other voltages and currents are:

$$i_a = -(i_b + i_c) \tag{21}$$

$$i_d = -(i_b + i_c) \tag{22}$$

$$i_{c\sigma} = -(i_b + i_c + i_L) \tag{23}$$

$$u_{r2} = u_{r3} = u_{r4} = 0 \tag{24}$$

$$u_{r1} = u_{r5} = u_{r6} = -u_d \, . \tag{25}$$

During the simulation the sign of i_b is to be checked, because in case of $i_b < 0$ a mode change appears. Then the simulation is to be continued by integrating the state equations of the next mode (No. 1). In modes 4 and 8 the same state equation system is valid. The assign of the different variables to each other is the same as in Table 1, except that in mode 4 i_c, while in mode 8 i_q, has to be checked.

The state equations for the odd-numbered modes will be determined considering the circuit of mode 1 (see Fig. 5). The non-conducting semiconductors are also drawn. The Kirchhoff equations are:

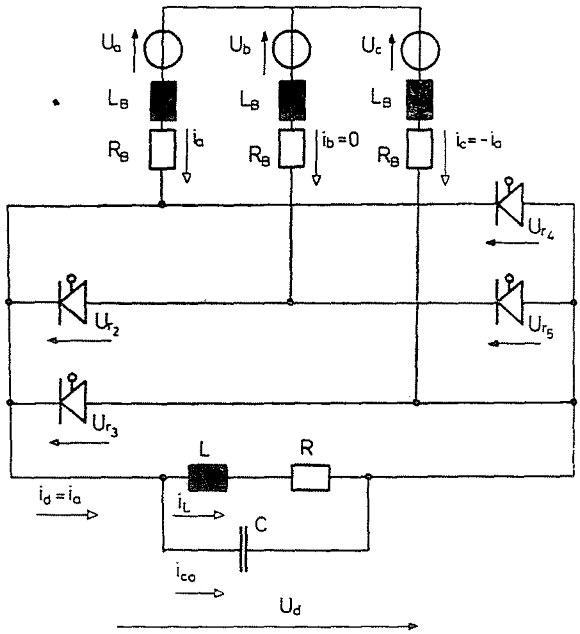

Fig. 5

$$u_a - u_c - L_B \frac{di_a}{dt} - R_B i_a - u_d - L_B \frac{di_a}{dt} - R_B i_a = 0 \tag{26}$$

$$i_b = 0 \tag{27}$$

$$u_d = R i_L + L \frac{di_L}{dt} \tag{28}$$

$$i_a - i_L = C \frac{du_d}{dt} . \tag{29}$$

The state equation are:

$$\dot{\mathbf{x}} = \mathbf{A}_2 \mathbf{x} + \mathbf{B}_2 \mathbf{u} \tag{30}$$

where $\mathbf{x}$ is the same as in Eq. (7) and

$$\mathbf{A}_2 = \begin{bmatrix} -\dfrac{R_B}{L_B} & 0 & 0 & -\dfrac{1}{2L_B} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\dfrac{R}{L} & \dfrac{1}{L} \\ \dfrac{1}{C} & 0 & -\dfrac{1}{C} & 0 \end{bmatrix} \tag{31}$$

$$\mathbf{B}_2 = \begin{bmatrix} \dfrac{1}{2L_B} & 0 & -\dfrac{1}{2L_B} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} . \tag{32}$$

It is noted that the sizes of matrices $\mathbf{A}_2$ and $\mathbf{B}_2$ were not reduced in order to obtain a simpler computer program. The expressions of the other voltages and currents are:

$$i_b = 0 \tag{33}$$

$$i_c = -i_a \tag{34}$$

$$i_d = i_a \tag{35}$$

$$i_{c\sigma} = i_a - i_L \tag{36}$$

$$u_{r1} = u_{r6} = 0 \tag{37}$$

$$u_{r3} = u_{r4} = -u_d \tag{38}$$

$$u_{r2} = u_b - u_a + L_B \frac{di_a}{dt} + R_B i_a \tag{39}$$

$$u_{r5} = u_c - u_b + L_B \frac{di_a}{dt} + R_B i_a . \tag{40}$$

During the simulation of sign of u_{r2} is to be checked, because if $u_{r2} > 0$ and the firing condition is fulfilled, a mode change appears. Then the simulation is to be continued by integrating the state equations of the next mode (No. 2). In the other odd-numbered modes the same state equations are valid. The assign of the variables to each other is seen in Table 2.

Table 2

Mode	Supply voltages			Currents			Reverse voltages						Reverse voltage to be checked
1	u_a	u_b	u_c	i_a	i_b	i_c	u_{r1}	u_{r2}	u_{r3}	u_{r4}	u_{r5}	u_{r6}	u_{r2}
3	u_b	u_a	u_c	i_b	i_a	i_c	u_{r2}	u_{r1}	u_{r3}	u_{r5}	u_{r4}	u_{r6}	u_{r4}
5	u_b	u_c	u_a	i_b	i_c	i_a	u_{r2}	u_{r3}	u_{r1}	u_{r5}	u_{r6}	u_{r4}	u_{r3}
7	u_c	u_b	u_a	i_c	i_b	i_a	u_{r3}	u_{r2}	u_{r1}	u_{r6}	u_{r5}	u_{r4}	u_{r5}
9	u_c	u_a	u_b	i_c	i_a	i_b	u_{r3}	u_{r1}	u_{r2}	u_{r6}	u_{r4}	u_{r5}	u_{r1}
11	u_a	u_c	u_b	i_a	i_c	i_b	u_{r1}	u_{r3}	u_{r2}	u_{r4}	u_{r6}	u_{r5}	u_{r6}

3. Computer algorithm

The program using the concerned logic was prepared for the computer RAZDAN-3 of the University Computing Centre. The input data of the program are the following: the effective value of the supply voltage, the frequency, the values of L_B, R_B, L, R, C, the phase angles of the supply voltage at zero time, the firing angles of the thyristors with respect to the zero-crossing (in case of diodes these are to be set to zero), the initial conducting condition of the semiconductors, the initial values of the phase currents, of the current on the loading inductance, of the voltage of the capacitance, finally the time and the step size of simulation. During the simulation the currents of the semiconductors are checked, and if any of them is found to be negative, the program gives an error message, because the operation of the circuit does not correspond to continuous conduction. The integrating method is the fourth-order Runge-Kutta method.

The advantage of the program is to be much faster, than any general program simulating semiconductor circuits, e.g. [4]. It is noted that if the circuit does not contain inductance or capacitance, then $L = 0$, $L_B = 0$ or $C = 0$ must not be given among the input data, because the program will stop with overflow. In this case L and L_B are to be given with values small enough to keep the integration stable. Similarly $R = 0$ must not be given, because then the integration will be unstable. In this case R has to be given with a value small enough to keep the integration stable. It is not a very correct solution, but the results are even so acceptable, because modelling the circuit anyway involves inaccuracy.

4. Example

The circuit to be simulated has the following element-values: $U_s = 220$ V, $L_B = 1$ mH, $R_B = 0.1\ \Omega$, $L = 5$ mH, $R = 1\ \Omega$, $C = 1000\ \mu$F. The simulation is started at the positive zero-crossing of the voltage of phase "a", and it is terminated 40 msec later. The step size of integration is 0.1 msec. At start, the semiconductors No 2 and 5 are assumed to be in conducting condition. The initial voltage and currents are zeros. The simulation was performed in two cases, first without any firing angle, and second with a firing angle of 30°

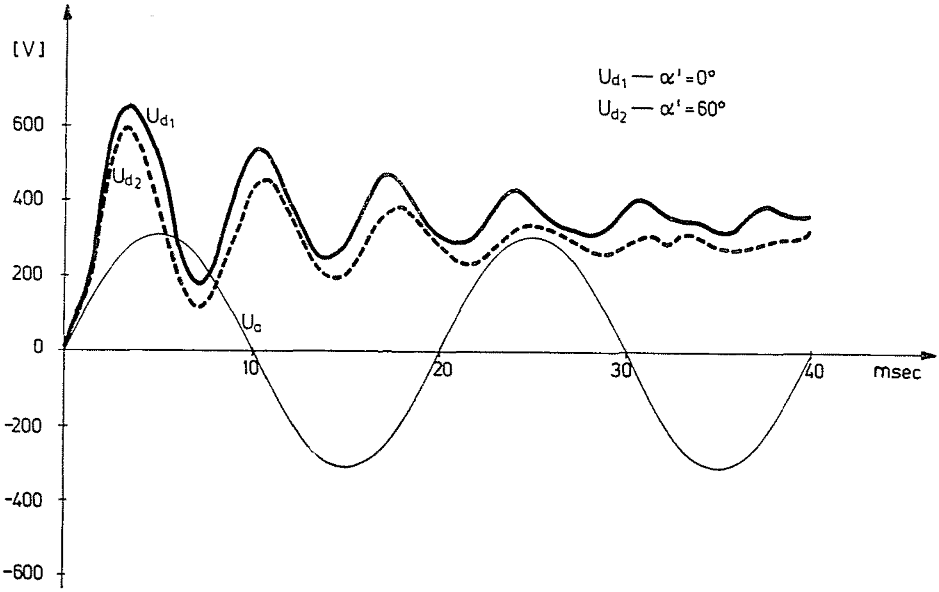

Fig. 6

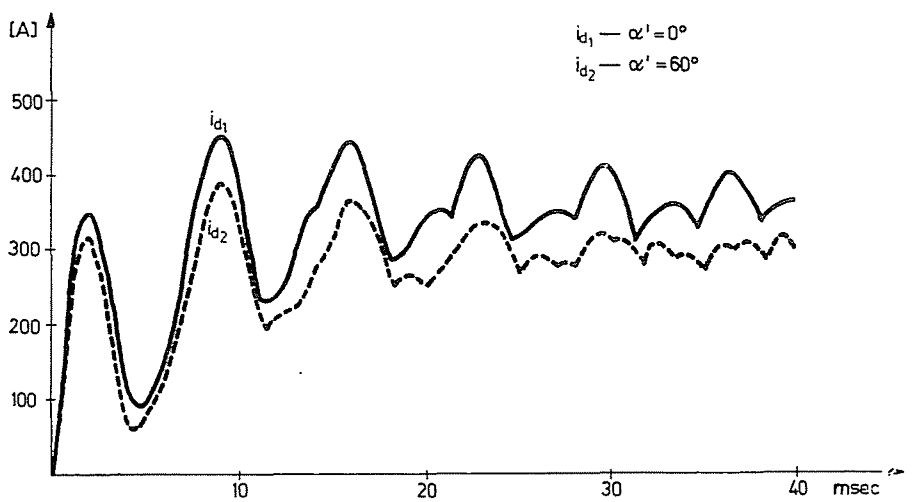

Fig. 7

measured from the natural commutation. The course of voltage u_d in both cases is seen in Fig. 6 in comparison with the course of voltage *ud*. The course of current i_d in both cases is seen in Fig. 7. The effect of the phase control is clearly visible.

Summary

This paper deals with the digital computer simulation of three-phase thyristor bridge circuits. The linkage reactance and the resistance of the supplying transformer are taken into consideration. The load consists of elements R, L, C. The simulation is performed using the state equations. Some details of the program using the algorithm are presented, and so are computer results.

References

1. HINGORANI, N. G.—HAY, J. L.—CROSBIE, R. E.: Dynamic simulation of h.v.d.c. transmission systems on digital computers. Proc. IEE, **113**, May, 1966.
2. HTSUI, J. S. C.—SHEPHERD, W.: Method of digital computation of thyristor switching circuits. Proc. IEE, **118**, August, 1971.
3. WILLIAMS, S.—SMITH, I. R.: Fast digital computation of 3-phase thyristor bridge circuits. Proc. IEE, **120**, July, 1973.
4. LAKATOS, L.: A new algorithm for simulating power semiconductor circuits. Periodica Polytechnica El. Eng. Vol. 21. No, 1.

Lóránt LAKATOS H-1521 Budapest