A METHOD FOR DETERMINING THE STEADY-STATE TEMPERATURE DISTRIBUTION IN THE WINDING DISCS OF OIL-COOLED TRANSFORMERS

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Summary

The calculation of the warming of transformers aims at determining the place and value of the highest operational temperature ("hot-spot" temperature).

The present study discusses an analytical method for the approximative determination of the two-dimensional temperature field in the discs of a given winding by means of matrix equations in the case of boundary conditions of the 3rd kind.

Introduction

The increasing power of transformer units makes the problems of operational safety and useful life more and more important. As a consequence, the calculation of the warming of transformers requires higher and higher accuracy.

Large oil-transformers are built with layer-type or with disc-type windings.

In recent years several papers of ALLEN and PREININGEROVA [1, 2, 3] discussed analytical methods for the calculation of the warming of layer-type windings. Similar problems have been dealt with also by PIVRNEC and HAVLICEK [4, 5].

A method for the calculation of the warming of disc-type windings has been elaborated by KISS [6].

The calculation of the warming of layer-type coils can be based — particularly in the case of forced convective cooling (OF) — on from certain view-points simpler model conditions (known average oil-speed, laminar flow, constant heat flow density on the surface of the winding).

With disc-type coils, the modelling of the thermal ambiency of the winding discs raises a considerably harder problem, since the boundary conditions of the 2nd kind cannot be applied (i.e. there is no uniform heat flow density on the surface), further no uniform internal heat source intensity can be assumed, and also the heat transfer conditions on the vertical and horizontal heat transferring surfaces of the winding discs are different [7].

For the calculation of the steady-state warming of disc-type transformers the possibility of modelling both with distributed and lumped parameters has been examined [8].

Our study discusses a two-dimensional calculation method based on the description with distributed parameters.

Hypotheses and model assumptions

Object of the examination is the — usually uppermost but one — winding disc exposed to thermally critical loading.

The term "winding disc" means a part of the coil consisting of turns wound on each other and bounded by oil channels.

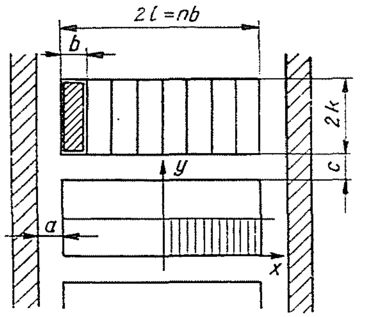

Fig. 1. The design of the winding disc and the main notations

The examination is based on the following hypotheses:

(1) The global material and energy balance relative to the whole transformer is not essentially influenced by a single disc (e.g. the one at the critical place).

(2) Following from (1), in steady state the ambiency of the disc is invariant, and the forced mass flow $\varnothing_m$ around it can be regarded as a determined value.

(3) The oil stream $\varnothing_m$ as well as the geometry and the operational parameters being known, a mixed average "ambient" temperature $t_w = t_{om}$ and a distribution of the heat transfer coefficient can be considered to be determined ($\alpha = \alpha(x,y)$).

The simplifying model conditions permitting an analytical description are as follows:

a) The oblong profile of the disc (Fig. 1) is regularly filled up by the conductors and the insulation.

b) The disc is of an inhomogeneous structure. In the directions of x and y, however, the general relationships allow the heat transfer coefficient to be interpreted as constant and equivalent [9]. Therefore the disc is replaced by a homogeneous material with the anisotropy corresponding to the values λ_x and λ_y, respectively.

c) On the disc surfaces opposite to each other the heat transfer conditions are identical in the directions of x and y, resp.

d) As a first approximation, the internal heat source distribution f_{hb} is assumed to be symmetrical to the centre-lines of the disc, and so the centre-lines are at the same time the symmetry axes of the temperature distribution.

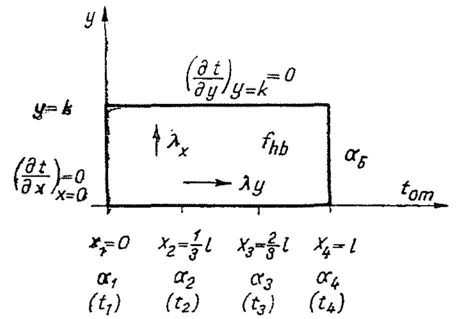

Fig. 2. A quarter of the winding disc as a space part under examination, with the conditions of calculation

e) From assumptions *c)* and *d)* follows that the temperature distribution is symmetrical in the disc, the maximum temperature arises at $x = 0$ and $y = 0$, and along the symmetry axes ($x = 0$ and $y = 0$) the boundary condition of the 0th kind is valid:

$$\left(\frac{\partial t}{\partial x}\right)_{x=0} = 0\,, \quad (q_x = 0); \tag{1}$$

$$\text{and} \left(\frac{\partial t}{\partial y}\right)_{y=0} = 0\,, \quad (q_y = 0)\,. \tag{2}$$

f) All this allows the examination to be restricted to one quarter of the winding disc. In Fig. 2 the space part examined and the boundary conditions mentioned are seen.

The tasks to be solved are as follows:

I. Determine the temperature $t_{\max} = t\,(0,\ k)$

II. Determine the heat flux density distribution in the plane of $y = 0$ (or at least the values of the surface heat density at $x = 0$, $x = \frac{1}{3}$, $x = \frac{2}{3}$ and $x = l$), required to determine the surface distribution $\frac{\partial t}{\partial y}$ in accordance with

$$q_h = -\lambda_x \frac{\partial t}{\partial y}. \tag{3}$$

Procedure of the solution

The solution was built up on the superposition principle, going from simpler assumptions towards those satisfying more and more complicated assumptions. The main steps of the solution are as follows:

In the case of $f_{hb} = 0$ and $\lambda = \lambda_x = \lambda_y$ the differential equation to be solved is homogeneous:

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0 \tag{4}$$

The second step consists in solving the homogeneous equation valid in the case of $\lambda_x = \lambda_y$:

$$\lambda_x \frac{\partial^2 t}{\partial x^2} + \lambda_y \frac{\partial^2 t}{\partial y^2} = 0 \tag{5}$$

and in generalizing the solution.

Further steps are the consideration of the internal heat source, the solution of the inhomogeneous equation, then the coupling of the homogeneous and inhomogeneous partial problems.

At last we examine the possibility of taking into consideration the internal heat source excess arising as an effect of the stray flux.

Solution of the homogeneous equation

Examine the solution of Eq. (4) for the space part according to Fig. 2, with the deviation in the boundary conditions that in the plane of $y = 0$ the heat distribution is provisorily regarded as given (boundary condition of the 1st kind). Let the distribution be given through the temperatures of the following four points:

$$\begin{aligned} t(0,0) &= t_1, \\ t\left(\frac{1}{3}l, 0\right) &= t_2, \\ t\left(\frac{2}{3}l, 0\right) &= t_3, \\ t(l, 0) &= t_4 \end{aligned} \tag{6}$$

At any intermediate place x the temperature is calculated from the above four values by an interpolation polynome of the 4th degree:

$$t(x,0) = c_1 + c_2x^2 + c_3x^3 + c_4x^4 \,. \tag{7}$$

In the polynome there is no member of the first degree since, according to (1),

$$\left(\frac{\partial t}{\partial x}\right)_{x=0} = 0 \,.$$

Write the linear relationship between the constants $c_1 \ldots c_4$ and the temperatures $t_1 \ldots t_4$ by using the matrix equation:

$$t = \mathbf{X}\, c \tag{8}$$

where

$$t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix}, \qquad c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \tag{9), (10}$$

are formal vectors, while matrix $\mathbf{X}$ is as follows:

$$\mathbf{X} = \begin{bmatrix} 1 & x_1^2 & x_1^3 & x_1^4 \\ 1 & x_2^2 & x_2^3 & x_2^4 \\ 1 & x_3^2 & x_3^3 & x_3^4 \\ 1 & x_4^2 & x_4^3 & x_4^4 \end{bmatrix} \tag{11}$$

Since the determinant of matrix $\mathbf{X}$ (the so-called Wandermonde type determinant) is non-zero, the equation

$$c = \mathbf{X}^{-1}\, t \tag{12}$$

is well-defined, and c can be produced a linear combination of the co-ordinates of t. The required temperature $t(0,k)$ and the derivatives can, as a first step be expressed by means of the components of c.

The solution of Eq. (4) for the space part considered, with the boundary conditions discussed, will be as follows [10]:

$$t(x,y) = \sum_{n=1}^{\infty} \frac{2\left[\left(\frac{\alpha_5}{\lambda}\right)^2 + \beta_n^2\right] \cos \beta_n x \cdot ch\beta_n (k-y)}{\left\{\left[\beta_n^2 + \left(\frac{\alpha_5}{\lambda}\right)^2\right] \cdot l + \frac{\alpha_5}{\lambda}\right\} \cdot ch\, \beta_n k} \int_0^l t(x,0) \cos \beta_n x \, d x \tag{13}$$

where the values for β_n $(n = 1, 2, \ldots)$ will be given by the positive roots of the trigonometric equation

$$\beta \operatorname{tg} \beta l = \frac{\alpha_5}{\lambda} \,. \tag{14}$$

After substituting the function $t(x,0)$ from relationship (7), then performing the integration and arranging the result according to the components of vector c one obtains the following equation:

$$t(x,y) = \sum_{n=1}^{\infty} \frac{2\left[\left(\frac{\alpha_5}{\lambda}\right)^2 + \beta_n^2\right] \cos \beta_n x \cdot ch\, \beta_n (k-y)}{\beta_n \left\{\left[\beta_n^2 + \left(\frac{\alpha_5}{\lambda}\right)^2\right] l + \frac{\alpha_5}{\lambda}\right\} ch\, \beta_n k} \cdot$$

$$\cdot \left\{(\sin \beta_l)\, c_1 + \right.$$

$$+ \left(\frac{l^2\beta_n^2 - 2}{\beta_n^2} \sin \beta_n l + \frac{2l}{\beta_n} \cos \beta_n l\right) c_2 + \tag{15}$$

$$+ \left(\frac{l^3\beta_n^2 - 6l}{\beta_n^2} \sin \beta_n l + \frac{3l^2\beta_n^2 - 6}{\beta_n^3} \cos \beta_n l + \frac{6}{\beta_n^3}\right) c_3 +$$

$$\left. + \left(\frac{l^4\beta_n^4 - 12l^2\beta_n^2 + 24}{\beta_n^4} \sin \beta_n l + \frac{4l^3\beta_n^2 - 24}{\beta_n^3} \cos \beta_n l\right) c_4\right\}$$

The substitutions $x = 0$ and $y = k$ from the solution (15) will give the temperature at the point required. From (15) the values of $\frac{\partial t}{\partial y}$ are formed to determine the heat flow densities arising at the points of temperature $t_1, \ldots t_4$.

The derivatives in the direction y are produced from four temperatures each, by deriving the interpolation polynomes with variable y fitted to the temperature, – essentially as a fixed combination of the four temperatures. The basic points for the polynome of the temperature change of direction y have been chosen in all places x as points with the co-ordinates $y_1 = 0$, $y_2 = 0{,}1\ k$, $y_3 = 0{,}\ 2\ k$ and $y_4 = k$.

Using the interpolar polynome of Lagrange, a third degree polynome can be fitted to the four basic points:

$$\begin{aligned} t(x,y) = & \frac{(y-y_2)(y-y_3)(y-y_4)}{(y_1-y_2)(y_1-y_3)(y_1-y_4)} t(x,y_1) + \\ & + \frac{(y-y_1)(y-y_3)(y-y_4)}{(y_2-y_1)(y_2-y_3)(y_2-y_4)} t(x,y_2) + \\ & + \frac{(y-y_1)(y-y_2)(y-y_4)}{(y_3-y_1)(y_3-y_2)(y_3-y_4)} t(x,y_3) + \\ & + \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_4-y_1)(y_4-y_2)(y_4-y_3)} t(x,y_4). \end{aligned} \tag{16}$$

The derivative of this function at $y = 0$ will be:

$$\left(\frac{\partial t(x,y)}{\partial y}\right)_{y=0} = \frac{1}{k}\left[-16\, t(x,y_1) + 22{,}2\, t(x,y_2) - \right. \\ \left. - 6{,}25\, t(x,y_3) + 0{,}027\, t(x,y_4)\right]. \tag{17}$$

By substituting the values of $t(x,y)$ calculated from Eq. (15) into (17) one can determine the partial derivative of the surface, further, according to (3), the required heat flow densities of the surface.

The solution of Eq. (5).

The difference between heat transfer coefficients in directions x and y ($\lambda_x \neq \lambda_y$) is taken into consideration in Eq. (5). By the introduction of generalized (dimensionless) variables the equation can be reduced to the form (4).

By introducing the transformation

$$x = \zeta l \tag{18}$$

and repeatedly applying the rules of differentiation, Eq. (5) will have the following form:

$$\frac{\partial^2 t}{\partial \zeta^2} + \frac{\partial^2 t}{\partial \left(\frac{y\sqrt{\lambda_x}}{l\sqrt{\lambda_y}}\right)^2} = 0\,. \tag{19}$$

Let the notion of the Carlslow number (Ca) be introduced in honour of the author of reference work [10]:

$$Ca = \frac{y}{l}\sqrt{\frac{\lambda_x}{\lambda_y}}\,. \tag{20}$$

By this, Eq. (19) transforms into (4):

$$\frac{\partial^2 t}{\partial \zeta^2} + \frac{\partial^2 t}{\partial Ca^2} = 0\,, \tag{21}$$

in which $0 \leq \zeta \leq 1$ (22)

$$0 \leq Ca \leq \frac{k}{l}\sqrt{\frac{\lambda_x}{\lambda_y}} = Ca_0\,. \tag{23}$$

On the heat-insulated borders as well as on the borders characterized

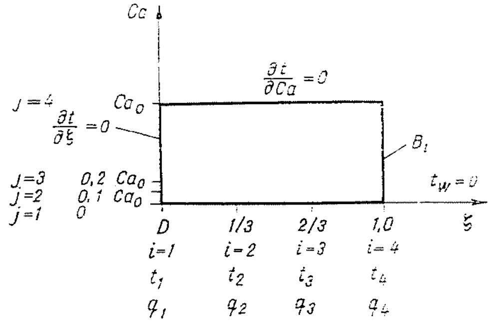

Fig. 3. Formulation of the task with the introduction of dimensionless variables

by the temperature function, the transformation does not cause any change. On the lateral face of $x = l$ the boundary condition of the 3rd kind

$$- \lambda_x \left(\frac{\partial t}{\partial x} \right)_{x=0} = \alpha_5 \, t \tag{24}$$

can be written, considering (18), also in the form

$$- \frac{\partial t}{\partial \zeta} = Bi.\, t\,. \tag{25}$$

where $Bi = \dfrac{\alpha_5 l}{\lambda_x}$.

With the respective boundary conditions (Fig. 3), and taking the transformations into account, the solution of Eq. (21), with due regard to (15), is to be written by means of the following vector equation:

$$t(\zeta_i,(a_j) = \left[a_1(\zeta_i,(a_j)\; a_2(\zeta_i,(a_j)\;\; a_3(\zeta_i,(a_j)\;\; a_4(\zeta_i,(a_j) \right] \cdot C = a^x \cdot C\,, \tag{26}$$

where the elements of the serial vector a^x are:

$$a_1(\zeta_i,\, \eta_j) = \lim_{M\to\infty} \sum_{n=1}^{M} b(\zeta_i,\, \eta_j)\; \sin \beta_n\,. \tag{27}$$

$$a_2(\zeta_i,\, \eta_j) = \lim_{M\to\infty} \sum_{n=1}^{M} b(\zeta_i,\, \eta_j) \left(\frac{\beta_n^2 - 2}{\beta_n^2} \sin \beta_n + \frac{2}{\beta_n} \cos\, \beta_n \right). \tag{28}$$

$$a_3(\zeta_i,\, \eta_j) = \lim_{M\to\infty} \sum_{n=1}^{M} b(\zeta_i,\, \eta_j) \left(\frac{\beta_n^2 - 6}{\beta_n^2} \sin \beta_n + \frac{3\beta_n^2 - 6}{\beta^3} \cos \beta_n + \frac{6}{\beta_n^3} \right). \tag{29}$$

$$a_4(\zeta_i,\, \eta_j) = \lim_{M\to\infty} \sum_{n=1}^{M} b(\zeta_i,\, \eta_j) \left(\frac{\beta_n^4 - 12\beta_n^2 + 24}{\beta_n^4} \sin \beta_n + \frac{4\beta_n^2 - 24}{\beta_n^3} \cos\, \beta_n \right). \tag{30}$$

In the relationships

$$b\ (\zeta_i, Ca_j) = \frac{2(Bi^2 + \beta_n^2)\cos \beta_n \zeta_i\, ch\,(Ca_0 - Ca_j)}{\beta_n(\beta_n^2 + Bi^2 + Bi)\, ch\, \beta_n\, Ca}\,, \tag{31}$$

and the values of β_n $(n = 1, 2, \ldots)$ are the positive roots of the trigonometric equation

$$\beta \operatorname{tg} \beta = Bi\,. \tag{32}$$

The temperature vector valid in the symmetry plane of $Ca = Ca_0$ (the first element being the maximum temperature to be found):

$$\begin{bmatrix} t\,(\zeta_1, Ca_0) \\ t\,(\zeta_2, Ca_0) \\ t\,(\zeta_3, Ca_0) \\ t\,(\zeta_4, Ca_0) \end{bmatrix} = \mathbf{A} \cdot c\,. \tag{33}$$

The elements of matrix A are formed by the co-ordinates of the serial vector a^x interpreted in Eq. (26):

$$\mathbf{A} = \begin{bmatrix} a^x(\zeta_1, Ca_0) \\ a^x(\zeta_2, Ca_0) \\ a^x(\zeta_3, Ca_0) \\ a^x(\zeta_4, Ca_0) \end{bmatrix} \tag{34}$$

Since on the boundary surface $Ca = 0$, the superficial temperature vector (6) has been considered to be known it will be expedient to go over from vector c to the vector of the surface temperature

$$t = \begin{bmatrix} t(\zeta_1, 0) \\ t(\zeta_2, 0) \\ t(\zeta_3, 0) \\ t(\zeta_4, 0) \end{bmatrix} \tag{35}$$

By going over from vector c to vector t one can determine the temperature vector t^{sup} (supremum) in the symmetry plane $Ca = Ca_0$:

$$t^{\mathrm{sup}} = \begin{bmatrix} t(\zeta_1, Ca_0) \\ t(\zeta_2, Ca_0) \\ t(\zeta_3, Ca_0) \\ t(\zeta_4, Ca_0) \end{bmatrix} = \mathbf{A}\, \zeta^{-1} \cdot t \tag{36}$$

where ζ is the equivalent of matrix $\mathbf{X}$ according to (11), with the proper substitution of $x_i = \zeta_i$.

Let henceforth the matrix $\mathbf{H}$ mean the sequence $\mathbf{A}\,\zeta^{-1}$:

$$\mathbf{H} = \mathbf{A}\,\zeta^{-1}. \tag{37}$$

Express now the vector of the surface heat flow density by means of vector t, using Eqs. (17) and (8) (details of the operation are omitted):

$$\begin{bmatrix} q(\zeta_1,0) \\ q(\zeta_2,0) \\ q(\zeta_3,0) \\ q(\zeta_4,0) \end{bmatrix} = -\sqrt{\frac{\lambda_x \lambda_y}{l}} \cdot \mathbf{TQ} \cdot t\,, \tag{38}$$

where matrix $\mathbf{TQ}$ serving for the calculation of vector q, on the basis of vector t, can be interpreted according to (17):

$$\mathbf{TQ} = \frac{1}{Ca_0}\left(-16\,\mathbf{E} + 22{,}\dot{2}\;\mathbf{D} - 6{,}25\,\mathbf{C} + 0{,}02\dot{7}\,\mathbf{P}\right) \tag{39}$$

in which

$$\mathbf{E} = \begin{bmatrix} a^x(\zeta_1,0) \\ a^x(\zeta_2,0) \\ a^x(\zeta_3,0) \\ a^x(\zeta_4,0) \end{bmatrix} \cdot \zeta^{-1} \tag{40/a}$$

$$\mathbf{D} = \begin{bmatrix} a^x(\zeta_1,0{,}1\,Ca) \\ a^x(\zeta_2,0{,}1\,Ca) \\ a^x(\zeta_3,0{,}1\,Ca) \\ a^x(\zeta_4,0{,}1\,Ca) \end{bmatrix} \cdot \zeta^{-1} \tag{40/b}$$

$$\mathbf{C} = \begin{bmatrix} a^x(\zeta_1,0{,}2\,Ca) \\ a^x(\zeta_2,0{,}2\,Ca) \\ a^x(\zeta_3,0{,}2\,Ca) \\ a^x(\zeta_4,0{,}2\,Ca) \end{bmatrix} \cdot \zeta^{-1} \tag{40/c}$$

$$\mathbf{B} = \begin{bmatrix} a^x(\zeta_1, Ca_0) \\ a^x(\zeta_2, Ca_0) \\ a^x(\zeta_3, Ca_0) \\ a^x(\zeta_4, Ca_0) \end{bmatrix} \cdot \zeta^{-1} = \mathbf{A} \cdot \zeta^{-1} \tag{40/d}$$

Considering Eqs. (40/a), (26) and (12), it becomes evident that the identity

$$t = \mathbf{E}\,t \tag{41}$$

must hold true, i.e. $\mathbf{E}$ is necessarily a unit matrix.

During the numerical calculations these facts make it possible to get evidence about the quality of the choice of the summing limit marked M in Equ. (27)...(31), i.e. to conclude to the relative error of the convergence on the basis of the individual deviation of the elements **E** from the elements of the unit matrix.

Solution of the inhomogeneous problem

In the winding disc there is an internal heat source of intensity f_{hb}. With adaptation to the above mentioned calculating method, and to simplify the calculation, the intensity of the internal heat source is given by a step function. A safety error will arise if the higher temperature is taken into account along every step:

$$f_{hb}(t) = \begin{cases} f_{h0}(1 + \beta_0\, t_1^{\text{sup}}), \text{ if } x_1 \leq x \leq x_2, \\ f_{h0}(1 + \beta_0\, t_2^{\text{sup}}), \text{ if } x_2 < x \leq x_3, \\ f_{h0}(1 + \beta_0\, t_3^{\text{sup}}), \text{ if } x_3 < x \leq x_4 \end{cases} \tag{42}$$

where β_0 is the temperature coefficient of the specific electrical resistance, and f_{h0} the intensity of the internal heat source at temperature t_w. The temperatures t^{sup} in the relationship mean those arising in the plane of $y = k$ of the winding disc. Thus the temperature changes of direction y will be disregarded, the partial problem will be of one dimension, the equation to be solved will be:

$$-\lambda_x \frac{\partial^2 t}{\partial x^2} = f_{hb} \tag{43}$$

and the boundary condition will be according to (24).

The differential equation is linear, and the internal heat source described by the step function can be given by a vector according to (42). Consequently, there must exist such a linear operator which produces the vector t^f of the steady state temperature from the known heat source vector. Using Equ. (42), writing the known solution of differential equation (43), with the use of transformation according to (18) and the notations of (25/b), after arranging, and omitting the details of calculation, one obtains the following matrix equation:

$$t^f = f_{h0}\, \beta_0\, \mathbf{FT}\, t^{\text{sup}} + t^{f0}, \tag{44}$$

$$\mathbf{FT} = \frac{l^2}{\lambda_x} \cdot$$

$$\begin{bmatrix} \zeta_2\left(1-\frac{\zeta_2}{2}+\frac{1}{Bi}\right) & (\zeta_3-\zeta_2)\left(1-\frac{\zeta_3+\zeta_2}{2}+\frac{1}{Bi}\right) & (1-\zeta_3)\left(\frac{1-\zeta_3}{2}+\frac{1}{Bi}\right) & 0 \\ \zeta_2\left(1-\zeta_2+\frac{1}{Bi}\right) & (\zeta_3-\zeta_2)\left(1-\frac{\zeta_3+\zeta_2}{2}+\frac{1}{Bi}\right) & (1-\zeta_3)\left(\frac{1-\zeta_3}{2}+\frac{1}{Bi}\right) & 0 \\ \zeta_2\left(1-\zeta_3+\frac{1}{Bi}\right) & (\zeta_3-\zeta_2)\left(1-\zeta_3+\frac{1}{Bi}\right) & (1-\zeta_3)\left(\frac{1-\zeta_3}{2}+\frac{1}{Bi}\right) & 0 \\ \zeta_2\frac{1}{Bi} & (\zeta_3-\zeta_2)\frac{1}{Bi} & (1-\zeta_3)\frac{1}{Bi} & 0 \end{bmatrix}$$

and t^{f0} means the temperature generated by the internal source part belonging to the temperature 0 °C:

$$t^{f0} = f_{h0} \cdot \frac{l^2}{\lambda_x} \begin{bmatrix} \frac{1}{Bi} + \frac{1}{2} \\ \frac{1}{Bi} + \frac{1-\zeta_2^2}{2} \\ \frac{1}{Bi} + \frac{1-\zeta_3^2}{2} \\ \frac{1}{Bi} \end{bmatrix} \tag{46}$$

Discussion of the partial solutions

The solution of the whole problem will be produced by joining the homogeneous and inhomogeneous partial problems, in the first step for the case of $t_w = 0$. Expressing the vector of the surface temperature from Eq. (38):

$$t = -\frac{l}{\sqrt{\lambda_x \lambda_y}} (\mathbf{TQ})^{-1} \cdot q \tag{47}$$

adding to it, as the solution of the homogeneous problem, the temperature vector given by (44) as the solution of the inhomogeneous problem, and denoting the sum again by t, the following relationship will be obtained:

$$t = -\frac{l}{\sqrt{\lambda_x \lambda_y}} (\mathbf{TQ})^{-1} q + f_0 \beta_0 \, \mathbf{FT} \, t^{\text{sup}} + t^{f0} \, . \tag{48}$$

The above equation yields a relationship between the unknown values of the surface temperature vector t, the heat density vector q, the temperature vector $t^{\sup}$ of the symmetry plane and a known characteristic: the temperature vector t^{f0} calculated by the use of an internal heat source belonging to the constant temperature.

The three unknown require two further equations. One of the two will be obtained by writing the boundary condition of the 3rd kind given on the plane of the equation $y = 0$ (see Fig. 2). Considering that $t_w = 0$, the superficial heat flow density and the temperature vector can be related by the matrix equation:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 \\ 0 & 0 & \alpha_3 & 0 \\ 0 & 0 & 0 & \alpha_4 \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix} \tag{49}$$

At last the relationship required between the vectors of the superficial temperature and of the maximum temperature is given by Eq. (36).

Substituting Equ. (36) and (49) into (48) and expressing the vector $t^{\sup}$ of the unknown maximum temperature, one gets

$$t^{\sup} = \mathbf{R} \cdot \mathbf{A} \cdot \zeta^{-1}\, t^{f0} \tag{50}$$

where

$$\mathbf{R} = \left[\mathbf{I} + \frac{l}{\sqrt{\lambda_x \lambda_y}} (\mathbf{TQ})^{-1} \langle \alpha_i \rangle - f_0 \cdot \beta_0 \cdot \mathbf{FT} \cdot \mathbf{A} \cdot \zeta^{-1}\right]^{-1} . \tag{51}$$

The maximum coil-temperature required will be obtained as the first co-ordinate of the temperature vector $t^{\sup}$. If $t \neq 0$, then the value of t_w must be added to the temperatures obtained from Equ. (50).

Accounting the excess of the internal heat source arising on the effect of stray fluxes

The heat source excess arising upon the effect of stray fluxes is not uniformly distributed and not symmetrical to the centre line of the winding, but it is stronger near the dispersion channel. As a consequence, conditions *d)* and *c)* of our describing model are not fulfilled for the heat source excess arising from the dispersed flux.

Giving up the symmetrical picture of distribution, a fair approximative solution can be deduced by the application of superposition, though the mathematical description of the problem is considerably more complicated [8]. To obviate the difficulties of the mathematical description one can use the simplification of evenly dividing the heat flow, arising from the internal heat source as an effect of the stray flux, to the original heat source distribution in the disc, i.e. the value of f_{h0} will be proportionally increased. This way the model described can be used without any change. The error made with this method is the residuum of two opposite effects: the transfer of a part of the excess heat source into the centre plane of the disc presumably increases the value of t_{max}, but the uniform distribution of the excess source decreases it. Considering that in the winding disc, according to our principles, the unevenness in the distribution of the internal heat source is within 6 per cent of the average value [6], the mentioned error of approximation in t_{max} usually amounts to a few per cent.

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