INDUCTION MOTORS WITH UNBALANCED ROTOR

By

P. VAS

United Electric Machine Works (EVIG)*

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Presented by Prof. Dr. GY. RETTER

1. Introduction

Equations governing the transient behaviour of induction motors with rotor asymmetry will be set up, for predicting the performance of the machine. The equations will be shown to be effective for studying special rotor asymmetries of squirrel cage induction motors. This assumption is necessary, as else computation time for squirrel cage machines with general rotor asymmetry would be great. Anyhow in practice such special asymmetries occur where adjacent rotor bars break. The derived differential equation also holds for slip ring machines, and is simplified to be valid for single phase machines with rotor asymmetries.

The steady state performance of the asymmetrical slip ring machine will be studied by the aid of a new equivalent circuit derived by using symmetrical component theory. Analogy will be shown between a known equivalent circuit of squirrel cage induction motors with rotor asymmetries, as well as with the equivalent circuits of salient pole synchronous machines. Also analogy will be pointed out existing between the equivalent circuits valid in case of other rotor asymmetries of slip ring induction motors and the one derived in this paper. These equivalent circuits can be easily modified for analyzing the constant speed transient operation of unbalanced induction motors. For the steady state case some calculated values will be shown, calculations were carried out with a digital computer.

A general method was given for the calculation of pulsating torques in a paper of KOVÁCS [1] instantaneous values of symmetrical components were used. In [2] the same author discussed the stationary performance of slip ring motors with special rotor asymmetry with the aid of α and β components.

In BARTON's paper [3] the positive and negative sequence equivalent circuits of slip ring motor with rotor asymmetry is being connected through a mutual impedance link. However it was not pointed out that if all three

* Based on research done at the Department of Electrical Machines, Technical University, Budapest.

rotor impedances differed but were purely resistive, then a simple equivalent circuit could be derived which couples the positive and negative sequence networks of the machine.

In the paper of BAJZA [4] the method of symmetrical components using ladder networks was applied for the case of induction motors with two side asymmetry. The rotor was symmetrical about the d, q axes.

The equations of performance were discussed in the paper by DESAI [5] for a single phase motor where some rotor bars were missing for the purpose of starting the machine.

The author of present paper studied the steady state operation of three phase induction motors with rotor assymmetries in the squirrel cage [6, 7]. Using the symmetrical component method, an equivalent circuit was derived and the use of the model was shown in case of rotor bars having different impedances in the same rotor, involving the practical case of some rotor bars broken.

2. Dynamic equations

The scheme of the asymmetrical machine is shown in Fig. 1. With the assumptions used in general electric machine theory [10] the equations of performance are as follows:

The voltage equation is:

$$\frac{d\Psi}{dt} = -\mathbf{R} \cdot \mathbf{L}^{-1} \cdot \Psi + u, \quad i = \mathbf{L}^{-1}\Psi \tag{1}$$

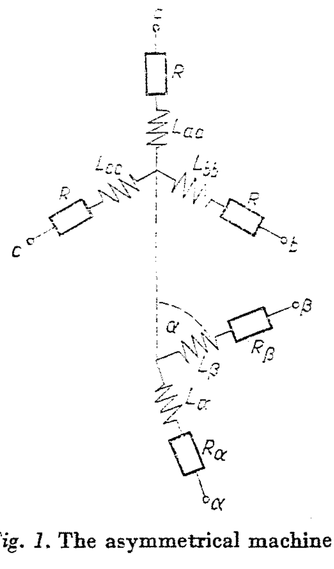

Fig. 1. The asymmetrical machine

The equation of motion

$$T = \Theta \cdot \frac{d^2\alpha}{dt^2} + T_L + f_V \, w_r \tag{2}$$

where the expressions of the matrixes are in Appendix 1, T is the electromagnetic torque, T_L is the load torque, Θ is the moment of inertia, f_V is the coefficient of viscous damping, ω_r is the speed of the rotor, and α is the rotor angle.

Eqs (1) and (2) govern the behaviour of the machine, with the necessary substitutions the equations of performance will be:

$$\begin{aligned} U \sin (w_s t + \varphi) &= R \cdot i_a + L_{aa} \frac{di_a}{dt} - \frac{1}{2} L_{ab} \frac{di_b}{dt} - \frac{1}{2} L_{ac} \frac{di_c}{dt} + \\ &+ M \cos \alpha \frac{d_{i\beta}}{dt} + M \sin \alpha \frac{di_\alpha}{dt} - Mi_{2\beta} \sin \alpha \frac{d\alpha}{dt} + Mi_{2\alpha} \frac{d\alpha}{dt} \cos \alpha \end{aligned} \tag{3}$$

$$\begin{aligned} U \sin (w_s t + 120 + \varphi) &= Ri_b + L_{bb} \frac{di_b}{dt} - \frac{1}{2} L_{ba} \frac{di_a}{dt} - \\ &- \frac{1}{2} L_{bc} \frac{di_c}{dt} + M \frac{di_\beta}{dt} \cos (\alpha - 120) + M \frac{di_\alpha}{dt} \times \\ &\times \sin (\alpha - 120) - Mi_\beta \sin (\alpha - 120) + \end{aligned} \tag{4}$$

$$+ Mi_\alpha \cos (\alpha - 120) \frac{d\alpha}{dt}$$

$$U \sin (w_s t - 120 + \varphi) = Ri_c + L_{cc} \frac{di_c}{dt} - \frac{1}{2} L_{ca} \frac{di_a}{dt} - \tag{5}$$

$$\begin{aligned} &- \frac{1}{2} L_{cb} \frac{di_b}{dt} + M \frac{di_\beta}{dt} \cos(\alpha - 240) + \\ &+ M \frac{di_\alpha}{dt} \sin (\alpha - 240) - Mi_\beta \frac{d\alpha}{dt} \sin (\alpha - 240) + \end{aligned} \tag{6}$$

$$+ Mi_\alpha \frac{d\alpha}{dt} \cos (\alpha - 240)$$

$$
\begin{aligned}
0 = R_\beta i_\beta &+ M \frac{\mathrm{d}i_a}{\mathrm{d}t}\cos\alpha - Mi_a \frac{\mathrm{d}\alpha}{\mathrm{d}t}\sin\alpha + M\frac{\mathrm{d}i_b}{\mathrm{d}t}\cos(\alpha-120) - \\
&- Mi_b\frac{\mathrm{d}\alpha}{\mathrm{d}t}\sin(\alpha-120) + M\frac{\mathrm{d}i_c}{\mathrm{d}t}\cos(\alpha-240) - \\
&- Mi_c\frac{\mathrm{d}\alpha}{\mathrm{d}t}\sin(\alpha-240) + L_\beta\frac{\mathrm{d}i_\beta}{\mathrm{d}t}
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
0 = R_\alpha i_\alpha &+ M\frac{\mathrm{d}i_a}{\mathrm{d}t}\sin\alpha + Mi_a\frac{\mathrm{d}\alpha}{\mathrm{d}t}\cos\alpha + M\frac{\mathrm{d}i_b}{\mathrm{d}t}\sin(\alpha-120) + \\
&+ \mathrm{M}\cos(\alpha-120)\, i_b\frac{\mathrm{d}\alpha}{\mathrm{d}t} + M\frac{\mathrm{d}i_c}{\mathrm{d}t}\sin(\alpha-240) + \\
&+ Mi_c\frac{\mathrm{d}\alpha}{\mathrm{d}t}\cos(\alpha-240) + L_\alpha\frac{\mathrm{d}i_\alpha}{\mathrm{d}t}
\end{aligned}
$$

and the electromagnetic torque can be derived by known methods from the general electric machine theory:

$$
\begin{aligned}
T = p\big[&-Mi_a i_\beta\sin\alpha + Mi_a i_\alpha\cos\alpha - Mi_b i_\alpha\sin(\alpha-120) + Mi_b i_\alpha\cos(\alpha-120) \\
&- Mi_b i_\alpha\cos(\alpha-120) - Mi_\beta i_c\sin(\alpha-240) + Mi_\alpha i_c\cos(\alpha-240) - \\
&- Mi_a i_\beta\sin\alpha - Mi_b i_\beta\sin(\alpha-120) - Mi_c i_\beta\sin(\alpha-240) + Mi_a i_\alpha\cos\alpha + \\
&+ Mi_b i_\alpha\cos(\alpha-120) + Mi_c i_\alpha\cos(\alpha-240)\big] = \Theta\frac{d^2\alpha}{\mathrm{d}t^2} + T_L + f_V w_r
\end{aligned}
$$

Eq. (8) can be split into two first-order differential equations, thus the problem is reduced to solve seven first order nonlinear differential equations for dependent variables i_a, i_b, i_c, i_α, i_β, ω_r and α.

Equations were solved by a digital computer using the Runge—Kutta method. Computation time of starting performance of a slip ring asynchronous machine with asymmetrical rotor resistances was less than in the case when poly-axis method was used by the author. Results in detail will be discussed in a subsequent paper where state variable equations of induction motors with two side asymmetries will be also presented, saturation of the main flux paths will be also considered.

Calculations were also carried out for induction motors where it was important to determine an optimum number of starting resistances with the amplitude of the pulsating torque as criterium for selecting the proper number of starting resistances. Results showed in the last stages pulsating torques with large amplitudes due to the fact that the difference between the three rotor phases was the greatest in these stages. The frequencies of these torques were $2sf_s$ (where f_s is the supply frequency).

It can be shown that in case of a single phase machine with d,q rotor asymmetry, the differential equations (3) to (8) will be always time dependent as no transformation to eliminate the rotor angle from the equations exists (due to two side asymmetry). Computerized calculations were carried out for the single phase machine with rotor asymmetries in the squirrel cage. Some bars were missing and similar results were obtained as in [6]. In a subsequent paper results of present model will be compared with those obtained when space harmonics are also considered.

3. A new equivalent circuit for steady state operation

In the following a new equivalent circuit is derived for a slip ring motor with all the three rotor resistances differing from each other. The equivalent circuit is shown to be similar to that derived in [6, 7] for the case of squirrel cage induction motor with rotor asymmetry. Analogy exists with the equivalent circuits of asymmetrical induction machines with asymmetrical rotor connections and asymmetrical rotor inductances.

It is important to express that the following method is not applicable to such a case where three rotor impedances differ. In that case symmetrical component theory could be also used, but the positive and negative sequence symmetrical component impedances of the asymmetrical motor are coupled by such a four-terminal network which consists of current or voltage generators controlled by voltage or current. The general impedance asymmetry is discussed in [8].

The slip ring machine with R_a, R_b and R_c external rotor resistances is shown in Fig. 2. Assumptions are the same as previously, and a positive sequence voltage $\bar{U}_{s1}$ is impressed on the stator, which has three phases and is fed by a three phase sinusoidal symmetrical voltage system.

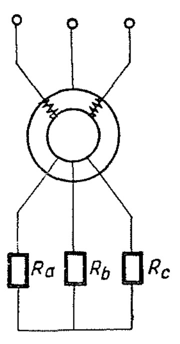

Fig. 2. Slip ring machine with rotor asymmetry

The symmetrical components of the external resistances are:

$$\begin{aligned} R_0 &= \frac{1}{3}(R_a + R_b + R_c) = k_0 3R_{r'} \\ R_1 &= \frac{1}{3}(R_a + aR_b + a^2R_c) = k_1 3R_r' e^{-j\delta} \\ R_2 &= \hat{R}_1 \end{aligned} \tag{10}$$

where the circumflex means a conjugate.

The symmetrical component rotor voltages are:

$$\begin{aligned} \overline{U}_{r1} &= R_0\bar{I}_{r1} + Re^{j\delta}\bar{I}_{r2} \\ \overline{U}_{r2} &= Re^{-j\delta}\bar{I}_{r1} + R_0\bar{I}_{r2} \end{aligned} \tag{11}$$

where the subscript r refers to the rotor and 1 and 2 are the symmetrical components, $R = 3k_1, R_r'$.

Introducing the complex instantaneous values, the time vectors of symmetrical component rotor currents in a reference frame spaced at α degrees from the stators "a" phase, are:

$$\bar{i}_{r1}^R = \bar{i}_{r1}e^{-j\alpha}\ , \quad \bar{i}_{r2}^R = \bar{i}_{r2}e^{j\alpha} \tag{12}$$

where

$$\bar{i}_{r1} = \bar{I}_{r1}e^{js\omega_s t}\ , \quad \bar{i}_{r2} = \bar{I}_{r2}e^{js\omega_s t}\ , \tag{13}$$

here s is the slip and ω_s is the angular frequency of the stator. Therefore the Park-vector equations are:

$$\begin{aligned} \overline{u}_{r1}^R e^{j\delta} &= R_0\bar{i}_{r1}^R e^{j\delta} + Re^{j\delta}\,\bar{i}_{r2}e^{-j\delta} \\ \overline{u}_{r2}^R e^{-j\delta} &= R\bar{i}_{r1}^R e^{-j\delta}e^{j\alpha} + R_0\bar{i}_{r2}^R e^{-j\cdot} \end{aligned} \tag{14}$$

so the symmetrical component voltage equations are:

$$\begin{aligned} \overline{U}_{r1}^R &= R_0 I_{r1}^R + R\bar{I}_{r2}^R\, e^{j(\delta-2\alpha)} \\ \overline{U}_{r2}^R &= R\bar{I}_{r1}^R \cdot e^{j(2\delta-\alpha)} + R_0\bar{I}_{r2}^R \end{aligned} \tag{15}$$

If the $\alpha = \delta/2$ transformation is applied, Eqs (15) are seen to be equations of a symmetrical four-terminal network, shown in Fig. 3. The complete equivalent circuit of the asymmetrical induction machine is therefore as in Fig. 4. The equivalent circuit can be easily modified with phase shifters, if the correct phase angles of the symmetrical component rotor currents and voltages are to be determined. The method is seen not to produce such

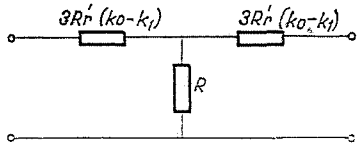

Fig. 3. Coupling T-network

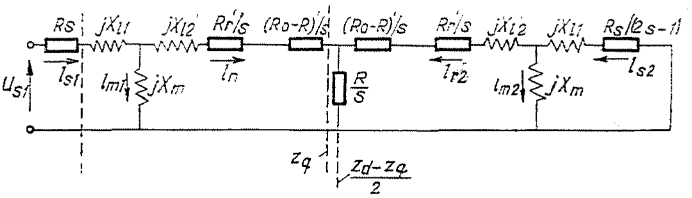

Fig. 4. Equivalent circuit of asymmetrical machine

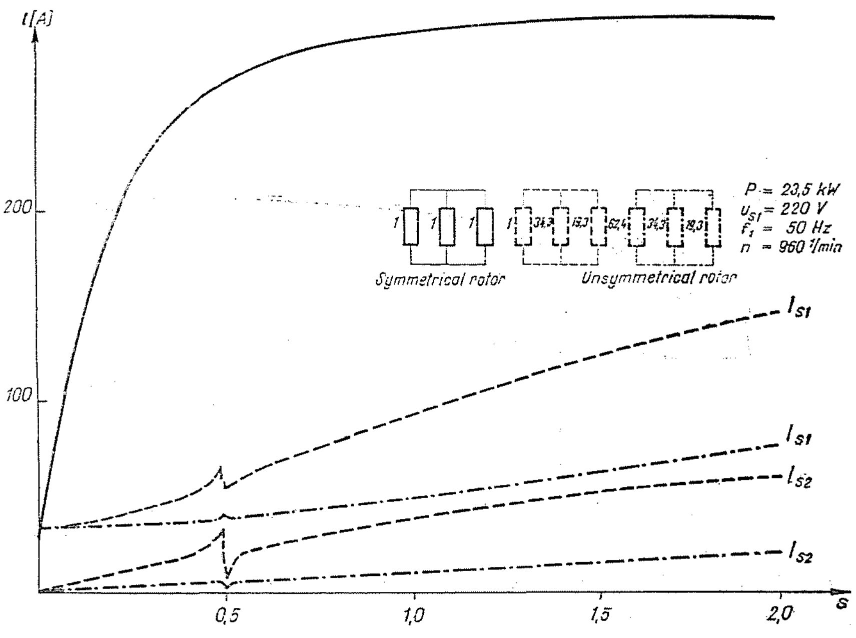

Fig. 5. Positive and negative sequence currents of the unbalanced slip ring machine

a coupling if there are three different impedances in the rotor. If the resistances of the rotor circuit are the same in all the phases but each rotor winding has a different number of turns, then a symmetrical T-network will couple the positive and negative sequence symmetrical component impedances of the machine. This is not surprising in all cases where the asymmetries are seen to be of d, q type. Comparing the derived equivalent circuit with the equivalent circuit of the salient pole synchronous machine, the d, q impedances of the asymmetrical machine result.

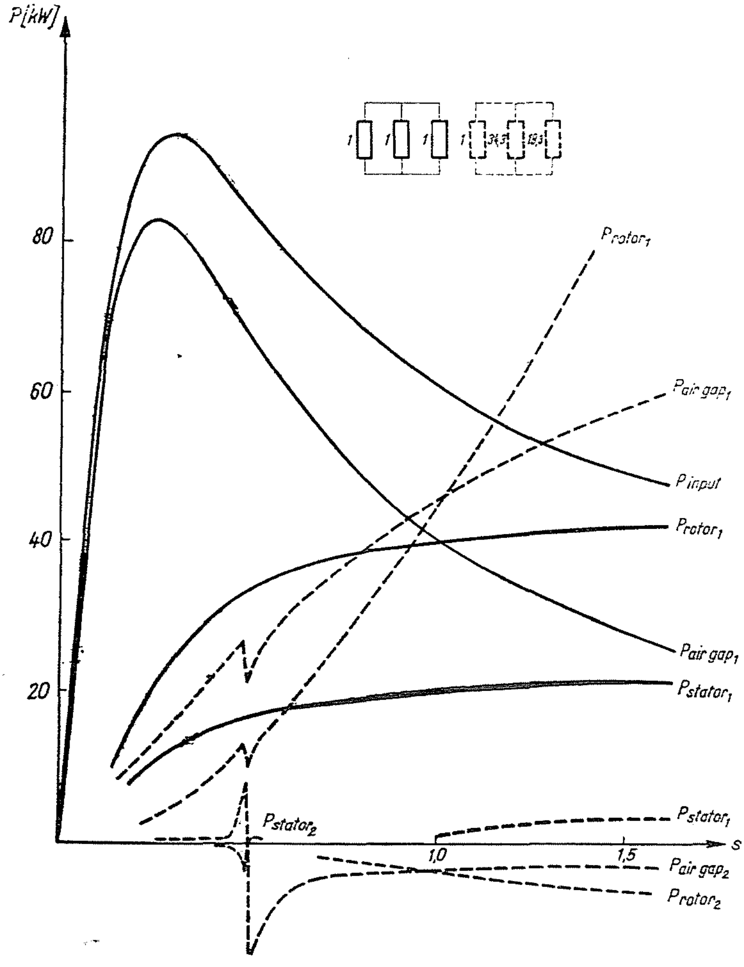

Fig. 6. Performance of an asymmetrical slip ring motor

As shown in [6, 7] the type of d, q asymmetry involves the case of adjacent broken rotor bars, so the equivalent circuit derived for squirrel cage motors with special rotor asymmetries is similar to the one derived in the foregoings, of course the coupling network is not resistive in that case.

For constant speed transients the derived equivalent circuits can be easily modified. However if the speed of the motor is not constant, general state variable differential equations can be set up by using the Park-vector method. This will be discussed in a subsequent paper.

The equivalent circuit of Fig. 4 was used to calculate the steady-state performance of an induction motor with unbalanced rotor resistances. Fig. 5 shows the positive and negative sequence currents of the unsymmetrical motor, and Fig. 6 shows the performance characteristics of the slip ring machine. For a high degree of asymmetry the equivalent circuit shows the exact symmetrical component rotor resistances not to be replaced by approximate effective resistances, anyhow the negative sequence components are greatly effective in such cases.

Appendix

The voltage and current column vectors of the machine are:

$$u = \begin{bmatrix} U\sin(\omega_s + \varphi) \\ U\sin(\omega_s + 120 + \varphi) \\ U\sin(\omega_s - 120 + \varphi) \\ 0 \\ 0 \end{bmatrix}; \quad i = \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_\beta \\ i_\alpha \end{bmatrix} \tag{1}$$

The resistance and flux matrixes are:

$$\mathbf{R} = \begin{bmatrix} R & 0 & 0 & 0 & 0 \\ 0 & R & 0 & 0 & 0 \\ 0 & 0 & R & 0 & 0 \\ 0 & 0 & 0 & R_\beta & 0 \\ 0 & 0 & 0 & 0 & R_\alpha \end{bmatrix};$$

$$\mathbf{L} = \begin{bmatrix} L_{aa} & -L_{ab}/2 & -L_{ac}/2 & M\cos\alpha & M\sin\alpha \\ -L_{ba}/2 & L_{bb} & -L_{bc}/2 & \mathrm{M}\cos(\alpha-120) & M\sin(\alpha-120) \\ -L_{ca}/2 & -L_{cb}/2 & L_{cc} & \mathrm{M}\cos(\alpha-240) & M\sin(\alpha-240) \\ \mathrm{M}\cos\alpha & M\cos(\alpha-120) & M\cos(\alpha-240) & L_\beta & 0 \\ M\sin\alpha & M\sin(\alpha-120) & M\sin(\alpha-240) & 0 & L_\alpha \end{bmatrix}$$

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Summary

The transient and steady-state operation of induction motors with asymmetrical rotor has been studied. Dynamic equations governing the behaviour of slip ring and squirrel cage machines have been set up, for predecting the performance of the asymmetrical machine. Equations are also valid for single phase machine with asymmetrical cage rotor.

The steady state performance of the slip ring machine has been studied by means of a new equivalent circuit, pointing out an analogy existing between the derived equivalent circuit and other equivalent circuits valid for other types of rotor asymmetries. Computerized calculations are also shown.

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Péter VAS, H-1521 Budapest