ANALYSIS OF STEADY STATE LINEAR NETWORKS CONTAINING CONTROLLED GENERATORS

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Introduction

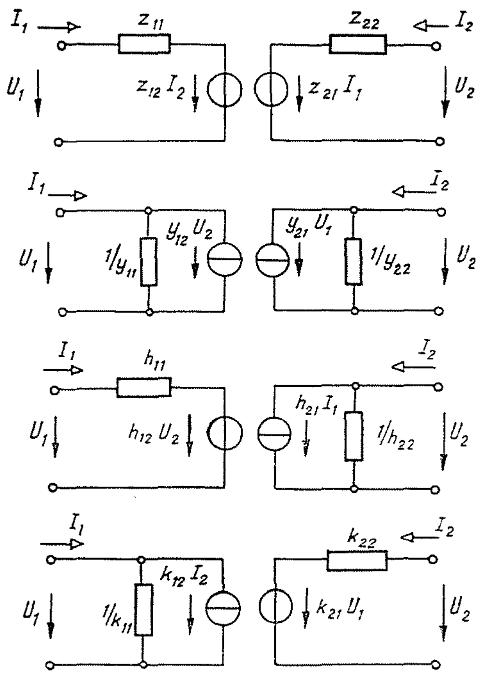
The graph theory method to be presented permits the analysis of networks in the steady state, which contain invariant linear two-poles and any kinds of invariant linear two-ports with the exception of the nullor. Hence the calculation described in this paper lends itself for the examination of the steady state of circuits simulable by two-poles and two-ports, of electronic networks considered practically linear.

Similar calculations are found in [1], to be commented later. To solve the given problem, the chain parameters of two-ports are used in [2]. In this way also a network containing nullor can be analysed except, if the network contains a two-port which cannot be characterized by chain parameters. The method described in [3] substitutes two-ports by the model formed of nullators and norators to analyse the resulting network.

Transient phenomena may be examined by state equations of the network, to be established according to [4].

Network equations

The problem being one of analysis thus the structure of the tested network and the characteristic parameters of its elements are considered to be known. Two ports, with the exception of the nullor, can be characterized by impedance, admittance, hybrid or inverse hybrid parameters, and on the basis of these the two-ports model containing controlled generators (Fig. 1) can be given. In our calculations two-ports are replaced by such equivalent circuits. The independent generators in the network are substituted by the Thevenin or Norton equivalent, and considered to consist of two branches, one containing an independent source, the other passive elements. Branches containing passive elements can be taken into consideration by their impedance or admittance.



$$U_1 = z_{11} I_1 + z_{12} I_2$$
$$U_2 = z_{21} I_1 + z_{22} I_2$$

$$I_1 = y_{11} U_1 + y_{12} U_2$$
$$I_2 = y_{21} U_1 + y_{22} U_2$$

$$U_1 = h_{11} I_1 + h_{12} U_2$$
$$I_2 = h_{21} I_1 + h_{22} U_2$$

$$I_1 = k_{11} U_1 + k_{12} I_2$$
$$U_2 = k_{21} U_1 + k_{22} I_2$$

Fig. 1

Branches of a graph of this network model are classified in four groups as follows.

Branches of group 1:

branches containing independent current source.

Branches of group 2:

branches containing controlled current generator,
branches containing controlled current source,
branches of control voltage,
breaks,
some branches consisting of passive elements.

Branches of group 3:

branches containing controlled voltage generator,
branches containing controlled voltage source,
branches of control current,
short circuits,
some branches consisting of passive elements.

Branches of group 4:

branches containing independent voltage sources.

Passive two-poles should be ranged among branches of groups 2 or 3 so, that branches belonging to groups 1 and 2 are links, while those of groups

3 and 4 twigs. In the calculation passive branches in group 2 are taken into consideration by their admittance, those in group 3 by their impedance. The number of branches classified in groups are b_1, b_2, b_3, b_4, resp. Branches of the network are numbered accordingly.

In accordance with the aforesaid, the primary and secondary sides of a two-port characterized by admittance parameters belong to group 2, while the primary and secondary sides of a two-port characterized by the impedance parameter matrix to group 3 (Fig. 1). If the two-port is given by hybrid parameters, then the primary side belongs to group 3, the secondary side to group 2, while in the case of inverse hybrid parameters, the primary side belongs to group 2, the secondary side to group 3 (Fig. 1).

It may happen that branches of the network cannot be grouped according to the preceding, since independent voltage sources, controlled voltage generators and short-circuits form a loop, or independent current sources, controlled current generators and breaks form a cut-set. In this case the tree is constructed by taking controlled generators by two branches, namely a controlled source and an impedance into consideration, and the impedance branch is placed in the other group. If nevertheless branches can not be grouped as required, then independent and controlled voltage sources form a loop, or independent and controlled current sources form a cut-set in the network, a contradictory.

Write first the relationship for current and voltage, respectively, for branches in groups 2 and 3. Current in branch i in group 2 (Fig. 2):

$$I_{2i} = y_{ii}U_{2i} + y_{ij}U_{2j} + \varkappa_{ik} I_{3k}, \tag{1}$$

where $y_{ii} = Y_i$ is the admittance in the branch, U_{2i} the voltage of the branch, $y_{ij}\,U_{2j}$ the source current of the voltage controlled current source in the branch, U_{2j} its control voltage, $\varkappa_{ik}I_{3k}$ the source current of the current controlled current source in the branch, while I_{3k} is the control current. U_{2j} is the voltage of branch j belonging to group 2, I_{3k} the current of branch k belonging to group 3. If branch i is passive, i.e. containing only admittance,

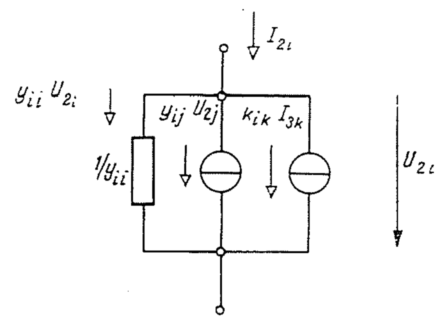


Fig. 2

then $y_{ij} = 0$ and $\varkappa_{ik} = 0$. If branch i contains only a voltage controlled current generator, then $\varkappa_{ik} = 0$, and finally, if it contains only a current controlled current generator, then $y_{ik} = 0$. On the basis of Eq. (1), column matrix $\mathbf{I}_2$ formed of currents of branches belonging to group 2:

$$\mathbf{I}_2 = \boldsymbol{Y}_2 \, \mathbf{U}_2 + \boldsymbol{K} \, \mathbf{I}_3 \tag{2}$$

where $\mathbf{U}_2$ and $\mathbf{I}_3$ are column matrices formed of voltages of branches in group 2, and of currents of branches in group 3, resp., $\boldsymbol{Y}_2$ is a quadratic matrix of order b_2, y_{ij} being the j-th element in the i-th row, the number of rows in matrix $\boldsymbol{K}$ is b_2, that of columns b_3, the k-th element of the i-th row is $\varkappa_{ik}$.

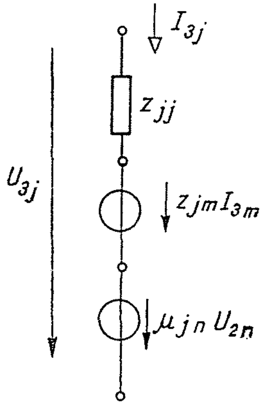


Fig. 3

Column matrix $\mathbf{U}_3$ formed of voltages of branches in group 3 can be written similarly. Voltage of the branch j (Fig. 3) is namely

$$U_{3j} = z_{jj} \, I_{3j} + z_{jm} \, I_{3m} + \mu_{jn} \, U_{2n} \tag{3}$$

where $z_{jj} = Z_j$ is the impedance in the branch, I_{3j} is the current of the branch, $z_{jm} \, I_{3m}$ is the source voltage of the current controlled voltage source in the branch, I_{3m} is its control current, $\mu_{jn} U_{2n}$ is the source voltage of the voltage controlled voltage source in the branch. I_{3m} is the current of branch m belonging to group 3, U_{2n} is the voltage of branch n belonging to group 2. On the basis of (3):

$$\mathbf{U}_3 = \boldsymbol{M} \, \mathbf{U}_2 + \boldsymbol{Z}_3 \, \mathbf{I}_3 \tag{4}$$

where $\boldsymbol{Z}_3$ is a quadratic matrix of order b_3, where z_{jm} is the m-th element in the j-th row, the number of rows of matrix $\boldsymbol{M}$ is b_3, that of columns b_2, the n-th element of the j-th row is μ_{jn}.

Relationships (2) and (4) are involved in writing the Kirchhoff equations of the network. The corresponding equations in [1] contain further terms, too. These are, however, useless calculations needing exclusively the models in Fig. 1.

With loop matrix $\boldsymbol{B}$ of the fundamental loop system generated by the tree selected according to the aforesaid, the linearly independent loop equations of the network are comprised in the matrix equation

$$\boldsymbol{B}\mathbf{U} = \begin{bmatrix} 1 & 0 & \boldsymbol{F}_{11} & \boldsymbol{F}_{12} \\ 0 & 1 & \boldsymbol{F}_{21} & \boldsymbol{F}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \\ \mathbf{U}_3 \\ \mathbf{U}_4 \end{bmatrix} = \mathbf{0} \tag{5}$$

where loop matrix $\boldsymbol{B}$ and column vector $\mathbf{U}$ of branch voltages are partitioned according to the four groups of branches. With a view on $\mathbf{U}_4 = \mathbf{U}_g$, column matrix of source voltages of independent voltage sources, (5) can be written as:

$$\mathbf{U}_1 + \boldsymbol{F}_{11}\,\mathbf{U}_3 + \boldsymbol{F}_{12}\,\mathbf{U}_g = \mathbf{0} \tag{6}$$

$$\mathbf{U}_2 + \boldsymbol{F}_{21}\,\mathbf{U}_3 + \boldsymbol{F}_{22}\,\mathbf{U}_g = \mathbf{0}\,. \tag{7}$$

Similarly, the linearly independent cut-set equations of the network are:

$$\boldsymbol{Q}\mathbf{I} = \begin{bmatrix} -\boldsymbol{F}_{11}^{+} & -\boldsymbol{F}_{21}^{+} & 1 & 0 \\ -\boldsymbol{F}_{12}^{+} & -\boldsymbol{F}_{22}^{+} & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \\ \mathbf{I}_4 \end{bmatrix} = \mathbf{0} \tag{8}$$

where $\boldsymbol{Q}$ is the cut-set matrix, $\mathbf{I}$ is the column matrix of branch currents, and $^{+}$ designates the transpose of the matrix. By applying the designation $\mathbf{I}_1 = \mathbf{I}_g$, (8) can be written in two equations, as

$$-\boldsymbol{F}_{11}^{+}\,\mathbf{I}_g - \boldsymbol{F}_{21}^{+}\,\mathbf{I}_2 + \mathbf{I}_3 = \mathbf{0} \tag{9}$$

$$-\boldsymbol{F}_{12}^{+}\,\mathbf{I}_g - \boldsymbol{F}_{22}^{+}\,\mathbf{I}_2 + \mathbf{I}_4 = \mathbf{0} \tag{10}$$

Substituting (2) into (9), further (4) into (7), united in a single equation

$$\begin{bmatrix} \mathbf{U}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} -\boldsymbol{F}_{21}^{+}\boldsymbol{Y}_2 & 1 - \boldsymbol{F}_{21}^{+}\boldsymbol{K} \\ 1 + \boldsymbol{F}_{21}\boldsymbol{M} & \boldsymbol{F}_{21}\boldsymbol{Z}_3 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{F}_{11}^{+} & 0 \\ 0 & -\boldsymbol{F}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_g \\ \mathbf{U}_g \end{bmatrix} \tag{11}$$

Hereby voltages of branches belonging to group 2, and currents of branches in group 3 have been expressed in terms of known quantities, determinable from Eqs (2) and (4), respectively. Their knowledge permits to calculate voltages of independent current sources from (6), currents of independent voltage sources from (10).

It should be mentioned that independent sources of networks are often exlusively voltage sources. In this case $\boldsymbol{F}_{11}$ is inexistent (with zero rows), the second matrix in the right-hand side of Eq. (11) contains, however, a block $\boldsymbol{0}$ with as many columns as in $\boldsymbol{F}_{22}$, and as many rows as there are branches in group 3. Similarly, if there is no voltage source in the network, then $\boldsymbol{F}_{22}$ is inexistent (with zero columns), and the second matrix in the right-hand side of (11) contains a block $\boldsymbol{0}$ with as many columns as in $\boldsymbol{F}_{11}$, and as many rows as there are branches in group 2. Accordingly, Eq. (11) has a simpler form in the mentioned cases.

The equations can be ordered so that the unknown values can be determined by inverting a matrix of lower order than before ([1], with error). To achieve this, branches are classified into six groups, namely

1. independent current sources;
2. controlled current generators;
   controlled current sources;
   branches of control voltage;
3. finite admittances;
4. finite impedances;
5. controlled voltage generators;
   controlled voltage sources;
   branches of control current;
6. independent voltage source.

Branches are ranged into groups 3 and 4 in such a way that branches belonging to groups 1, 2, 3 are links, while those belonging to groups 4, 5, 6, twigs. The number of branches in each group are b_1, b_2, ..., b_6. Branches are numbered accordingly.

In this case column matrices $\mathbf{I}_2$ and $\mathbf{U}_5$ formed of currents of branches in group 2, and of voltages of branches in group 5, can be written in terms of column matrices $\mathbf{U}_2$ and $\mathbf{I}_5$ of branch voltages in group 2, and branch currents of groups 5, respectively, similarly to (2) and (4):

$$\mathbf{I}_2 = \boldsymbol{Y}_2 \mathbf{U}_2 + \boldsymbol{K} \mathbf{I}_5 \tag{12}$$

and

$$\mathbf{U}_5 = \boldsymbol{M} \mathbf{U}_2 + \boldsymbol{Z}_5 \mathbf{I}_5 \tag{13}$$

Here the order of $\boldsymbol{Y}_2$, the number of rows in $\boldsymbol{K}$ and of columns in $\boldsymbol{M}$ is b_2, while the order of $\boldsymbol{Z}_5$, the number of columns in $\boldsymbol{K}$ and that of rows in $\boldsymbol{M}$ is b_5.

Branch currents and branch voltages in group 3 and 4, respectively, are related by Ohm's law:

$$\mathbf{I}_3 = \boldsymbol{Y}_3 \mathbf{U}_3 \tag{14}$$

$$\mathbf{U}_4 = \boldsymbol{Z}_4 \mathbf{I}_4 \tag{15}$$

where $\boldsymbol{Y}_3$ is the admittance matrix of branches in group 3, and $\boldsymbol{Z}_4$ the impedance matrix of branches in group 4.

Matrices partitioned according to the six groups of branches, are used for writing Kirchhoff's equations for the fundamental loop and cut-set system generated by the selected tree. Designating $\mathbf{U}_6 = \mathbf{U}_g$ and $\mathbf{I}_1 = \mathbf{I}_g$:

$$\begin{bmatrix} 1 & 0 & 0 & \boldsymbol{F}_{11} & \boldsymbol{F}_{12} & \boldsymbol{F}_{13} \\ 0 & 1 & 0 & \boldsymbol{F}_{21} & \boldsymbol{F}_{22} & \boldsymbol{F}_{23} \\ 0 & 0 & 1 & \boldsymbol{F}_{31} & \boldsymbol{F}_{32} & \boldsymbol{F}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \\ \mathbf{U}_3 \\ \mathbf{U}_4 \\ \mathbf{U}_5 \\ \mathbf{U}_g \end{bmatrix} = \mathbf{0} \tag{16}$$

$$\begin{bmatrix} -\boldsymbol{F}_{11}^{+} & -\boldsymbol{F}_{21}^{+} & -\boldsymbol{F}_{31}^{+} & 1 & 0 & 0 \\ -\boldsymbol{F}_{12}^{+} & -\boldsymbol{F}_{22}^{+} & -\boldsymbol{F}_{32}^{+} & 0 & 1 & 0 \\ -\boldsymbol{F}_{13}^{+} & -\boldsymbol{F}_{23}^{+} & -\boldsymbol{F}_{33}^{+} & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_g \\ \mathbf{I}_2 \\ \mathbf{I}_3 \\ \mathbf{I}_4 \\ \mathbf{I}_5 \\ \mathbf{I}_6 \end{bmatrix} = \mathbf{0} \tag{17}$$

whence:

$$\mathbf{U}_1 + \boldsymbol{F}_{11} \mathbf{U}_4 + \boldsymbol{F}_{12} \mathbf{U}_5 + \boldsymbol{F}_{13} \mathbf{U}_g = \mathbf{0} \tag{18}$$

$$\mathbf{U}_2 + \boldsymbol{F}_{21} \mathbf{U}_4 + \boldsymbol{F}_{22} \mathbf{U}_5 + \boldsymbol{F}_{23} \mathbf{U}_g = \mathbf{0} \tag{19}$$

$$\mathbf{U}_3 + \mathbf{F}_{31} \mathbf{U}_4 + \mathbf{F}_{32} \mathbf{U}_5 + \mathbf{F}_{33} \mathbf{U}_g = \mathbf{0} \tag{20}$$

$$-\boldsymbol{F}_{11}^{+} \mathbf{I}_g - \boldsymbol{F}_{21}^{+} \mathbf{I}_2 - \boldsymbol{F}_{31}^{+} \mathbf{I}_3 + \mathbf{I}_4 = \mathbf{0} \tag{21}$$

$$-\boldsymbol{F}_{12}^{+} \mathbf{I}_g - \boldsymbol{F}_{22}^{+} \mathbf{I}_2 - \boldsymbol{F}_{32}^{+} \mathbf{I}_3 + \mathbf{I}_5 = \mathbf{0} \tag{22}$$

$$-\mathbf{F}_{13}^{+} \mathbf{I}_g - \mathbf{F}_{23}^{+} \mathbf{I}_2 - \mathbf{F}_{33}^{+} \mathbf{I}_3 + \mathbf{I}_6 = \mathbf{0} \tag{23}$$

From these equations $\mathbf{U}_3$, $\mathbf{I}_3$, or $\mathbf{U}_4$, $\mathbf{I}_4$ can be eliminated.

Eliminating $\mathbf{U}_3$ and $\mathbf{I}_3$ leads to an equation of the form

$$\boldsymbol{N}_1 \begin{bmatrix} \mathbf{U}_2 \\ \mathbf{I}_4 \\ \mathbf{I}_5 \end{bmatrix} = \boldsymbol{N}_2 \begin{bmatrix} \mathbf{I}_g \\ \mathbf{U}_g \end{bmatrix} \tag{24}$$

where

$$N_1 = \begin{bmatrix} 1 + F_{22}M & F_{21}Z_4 & F_{22}Z_5 \\ F_{31}^{+}Y_3F_{32}M - F_{21}^{+}Y_2 & F_{31}^{+}Y_3F_{31}Z_4 + 1 & F_{31}^{+}Y_3F_{32}Z_5 - F_{21}^{+}K \\ F_{32}^{+}Y_3F_{32}M - F_{22}^{+}Y_2 & F_{32}^{+}Y_3F_{31}Z_4 & F_{32}^{+}Y_3F_{32}Z_5 - F_{22}^{+}K + 1 \end{bmatrix} \quad (25)$$

$$N_2 = \begin{bmatrix} 0 & -F_{23} \\ F_{11}^{+} & -F_{31}^{+}Y_3F_{33} \\ F_{12}^{+} & -F_{32}^{+}Y_3F_{33} \end{bmatrix} \quad (26)$$

(24) yields $\mathbf{U}_2$, $\mathbf{I}_4$, $\mathbf{I}_5$ and these, combined with relationships (18) to (23), express all the required branch currents and branch voltages of the network.

The calculation is similar if $\mathbf{U}_4$ and $\mathbf{I}_4$ are eliminated, leading to:

$$P_1 \begin{bmatrix} \mathbf{U}_2 \\ \mathbf{U}_3 \\ \mathbf{I}_5 \end{bmatrix} = P_2 \begin{bmatrix} \mathbf{I}_g \\ \mathbf{U}_g \end{bmatrix} \quad (27)$$

Here

$$P_1 = \begin{bmatrix} F_{22}Y_2 & F_{32}^{+}Y_3 & F_{22}^{+}K - 1 \\ 1 + F_{21}Z_4F_{21}^{+}Y_2 + F_{22}M & F_{21}Z_4F_{31}^{+}Y_3 & F_{21}Z_4F_{21}^{+} + F_{22}Z_5 \\ F_{31}Z_4F_{21}^{+}Y_2 + F_{32}M & 1 + F_{31}Z_4F_{31}^{+}Y_3 & F_{31}Z_4F_{21}^{+} + F_{32}Z_5 \end{bmatrix} \quad (28)$$

$$\mathbf{P}_2 = \begin{bmatrix} -F_{12}^{+} & 0 \\ -F_{21}Z_4F_{11}^{+} & -F_{23} \\ -F_{31}Z_4F_{11}^{+} & -F_{33} \end{bmatrix} \quad (29)$$

Thus $\mathbf{U}_2$, $\mathbf{U}_3$, and $\mathbf{I}_5$ can be calculated from (27), currents and voltages of the other branches, from (18) to (23).

Eliminating $\mathbf{U}_3$, $\mathbf{I}_3$, or $\mathbf{U}_4$, $\mathbf{I}_4$, the problem can be solved by inverting a matrix of lower order than in case of Eq. (11). Among branch voltages and branch currents in groups 3 and 4 it is advisable to eliminate those, useless for the given problem, or in which the number of branches is higher.

Example

The latter calculation is demonstrated on a problem. Determine the complex voltage transfer coefficient U_5/U_g of the circuit shown in Fig. 4, if the voltage U_g has the angular frequency ω. Resistances R_1, R_2, R_3, R_4, R_5, R_g, capacitances C_5 and C_g, and the hybrid parameters of the two transistors $h_{11}^{(1)}$, $h_{12}^{(1)} \approx 0$, $h_{21}^{(1)}$, $h_{22}^{(1)}$, and $h_{11}^{(2)}$, $h_{12}^{(2)} \approx 0$, $h_{21}^{(2)}$, $h_{22}^{(2)}$, respectively, are considered as being given.

On the basis of Fig. 1, the equivalent circuit of the transistors containing current controlled current generators, considering the direct voltage generator as being short-circuited, yields the network model in Fig. 5, with graph shown in Fig. 6.

For the calculation branches are classified into six groups. No branch of the circuit is in group 1. Branches 1, 2 belong to group 2, branches 3, 4 to group 3, branches 5, 6 to group 4, branches 7, 8 to group 5, branch 9 to group 6. This means at the same time that the tree consisting of branches 5, 6, 7, 8, 9 of the graph is selected for the calculation.

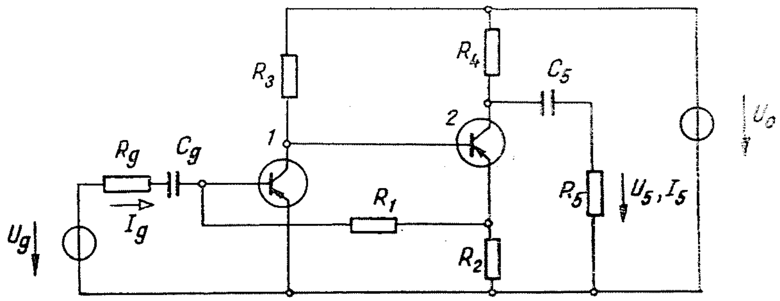


Fig. 4.

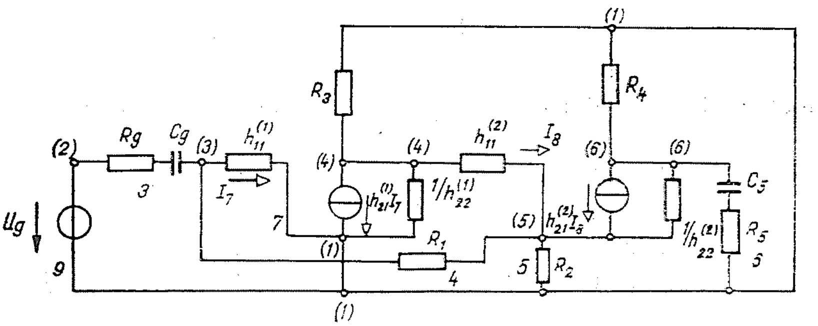


Fig. 5.

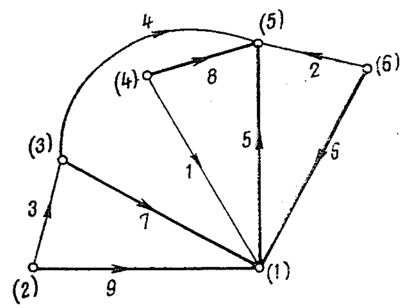


Fig. 6.

Matrices necessary for the calculation:

$$\boldsymbol{Y}_2 = \begin{bmatrix} h_{22}^{(1)} + \dfrac{1}{R_3} & 0 \\ 0 & h_{22}^{(2} \end{bmatrix}$$

$$\boldsymbol{K} = \begin{matrix} \\ 1 \\ 2 \end{matrix} \begin{matrix} \begin{matrix} 7 & \quad 8 \end{matrix} \\ \begin{bmatrix} h_{21}^{(1)} & 0 \\ 0 & h_{21}^{(2)} \end{bmatrix} \end{matrix}$$

$$\boldsymbol{M} = \boldsymbol{0}$$

$$\boldsymbol{Z}_5 = \begin{bmatrix} h_{11}^{(1)} & 0 \\ 0 & h_{11}^{(2} \end{bmatrix}$$

$$\boldsymbol{Y}_3 = \begin{bmatrix} \dfrac{1}{R_g} + j\omega C_g & 0 \\ 0 & \dfrac{1}{R_1} \end{bmatrix}$$

$$\boldsymbol{Z}_4 = \begin{bmatrix} R_2 & 0 \\ 0 & R_4 \times \left(R_5 + \dfrac{1}{j\omega C_5} \right) \end{bmatrix}$$

The matrix of the fundamental loop system generated by the selected tree is:

$$\boldsymbol{B} = \left[\begin{array}{cc|cc|cc|cc|c} 1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & -1 & 0 & 0 \end{array} \right]$$

This means that

$$\boldsymbol{F}_{11} = \boldsymbol{0}\,; \qquad \boldsymbol{F}_{12} = \boldsymbol{0}\,; \qquad \boldsymbol{F}_{13} = \boldsymbol{0}\,;$$

$$\boldsymbol{F}_{21} = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}; \quad \boldsymbol{F}_{22} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}; \quad \boldsymbol{F}_{23} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\boldsymbol{F}_{31} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}; \quad \boldsymbol{F}_{32} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}; \quad \boldsymbol{F}_{33} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

From these, on the basis of (25) and (26)

(Continuation on Supplement)

$$N_1 = \begin{bmatrix} 1 & 0 & R_2 & 0 & 0 & -h_{11}^{(2)} \\ 0 & 1 & -R_2 & -\left[R_4 \times \left(R_5 + \dfrac{1}{j\omega C_5}\right)\right] & 0 & 0 \\ -h_{22}^{(1)} - \dfrac{1}{R_3} & h_{22}^{(2)} & 1 + \dfrac{R_2}{R_1} & 0 & \dfrac{1}{R_1} h_{11}^{(1} - h_{21}^{(1} & h_{21}^{(2} \\ 0 & h_{22}^{(2)} & 0 & 1 & 0 & h_{21}^{(2)} \\ 0 & 0 & \dfrac{R_2}{R_1} & 0 & 1 + h_{11}^{(1}\left(\dfrac{1}{R_g} + j\omega C_g + \dfrac{1}{R_1}\right) & 0 \\ h_{22}^{(1)} + \dfrac{1}{R_3} & 0 & 0 & 0 & h_{21}^{(1} & 1 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{1}{R_g} + j\omega C_g \\ 0 \end{bmatrix}$$

That is, (24), involving $\mathbf{U}_g = U_g$, yields:

$$\begin{bmatrix} \mathbf{U}_2 \\ \mathbf{I}_4 \\ \mathbf{I}_5 \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & R_2 & 0 & 0 & -h_{11}^{(2)} \\ 0 & 1 & -R_2 & -\left[R_4 \times \left(R_5 + \dfrac{1}{j\omega C_5}\right)\right] & 0 & 0 \\ -h_{22}^{(1)} - \dfrac{1}{R_3} & h_{22}^{(2)} & 1 + \dfrac{R_2}{R_1} & 0 & \dfrac{1}{R_1} h_{11}^{(1} - h_{21}^{1} & h_{21}^{(2} \\ 0 & h_{22}^{(2)} & 0 & 1 & 0 & h_{21}^{(2)} \\ 0 & 0 & \dfrac{R_2}{R_1} & 0 & 1 + h_{11}^{(1}\left(\dfrac{1}{R_g} + j\omega C_g + \dfrac{1}{R_1}\right) & 0 \\ h_{22}^{(1)} + \dfrac{1}{R_3} & 0 & 0 & 0 & h_{21}^{(1)} & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{1}{R_g} + j\omega C_g \\ 0 \end{bmatrix}$$

(Continuation on p. 139)

From this I_5 can be calculated and I_5R_5/U_g is the required complex voltage transfer coefficient.

Summary

The described method is suited for the analysis of a linear invariant network containing two-poles and two-ports. The active and extreme parameter two-ports of the network are taken into consideration by a model containing controlled sources. To write the Kirchhoff's equations, the concepts of graph theory are used. Two possibilities of reducing the number of unknown values are presented. Application is demonstrated on an example.

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