MODELLING ASYNCHRONOUS MACHINES BY ELECTRIC CIRCUITS

By

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(Received January 21, 1976)

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Network models are generally used for the examination of steady-state operational conditions of asynchronous machines. These models are known as equivalent circuits [1], [2]. In some publications [1], [3], equivalent circuits are indicated also for transient states of operation. In the followings a network model of the three-phase asynchronous machine will be presented, describing relationship both between currents and voltages and between moment and angular velocity. Thus electromagnetic and mechanical processes can be examined on a single model. Modelling mechanical processes involves the so-called dynamical analogy between electric networks and certain mechanical systems [4], [5], [6]. Models established on the basis of dynamical analogy have been published by MEISEL [4] for direct current machines, and by SEELY [5] for electromechanical converters.

Equations for three-phase asynchronous machines

Equations for symmetrical three-phase asynchronous machines of structure (Fig. 1) apply the following symbols.

A, B, C subscripts for currents, voltages, and fluxes, of the stator coils;

a, b, c Subscripts for currents, voltages, and fluxes, of the rotor coils;

ω angular velocity of the rotor, with respect to a two-pole machine;

α angle included by the axes of coils on the stator and rotor, pertaining to identical phases;

p_0 number of pairs of poles;

ω_g actual, geometric angular velocity of the rotor $\left(\omega_g = \frac{\omega}{p_0}\right)$;

L_s self-induction coefficient of stator coils;

L_r self-induction coefficient of rotor coils;

L_{ks} mutual induction coefficient of stator coils;

L_{kr} mutual induction coefficient of rotor coils;

L_{sr} maximum value of the mutual induction coefficient of stator and rotor coils;

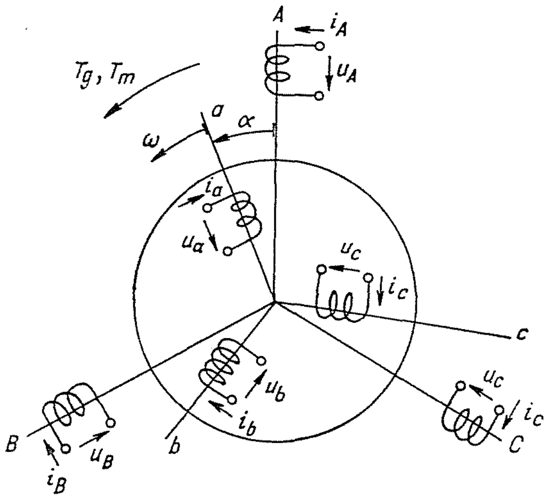


Fig. 1

R_s resistance of stator coils;
R_r resistance of rotor coils;
J moment of inertia of the rotor;
K viscous friction coefficient of the rotor;
T_m external moment acting on the rotor;
T_g electric moment generated by the machine.

Starting a conditions for the equations are:

1. Magnetic nonlinearities due to saturation are negligible hence the correlation between currents and fluxes is linear.
2. Friction moment braking the shaft of the machine is considered to be proportional to angular velocity.
3. Both stator and rotor bear symmetrical structure three-phase winding of providing a magnetic field of sinusoidal distribution in the air-gap of the machine.

Basic equations of the asynchronous machine are obtained by writing the Kirchhoff equations for stator and rotor coils, and the movement equation for the rotation of the rotor, according to reference directions in Fig. 1.

Precise writing of equations can be found in several books on the subject, therefore details are not discussed here.

Relationships in matrix form are:

$$u(t) = \mathbf{R}i(t) + \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{\Psi}(t)\,. \tag{1}$$

$$\frac{1}{p_0}\; J\frac{\mathrm{d}}{\mathrm{d}t}\omega(t) + \frac{K}{P_0}\omega(t) - T_m(t) = T_g(t) \tag{2}$$

Matrices in Eq. (1) are the following:

$$\boldsymbol{u}(t)=\begin{bmatrix} u_A(t)\\ u_B(t)\\ u_C(t)\\ \hline u_a(t)\\ u_b(t)\\ u_c(t)\end{bmatrix}=\begin{bmatrix}\boldsymbol{u}_s\\ \\ \boldsymbol{u}_r\end{bmatrix},\quad \boldsymbol{i}(t)=\begin{bmatrix} i_A(t)\\ i_B(t)\\ i_C(t)\\ \hline i_a(t)\\ i_b(t)\\ i_c(t)\end{bmatrix}=\begin{bmatrix}\boldsymbol{i}_s\\ \\ \boldsymbol{i}_r\end{bmatrix},\quad \boldsymbol{\psi}(t)=\begin{bmatrix} \psi_A(t)\\ \psi_B(t)\\ \psi_C(t)\\ \hline \psi_a(t)\\ \psi_b(t)\\ \psi_c(t)\end{bmatrix}=\begin{bmatrix}\boldsymbol{\psi}_s\\ \\ \boldsymbol{\psi}_r\end{bmatrix}$$

$$\mathbf{R}=\begin{bmatrix}\mathbf{R}_s & \mathbf{0}\\ \mathbf{0} & \mathbf{R}_r\end{bmatrix}=\begin{bmatrix}R_s\mathbf{1} & \mathbf{0}\\ \mathbf{0} & R_r\mathbf{1}\end{bmatrix}, \tag{3}$$

where $\mathbf{1}$ denotes the unit matrix of third order, $\mathbf{0}$ the zero matrix. According to condition, 1, fluxes are expressed in terms of currents as follows:

$$\begin{aligned}\boldsymbol{\psi}_s &= \mathbf{L}_s\,\boldsymbol{i}_s+\mathbf{L}_{sr}\,\boldsymbol{i}_r\\ \boldsymbol{\psi}_r &= \mathbf{L}_r\,\boldsymbol{i}_r+\mathbf{L}_{sr}^*\,\boldsymbol{i}_s\,.\end{aligned} \tag{4}$$

where * denotes transposition, and

$$\mathbf{L}_s=\begin{bmatrix}L_s & L_{ks} & L_{ks}\\ L_{ks} & L_s & L_{ks}\\ L_{ks} & L_{ks} & L_s\end{bmatrix},\qquad \mathbf{L}_r=\begin{bmatrix}L_r & L_{kr} & L_{kr}\\ L_{kr} & L_r & L_{kr}\\ L_{kr} & L_{kr} & L_r\end{bmatrix},$$

$$\mathbf{L}_{sr}=\begin{bmatrix}L_{sr}\cos\alpha & L_{sr}\cos(120°+\alpha) & L_{sr}\cos(120°-\alpha)\\ L_{sr}\cos(120°-\alpha) & L_{sr}\cos\alpha & L_{sr}\cos(120°+\alpha)\\ L_{sr}\cos(120°+\alpha) & L_{sr}\cos(120°-\alpha) & L_{sr}\cos\alpha\end{bmatrix}. \tag{5}$$

It should be noted that in consequence of the rotational symmetry of the structure of the machine, the self-induction and mutual induction coefficients of the coils both on stator and rotor are independent of angle α, while the coefficients of mutual induction of stator and rotor coils depend on α. These latter factors are known to be proportional to the cosine of the angle included by the coil axes [1]. Accordingly the elements of $\mathbf{L}_{sr}$ can be determined on the basis of Fig. 1, where individual phase coils are represented by solenoids. Using Eq. (4),

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_s &= \mathbf{L}_s\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_s+\left(\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{L}_{sr}\right)\boldsymbol{i}_r+\mathbf{L}_{sr}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_r\,,\\ \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_r &= \mathbf{L}_r\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_r+\left(\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{L}_{sr}\right)\boldsymbol{i}_s+\mathbf{L}_{sr}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_s\,.\end{aligned} \tag{6}$$

For the derivative of $\mathbf{L}_{sr}$ with respect to time, the following relationship is obtained.

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{L}_{sr}=\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\mathrm{d}}{\mathrm{d}\alpha}\mathbf{L}_{sr}=\omega L_{sr}\begin{bmatrix}-\sin\alpha & -\sin(120^\circ+\alpha) & \sin(120^\circ-\alpha)\\ \sin(120^\circ-\alpha) & -\sin\alpha & -\sin(120^\circ+\alpha)\\ -\sin(120^\circ+\alpha) & \sin(120^\circ-\alpha) & -\sin\alpha\end{bmatrix} \tag{7}$$

In the following the notation

$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\mathbf{L}_{sr}=\mathbf{M} \tag{8}$$

will be applied.

Supposing currents to be constant electric moment arising in the machine can be determined from magnetic energy [8] as:

$$T_g=\frac{\mathrm{d}W_m}{\mathrm{d}\alpha}=p_0\frac{1}{2}\left[\boldsymbol{i}_s^*\left(\frac{\mathrm{d}}{\mathrm{d}\alpha}\mathbf{L}_s\right)\boldsymbol{i}_s+\boldsymbol{i}_r^*\left(\frac{\mathrm{d}}{\mathrm{d}\alpha}\mathbf{L}_r\right)\boldsymbol{i}_r+2\boldsymbol{i}_s^*\left(\frac{\mathrm{d}}{\mathrm{d}\alpha}\mathbf{L}_{sr}\right)\boldsymbol{i}_r\right]. \tag{9}$$

That is:

$$T_g=\boldsymbol{i}_s^*\,\mathbf{M}\,\boldsymbol{i}_r\,p_0\,. \tag{10}$$

Substituting relationship (6) into starting equation (1), we obtain a common form of the equations of the asynchronous machine:

$$\begin{aligned}\boldsymbol{u}_s(t)&=\mathbf{R}_s\boldsymbol{i}_s+\mathbf{L}_s\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_s+\omega\mathbf{M}\,\boldsymbol{i}_r+\mathbf{L}_{sr}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_r\,,\\ \boldsymbol{u}_r(t)&=\mathbf{R}_r\boldsymbol{i}_r+\mathbf{L}_r\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_r+\omega\mathbf{M}^*\boldsymbol{i}_s+\mathbf{L}_{sr}^*\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_s\,,\end{aligned} \tag{11}$$

$$p_0\boldsymbol{i}_s^*\,\mathbf{M}\,\boldsymbol{i}_r=\frac{J}{p_0}\frac{\mathrm{d}\omega}{\mathrm{d}t}+\frac{K\omega}{p_0}-T_m\,. \tag{12}$$

The analytical solution of the above equations with respect to currents and angular velocity is difficult on account of variable parameters and of the nonlinearity. A suitable transformation permits to eliminate variable parameters. Transformations of this kind are discussed in most books on the subject, at slight differences. In the following chapter, the most important relationships for and fundamentals of co-ordinate transformations used in the theory of electric machines will be recapitulated according to [9], [10], [11], [12].

Transformation of currents and voltages in three-phase machines

In the examination of three-phase machines, transformation is of use both for reducing Eqs. (11), and —it will be seen later—, for the introduction of the network model. Determination of transformation correlations start from the following conditions. Starting assumption in determining the transformation relationships are:

1. Currents actually arising in the three-phase machine and currents determined by transformation, produce identical magnetic fields in the air-gap of the machine.
2. Currents and voltages determined by transformation supply a power identical with that produced by actual currents and voltages of the three-phase machine (power invariance).

The first condition permits to determine the transformation relationship for currents permitting, in turn to determine voltage transformation utilizing the second condition.

Let $\boldsymbol{i}$ and $\boldsymbol{u}$ denote column vectors of currents and voltages, respectively, in the original system, transformed quantities being denoted by a comma ('). Transformation matrices are denoted by $\mathbf{C}_i$ for currents, and by $\mathbf{C}_u$ for voltages. Then

$$\boldsymbol{i} = \mathbf{C}_i \boldsymbol{i}' \qquad \boldsymbol{u} = \mathbf{C}_u \boldsymbol{u}' \tag{13}$$

On the basis of power invariance,

$$\boldsymbol{i}^* \boldsymbol{u} = \boldsymbol{i}'^* \boldsymbol{u}' = \boldsymbol{i}'^* \mathbf{C}_i^* \mathbf{C}_u \boldsymbol{u}\,, \tag{14}$$

yielding one possible form of the transformation matrix of voltages:

$$\mathbf{C}_u = \mathbf{C}_i^{*-1}. \tag{15}$$

Transformation of currents and voltages can be achieved by the same relationship, i.e.

$$\mathbf{C}_u = \mathbf{C}_i = \mathbf{C}\,. \tag{16}$$

Condition (15) for power invariance is given in this case by

$$\mathbf{C} = (\mathbf{C}^*)^{-1} \tag{17}$$

That is,

$$\mathbf{C}^{-1} = \mathbf{C}^*\,. \tag{18}$$

In the following a transformation meeting condition (18) will be determined.

Magnetic field strength generated in the air-gap of the three-phase machine is of the same distribution as magnetizing force supposed the width of air-gap to be constant, and rotor and stator to have infinite permeability. In the following these conditions are supposed to be satisfied. In this case it is sufficient to determine resultant magnetizing force generated by the three-phase windling. In the reference system the complex space vector of resultant magnetizing force generated by stator currents (Fig. 2) is:

$$\theta_s = N_s[i_A + i_B \cos 120^\circ + i_C \cos(-120^\circ) + j i_B \sin 120^\circ + + j i_C \sin(-120^\circ)] , \tag{19}$$

where N_s is the effective number of turns per phase in stator windling. Let

$$a = e^{j120^\circ} , \tag{20}$$

we obtain

$$\theta_s = N_s (i_A + i_B a + i_C a^2) . \tag{21}$$

In the calculations it is sufficient to use current space vector

$$I_s = K_0 (i_A + i_B a + i_C a^2) \tag{22}$$

proportional to resultant magnetizing force. As known, for stator currents forming a symmetrical three-phase system:

$$i_A = I_m \cos \omega_0 t, \; i_B = I_m \cos(\omega_0 t - 120^\circ), \; i_C = I_m \cos(\omega_0 t + 120^\circ) , \tag{23}$$

we have

$$I_s = K_0 \frac{3}{2} I_m e^{j\omega_0 t}, \tag{24}$$

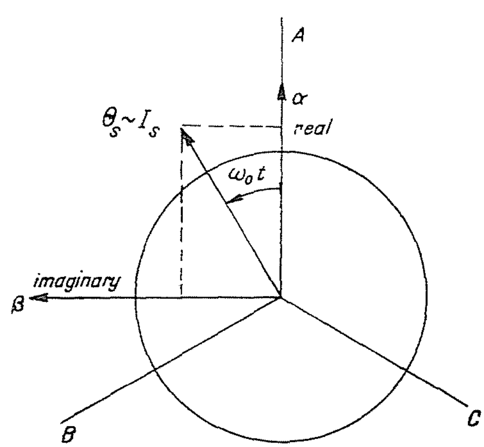


Fig. 2

a rotating magnetific field characterized by a current vector of constant magnitude, of angular velocity ω_0. Definition (22) is valid also for currents with arbitrary time function, but here both magnitude and angular velocity of I_s are functions of time. Remind that some books [12], [13] refer to the current space vector as with Park vector on. Evidently current space vector is unequivocally given by two perpendicular components, what is simpler than by phase currents, similarly characterizing space vector I_s unequivocally. Thus e.g. in system (α,β) Fig. 2. in the case of symmetrical currents

$$i_{s_\alpha} = I_s \cos \omega_0 t \, ,$$
$$i_{s_\beta} = I_s \sin \omega_0 t \, . \tag{25}$$

For the sake of generality, employ in place of system of co-ordinates (α, β) will be replaced by (u, v) (Fig. 3) rotating at an arbitrary angular velocity with respect to system (α, β) pertaining to the stator. Using formula

$$I_{s\,(u,\,v)} = I_s \, e^{-j\omega_k t} \, , \tag{26}$$

and relationship (22) after separation of real and imaginary parts the current space vector is given in the system (u, v) by relationships

$$i_{su} = K_0[i_A \cos \omega_k t + i_B \cos(\omega_k t - 120°) + i_C \cos(\omega_k t + 120°)]$$
$$i_{sv} = K_0[-i_A \sin \omega_k t - i_B \sin(\omega_k t - 120°) - i_C \sin(\omega_k t + 120°)] \, . \tag{27}$$

In final accent current transformation essentially corresponds to indicating space vector I_s by means of its components in a system of rectangular co-ordinates, rather then in a system of "phase" co-ordinates (A, B, C). For

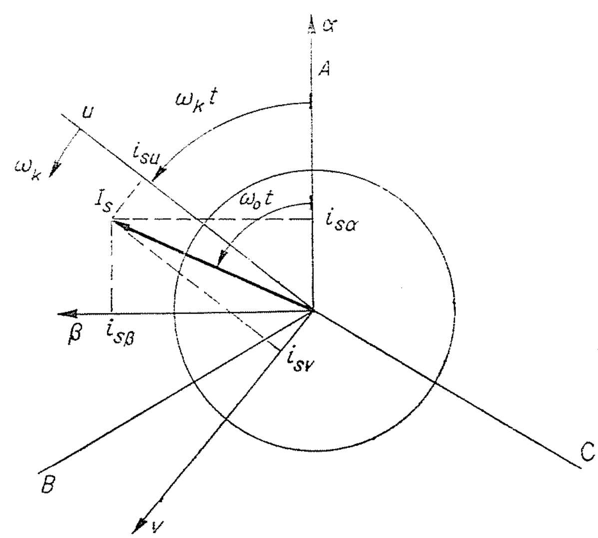


Fig. 3

the general case, this system of co-ordinates is called the system (u, v). Physically, transformation is interpreted as the substitution of three-phase currents and three-phase coiling by two-phase currents and coils producing an excitation identical with that by the three-phase system [12]. In the preceding, only the vector of stator currents has been given, but evidently, interpretation of the space vector of rotor currents is analogous. In the following, determination of phase currents in case of a given value of the space vector in system of co-ordinates (u, v) will be considered. For the unequivocal determination of the three-phase currents two equations (27) are not sufficient, another relationship linearly independent of these is needed.

Rearranging (26) we obtain:

$$\begin{aligned} Re\, I_{s(uv)}\, e^{j\omega_k t} &= Re\, I_s \\ Re\, I_{s(uv)}\, a^2\, e^{j\omega_k t} &= Re\, a^2\, I_s \\ Re\, I_{s(uv)}\, a\, e^{j\omega_k t} &= Re\, a\, I_s\,. \end{aligned} \tag{28}$$

Substituting expression (22) for the current space vector and taking the real parts, we have:

$$\begin{aligned} i_{su} \cos \omega_k t - i_{sv} \sin \omega_k t &= K_0 \left[i_A - \frac{1}{2}(i_B + i_C) \right] = K_0 \left[\frac{3}{2} i_A - \frac{1}{2}(i_A + i_B + i_C) \right] \\ i_{su} \cos(\omega_k t - 120°) - i_{sv} \sin(\omega_k t - 120°) &= K_0 \left[\frac{3}{2} i_B - \frac{1}{2}(i_A + i_B + i_C) \right] \\ i_{su} \cos(\omega_k t + 120°) - i_{sv} \sin(\omega_k t + 120°) &= K_0 \left[\frac{3}{2} i_C - \frac{1}{2}(i_A + i_B + i_C) \right] \end{aligned} \tag{29}$$

with the sum of phase currents in the left-hand side. According to considerations in [10], $i_A + i_B + i_C$ cannot be expressed as linear combination of i_{su} and i_{sv}, hence

$$i_{so} = \frac{1}{\sqrt{3}}(i_A + i_B + i_C) \tag{30}$$

is linearly independent of (27). Substituting zero-order current i_{s0} into (29), we obtain, after arranging, for phase currents:

$$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = K_0 \begin{bmatrix} \cos \omega_k t & -\sin \omega_k t & \frac{1}{\sqrt{3K_0}} \\ \cos(\omega_k t - 120°) & -\sin(\omega_k t - 120°) & \frac{1}{\sqrt{3K_0}} \\ \cos(\omega_k t + 120°) & -\sin(\omega_k t + 120°) & \frac{1}{\sqrt{3K_0}} \end{bmatrix} \begin{bmatrix} i_{su} \\ i_{sv} \\ i_{so} \end{bmatrix} \tag{31}$$

the inverse of transformation (31), supplied jointly by (27) and (30), being:

$$\begin{bmatrix} i_{su} \\ i_{sv} \\ i_{so} \end{bmatrix} = \frac{2}{3K_0} \begin{bmatrix} \cos \omega_k t & \cos(\omega_k t - 120°) & \cos(\omega_k t + 120°) \\ -\sin \omega_k t & -\sin(\omega_k t - 120°) & -\sin(\omega_k t + 120°) \\ K_0 \frac{\sqrt{3}}{2} & K_0 \frac{\sqrt{3}}{2} & K_0 \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} \tag{32}$$

Condition (18) for power invariance is satisfied if

$$K_0 = \sqrt{\frac{2}{3}}\,. \tag{33}$$

In this case, transformation matrix in (31) becomes:

$$\mathbf{C} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos \omega_k t & -\sin \omega_k t & \frac{1}{\sqrt{2}} \\ \cos(\omega_k t - 120°) & -\sin(\omega_k t - 120°) & \frac{1}{\sqrt{2}} \\ \cos(\omega_k t + 120°) & -\sin(\omega_k t + 120°) & \frac{1}{\sqrt{2}} \end{bmatrix}. \tag{34}$$

Transformation of rotor currents of the three-phase machine is analogous to that of stator currents. Remind in transformation that axes of stator and rotor coils do not coincide on account of the rotation of the rotor but, they include an angle α (Fig. 1). Thus, transformation matrix of rotor currents:

$$\mathbf{C}_r = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\omega_k t - \alpha) & -\sin(\omega_k t - \alpha) & \frac{1}{\sqrt{2}} \\ \cos(\omega_k t - \alpha - 120°) & -\sin(\omega_k t - \alpha - 120°) & \frac{1}{\sqrt{2}} \\ \cos(\omega_k t - \alpha + 120°) & -\sin(\omega_k t - \alpha + 120°) & \frac{1}{\sqrt{2}} \end{bmatrix}. \tag{35}$$

Rotor currents are transformed, as shown above, into a co-ordinate system common with stator currents, causing transformed currents to vary identically with time e.g. in steady state we obtain sinusoidally varying currents of angular frequency $\omega_0 - \omega_k$ on both stator and rotor. By virtue of (16), transformation being identical for currents and voltages, what has been said so far is valid also for voltages of the three-phase machine.

We note that Eqs. (28) geometrically mean to determine the projection of the space vector on the axis of phase coils. For the sake of illustrativeness, the projection of the space vector on the axis of phase coils can be said to give phase currents Fig. 4, [1], [10], evidently, however this relationship is only correct for $i_0 = 0$. Accordingly phase currents are unequivocally determined by current space vector only together with zero order current.

This fact will be express by referring to system (u, v) as system $(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{o})$ in the following.

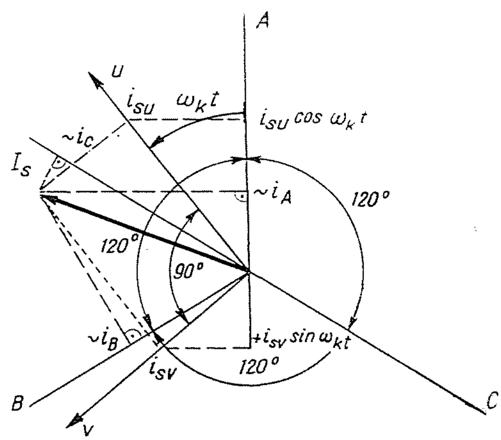


Fig. 4

It is advisible the choose the value of angular velocity ω_k of system (u, v, o) in dependence of the character of the problem. The most frequent cases are the following.

1. $\omega_k = 0$. In this case system (u, v, o) is transformed into system (α, β, o) where currents and voltages have an angular frequency ω_0.

2. $\omega_k = \omega$. In this case the system of co-ordinates is $(\boldsymbol{d}, \boldsymbol{q}, \boldsymbol{o})$, for a rotor of assymetrical structure advisably used. Transformation by (34) and (35) results the so-called Park—Gorew transformation.

3. For $\omega_k = \omega_0$, we obtain the system (x, y, o), characterized by an angular frequency $\omega_0 - \omega_k = 0$ for all currents and voltages.

Transformation of equations of the three-phase ansynchronous machine

Applying the transformation introduced above for currents and voltages of the asynchoronous machine yields:

$$\boldsymbol{u}_s = \mathbf{C}\, \boldsymbol{u}'_s \quad \text{and} \quad \boldsymbol{i}_s = \mathbf{C}\, \boldsymbol{i}'_s\,, \tag{36}$$

where

$$u'_s = \begin{bmatrix} u_{su} \\ u_{sv} \\ u_{so} \end{bmatrix} \quad \text{and} \quad i'_s = \begin{bmatrix} i_{su} \\ i_{sv} \\ i_{so} \end{bmatrix}, \tag{37}$$

further

$$u_r = \mathbf{C}_r\, u'_r \quad \text{and} \quad i_r = \mathbf{C}_r\, i'_r\,, \tag{38}$$

where

$$u'_r = \begin{bmatrix} u_{ru} \\ u_{rv} \\ u_{ro} \end{bmatrix} \quad \text{and} \quad i'_r = \begin{bmatrix} i_{ru} \\ i_{rv} \\ i_{ro} \end{bmatrix}. \tag{39}$$

Substituting transformed currents and voltages into Eqs. (11) and (12):

$$\begin{aligned} u'_s &= \mathbf{C}^*\mathbf{R}_s\mathbf{C}i'_s + \mathbf{C}^*\mathbf{L}_s \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{C}i'_s) + \omega\mathbf{C}^*\mathbf{M}\mathbf{C}_r i'_r + \mathbf{C}^*\mathbf{L}_{sr}\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{C}_r i'_r) \\ u'_r &= \mathbf{C}_r^*\mathbf{R}_r\mathbf{C}_r i'_r + \mathbf{C}_r^*\mathbf{L}_r \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{C}_r i'_r) + \omega\mathbf{C}_r^*\mathbf{M}^*\mathbf{C}i'_s + \mathbf{C}_r^*\mathbf{L}_{sr}^*\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{C}i'_s) \end{aligned} \tag{40}$$

$$p_0 i'^*_s \mathbf{C}^*\mathbf{M}\mathbf{C}_r i'_r = \frac{J}{p_0}\frac{\mathrm{d}\omega}{\mathrm{d}t} + \frac{K}{p_0}\omega - T_m\,. \tag{41}$$

Coefficient matrices in (40) become:

$$\mathbf{C}^*\mathbf{R}_s\mathbf{C} = \mathbf{R}_s\,, \tag{42}$$

$$\mathbf{C}^*\mathbf{L}_s\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{C}i'_s) = \mathbf{C}^*\mathbf{L}_s\mathbf{C}\frac{\mathrm{d}}{\mathrm{d}t}i'_s + \mathbf{C}^*\mathbf{L}_s\left(\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{C}\right)i'_s \tag{43}$$

where

$$\mathbf{C}^*\mathbf{L}_s\mathbf{C} = \begin{bmatrix} L_s - L_{ks} & 0 & 0 \\ 0 & L_s - L_{ks} & 0 \\ 0 & 0 & L_s + 2L_{ks} \end{bmatrix} \tag{44}$$

and

$$\mathbf{C}^*\mathbf{L}_s\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{C} = \omega_k(L_s - L_{ks})\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{45}$$

Further

$$\omega\mathbf{C}^*\mathbf{M}\mathbf{C}_r = \frac{3}{2}\omega L_{rs}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{46}$$

$$\mathbf{C}^*\mathbf{L}_{sr}\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{C}_r i'_r) = \mathbf{C}^*\mathbf{L}_{sr}\mathbf{C}_r\frac{\mathrm{d}}{\mathrm{d}t}i'_r + \mathbf{C}^*\mathbf{L}_{sr}\left(\frac{\mathrm{d}}{\mathrm{d}t}C_r\right)i'_r\,, \tag{47}$$

where

$$\mathbf{C}^*\mathbf{L}_{sr}\mathbf{C}_r = \frac{3}{2} L_{sr} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{48}$$

$$\mathbf{C}^*\mathbf{L}_{sr} \frac{\mathrm{d}}{\mathrm{d}t} \mathbf{C}_r = \frac{3}{2} (\omega_k - \omega) L_{sr} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{49}$$

Similar operations in the voltage equation of the rotor lead to the following coefficients:

$$\mathbf{C}_r^*\mathbf{R}_r\mathbf{C}_r = \mathbf{R}_r \tag{50}$$

$$\mathbf{C}_r^*\mathbf{L}_r \frac{\mathrm{d}}{\mathrm{d}t} (\mathbf{C}_r \boldsymbol{i}_r') = \mathbf{C}_r^*\mathbf{L}_r\mathbf{C}_r \frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{i}_r' + \mathbf{C}_r^*\mathbf{L}_r \left(\frac{\mathrm{d}}{\mathrm{d}t} \mathbf{C}_r \right) \boldsymbol{i}_r' , \tag{51}$$

where

$$\mathbf{C}_r^*\mathbf{L}_r\mathbf{C}_r = \begin{bmatrix} L_r - L_{kr} & 0 & 0 \\ 0 & L_r - L_{kr} & 0 \\ 0 & 0 & L_r + 2L_{kr} \end{bmatrix}, \tag{52}$$

and

$$\mathbf{C}_r^*\mathbf{L}_r \frac{\mathrm{d}}{\mathrm{d}t} \mathbf{C}_r = (\omega_k - \omega)(L_r - L_{kr}) \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{53}$$

Further

$$\omega \mathbf{C}_r^*\mathbf{M}^*\mathbf{C} = \frac{3}{2} L_{sr}\omega \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{54}$$

$$\mathbf{C}_r^*\mathbf{L}_{sr}^* \frac{\mathrm{d}}{\mathrm{d}t} (\mathbf{C}\boldsymbol{i}_s') = \mathbf{C}_r^*\mathbf{L}_{sr}^*\mathbf{C} \frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{i}_s' + \mathbf{C}_r^*\mathbf{L}_{sr}^* \left(\frac{\mathrm{d}}{\mathrm{d}t} \mathbf{C} \right) \boldsymbol{i}_s' \tag{55}$$

where

$$\mathbf{C}_r^*\mathbf{L}_{sr}^*\mathbf{C} = \frac{3}{2} L_{sr} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{56}$$

$$\mathbf{C}_r^*\mathbf{L}_{sr}^* \left(\frac{\mathrm{d}}{\mathrm{d}t} \mathbf{C} \right) = \frac{3}{2} \omega_k L_{sr} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{57}$$

Introducing:

$$L_{01} = L_s + 2L_{ks} , \qquad L_{02} = L_r + 2L_{kr} , \qquad M_{12} = \frac{3}{2} L_{sr} ,$$
$$L_1 = L_s - L_{ks} , \qquad L_2 = L_r - L_{kr} . \tag{58}$$

Substituting these and the transformed coefficients into (40) and (41), we obtain:

$$\begin{bmatrix} u_{su} \\ u_{sv} \\ u_{s0} \\ u_{ru} \\ u_{rv} \\ u_{r0} \end{bmatrix} = \begin{bmatrix} R_s+L_1\frac{d}{dt} & -\omega_k L_1 & 0 & M_{12}\frac{d}{dt} & -\omega_k M_{12} & 0 \\ \omega_k L_1 & R_s+L_1\frac{d}{dt} & 0 & \omega_k M_{12} & M_{12}\frac{d}{dt} & 0 \\ 0 & 0 & R_s+L_0\frac{d}{dt} & 0 & 0 & 0 \\ M_{12}\frac{d}{dt} & -(\omega_k-\omega)M_{12} & 0 & R_r+L_2\frac{d}{dt} & -(\omega_k-\omega)L_2 & 0 \\ (\omega_k-\omega)M_{12} & M_{12}\frac{d}{dt} & 0 & (\omega_k-\omega)L_2 & R_r+L_2\frac{d}{dt} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_r+L_{02}\frac{d}{dt} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{su} \\ i_{sv} \\ i_{s0} \\ i_{ru} \\ i_{rv} \\ i_{r0} \end{bmatrix} \tag{59}$$

$$p_0 M_{12}(i_{sv}i_{ru} - i_{su}i_{rv}) = \frac{J}{p_0}\frac{d\omega}{dt} + \frac{K}{p_0} - T_m \tag{60}$$

(59) exhibits constant induction coefficients. Therefore the system of equations is simpler to solve that (11), and the network model can be established from network elements with invariant parameters.

It should be noted that in certain cases it is advisable to write the above equations by replacing current and voltage space vectors in place of their components. Accordingly, the equations for the asynchronous machine become:

$$\begin{bmatrix} U_s \\ U_r \end{bmatrix} = \begin{bmatrix} R_s + L_1\frac{d}{dt} + j\omega_k L_1 & M_{12}\frac{d}{dt} + j\omega_k M_{12} \\ M_{12}\frac{d}{dt} + j(\omega_k-\omega)M_{12} & R_r + L_2\frac{d}{dt} + j(\omega_k - \omega)L_2 \end{bmatrix} \begin{bmatrix} I_s \\ I_r \end{bmatrix} \tag{61}$$

$$\begin{bmatrix} U_{s0} \\ U_{r0} \end{bmatrix} = \begin{bmatrix} R_s + L_{01}\frac{d}{dt} & 0 \\ 0 & R_r + L_{02}\frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{s0} \\ i_{r0} \end{bmatrix},$$

$$p_0 M_{12} Im(I_s \tilde{I}_r) = \frac{J}{p_0}\frac{d\omega}{dt} + \frac{K}{p_0}\omega - T_m\,. \tag{62}$$

(Symbol ~ denotes the conjugate.)

The correctness of the above relationships is simple to prove by substituting space vector components.

Establishment of the network model

To establish a linear network model for the three-phase asynchronous machine according to relationships (59) and (60), the involved mechanical quantities are substituted by analogous electric magnitudes and the equations linearized. For the method of substituting mechanical quantities let us refer to [7].
Linearization will be made by assuming currents and voltages in (59), as well as mechanical variables to be:

$$\begin{aligned} u'_s &= U'_s + \bar{u}'_s, & i'_s &= I'_s + \bar{i}'_s, \\ u'_r &= U'_r + \bar{u}'_r, & i'_r &= I'_r + \bar{i}'_r, \\ \omega &= \Omega + \bar{\omega}, & T_m &= T_M + \bar{t}_m \end{aligned} \tag{63}$$

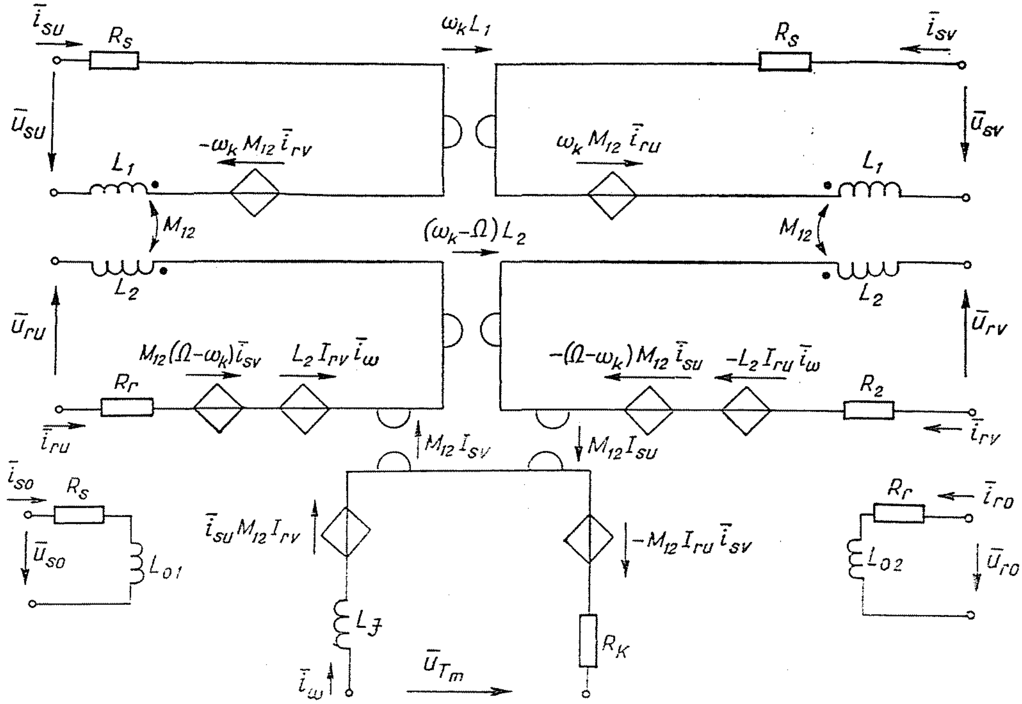
where capitals denote steady values, dashes above deviations there from. These relationships substituted into (59) and (60), and neglecting second-order deviations, we obtain the Eq. (64) (see p. 171) for the deviation from steady state, if voltage u_{Tm} corresponds to external mechanical moment and current i_ω to angular velocity of the rotor.
It should be noted, that after having substituted mechanical by electric quantities the mechanical equation and the circuit equations are considered as a single system of equations. Inductivity and resistance values introduced in (64), as well as voltage corresponding to moment are given by:

$$L_j = \frac{J}{p_0^2}, \quad R_k = \frac{K}{p_0^2}, \quad u_{Tm} = U_{Tm} + \bar{u}_{Tm} = \frac{T_m}{p_0}. \tag{65}$$

The network model constructed on the basis of relationship (64) is shown in Fig. 5. The realization is easy to verify by decomposing coefficient matrix (64) to three terms, being the symmetrical part of the coefficient matrix, the second the antisymmetrical part, while the third contains all the other. The first term can be realized by resistance and inductivities, the second by gyrators, while the third by controlled sources. Suitably connected, these yield the circuit shown in Fig. 5.

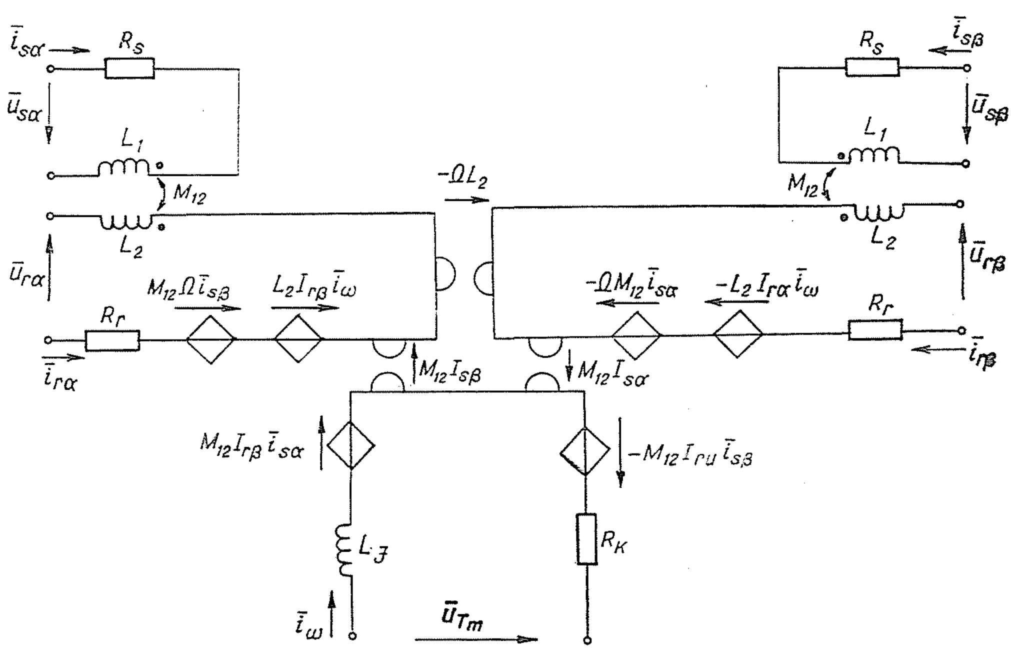
Examine case $\omega_k = 0$, where the asynchronous machine is examined in system (α, β, o) pertaining to the stator. The network model for this case is shown in Fig. 6. Since zero-order circuits are not connected to the other circuits, they can be examined in themselves. For the sake of simplicity,

$$
\begin{bmatrix} \bar{u}_{su} \\ \bar{u}_{sv} \\ \bar{u}_{s0} \\ \bar{u}_{ru} \\ \bar{u}_{rv} \\ \bar{u}_{r0} \\ \bar{u}_{Tm} \end{bmatrix}
=
\begin{bmatrix}
R_s+L_1\frac{\mathrm{d}}{\mathrm{d}t} & -\omega_k L_1 & 0 & M_{12}\frac{\mathrm{d}}{\mathrm{d}t} & -\omega_k M_{12} & 0 & 0 \\
\omega_k L_1 & R_s+L_1\frac{\mathrm{d}}{\mathrm{d}t} & 0 & \omega_k M_{12} & M_{12}\frac{\mathrm{d}}{\mathrm{d}t} & 0 & 0 \\
0 & 0 & R_s+L_{01}\frac{\mathrm{d}}{\mathrm{d}t} & 0 & 0 & 0 & 0 \\
M_{12}\frac{\mathrm{d}}{\mathrm{d}t} & -(\omega_k-\Omega)M_{12} & 0 & R_r+L_2\frac{\mathrm{d}}{\mathrm{d}t} & -(\omega_k-\Omega)L_2 & 0 & \begin{matrix} M_{12}I_{sv}+ \\ +L_2 I_{rv} \end{matrix} \\
(\omega_k-\Omega)M_{12} & M_{12}\frac{\mathrm{d}}{\mathrm{d}t} & 0 & (\omega_k-\Omega)L_2 & R_r+L_2\frac{\mathrm{d}}{\mathrm{d}t} & 0 & \begin{matrix} -M_{12}I_{su}- \\ -L_2 I_{ru} \end{matrix} \\
0 & 0 & 0 & 0 & 0 & R_r+L_{02}\frac{\mathrm{d}}{\mathrm{d}t} & 0 \\
M_{12}I_{rv} & -M_{12}I_{ru} & 0 & -M_{12}I_{su} & M_{12}I_{sv} & 0 & R_k+L_j\frac{\mathrm{d}}{\mathrm{d}t}
\end{bmatrix}
\begin{bmatrix} \bar{i}_{su} \\ \bar{i}_{sv} \\ \bar{i}_{s0} \\ \bar{i}_{ru} \\ \bar{i}_{rv} \\ \bar{i}_{r0} \\ \bar{i}_{\omega} \end{bmatrix}
\tag{64}
$$



$\bar{i}_{su}$
R_s
$\omega_k L_1$
R_s
$\bar{i}_{sv}$
$\bar{u}_{su}$
$-\omega_k M_{12} \bar{i}_{rv}$
$\omega_k M_{12} \bar{i}_{ru}$
$\bar{u}_{sv}$
L_1
L_1
M_{12}
$(\omega_k-\Omega) L_2$
M_{12}
L_2
L_2
$\bar{u}_{ru}$
$M_{12}(\Omega-\omega_k)\bar{i}_{sv}$
$L_2 I_{rv} \bar{i}_\omega$
$-(\Omega-\omega_k) M_{12} \bar{i}_{su}$
$-L_2 I_{ru} \bar{i}_\omega$
$\bar{u}_{rv}$
R_r
R_2
$\bar{i}_{ru}$
$M_{12} I_{sv}$
$M_{12} I_{su}$
$\bar{i}_{rv}$
$\bar{i}_{so}$
R_s
R_r
$\bar{i}_{r0}$
$\bar{u}_{so}$
L_{01}
$\bar{i}_{su} M_{12} I_{rv}$
$-M_{12} I_{ru} \bar{i}_{sv}$
L_{02}
$\bar{u}_{r0}$
L_J
R_K
$\bar{u}_{Tm}$
$\bar{i}_\omega$

Fig. 5



$\bar{i}_{s\alpha}$
R_s
R_s
$\bar{i}_{s\beta}$
$\bar{u}_{s\alpha}$
$\bar{u}_{s\beta}$
L_1
L_1
M_{12}
$-\Omega L_2$
M_{12}
L_2
L_2
$\bar{u}_{r\alpha}$
$M_{12}\Omega \bar{i}_{s\beta}$
$L_2 I_{r\beta} \bar{i}_\omega$
$-\Omega M_{12} \bar{i}_{s\alpha}$
$-L_2 I_{r\alpha} \bar{i}_\omega$
$\bar{u}_{r\beta}$
R_r
R_r
$\bar{i}_{r\alpha}$
$M_{12} I_{s\beta}$
$M_{12} I_{s\alpha}$
$\bar{i}_{r\beta}$
$M_{12} I_{r\beta} \bar{i}_{s\alpha}$
$-M_{12} I_{ru} \bar{i}_{s\beta}$
L_J
R_K
$\bar{i}_\omega$
$\bar{u}_{Tm}$

Fig. 6

these will be omitted. Since in most practical cases $i_0 = 0$, phenomena in zero order circuits need in general, not be examined. Remind that the network in Fig. 6 is of the same construction as that for the direct-current basic machine in [7] equations of three-phase machines in the system (α, β, o) being known to be essentially identical with those of the direct current basic machine. Provided the angular velocity of the rotor is constant, i. e. $\bar{i}_\omega = 0$, the network model in Fig. 7 can be established on the basis of Fig. 6, valid for the full range of currents and voltages (i.e. not only for the deviations). Provided ω = constant, and there are no zero-order currents, it is sufficient to examine the part of Eq. (61) relating to space vectors, by means of a model where space vectors are directly considered as variables, such as that in Fig. 8, or the similar equivalent network in [1]. It should be noted, however, that this model cannot be generalized so as to contain mechanical variables too, of the mechanical equation in form (62) including space vectors cannot be made to a model like as against relationship (60).

Performing the transformation of currents and voltages, of the previous models according to matrices (34) or (35), we obtain currents and voltages actually arising in the machine. The above models can be replaced

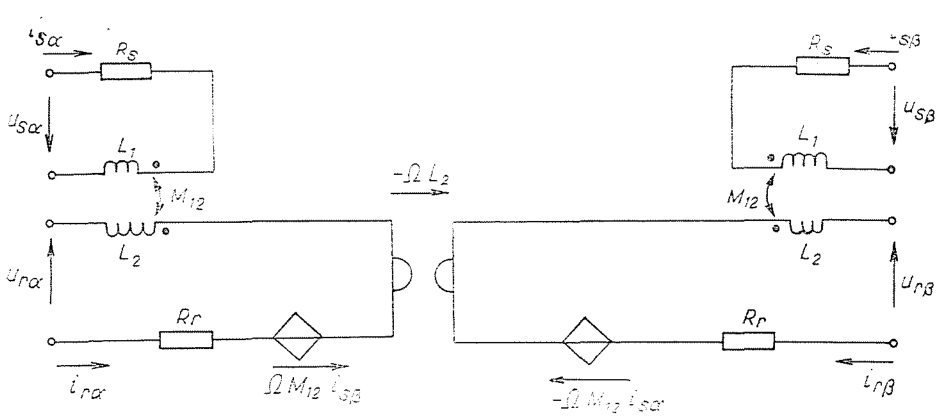


Fig. 7

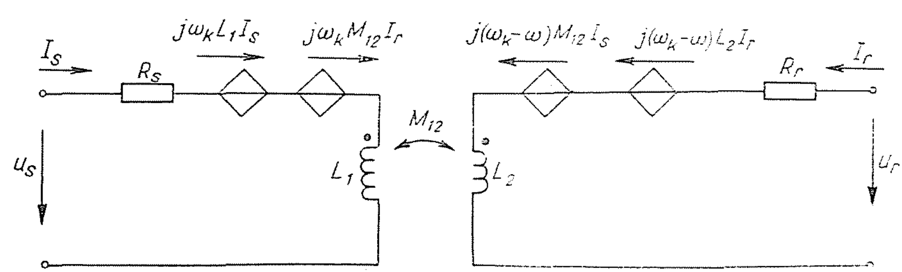


Fig. 8

by there permitting to determined actual stator currents. In the followings the construction of a model of this kind will be shown. Let us assume the angular velocity of the rotor to be constant and examined currents of the asynchronous machine in a system of co-ordinates pertaining to the stator, i.e. $\omega = \Omega$ and $\omega_k = 0$. In this case Eq. (64) becomes:

$$\begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \\ u_{so} \\ \hline u_{r\alpha} \\ u_{r\beta} \\ u_{ro} \\ \hline u_{Tm} \end{bmatrix} = \left[\begin{array}{ccc|cccc} R_s + L_1 \frac{d}{dt} & 0 & 0 & M_{12}\frac{d}{dt} & 0 & 0 & 0 \\ 0 & R_s + L_1\frac{d}{dt} & 0 & 0 & M_{12}\frac{d}{dt} & 0 & 0 \\ 0 & 0 & R_s + L_{01}\frac{d}{dt} & 0 & 0 & 0 & 0 \\ \hline M_{12}\frac{d}{dt} & \Omega M_{12} & 0 & R_r + L_2\frac{d}{dt} & \Omega L_2 & 0 & 0 \\ -\Omega M_{12} & M_{12}\frac{d}{dt} & 0 & -\Omega L_2 & R_r + L_2\frac{d}{dt} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_r + L_{02}\frac{d}{dt} & 0 \\ \hline M_{12} i_{r\beta} & -M_{12} i_{r\alpha} & 0 & 0 & 0 & 0 & R_K \end{array}\right] \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ i_{so} \\ \hline i_{r\alpha} \\ i_{r\beta} \\ i_{ro} \\ \hline I_\Omega \end{bmatrix} \tag{66}$$

For the sake of conciseness, (66) will be written by means of hypermatrices. Hypermatrices in (66) are separated by dotted lines. Thus, (66) in hypermatrix form:

$$\begin{bmatrix} u_s' \\ u_r' \\ u_{Tm} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{ss} & \mathbf{Z}_{sr} & O \\ \mathbf{Z}_{rs} & \mathbf{Z}_{rr} & O \\ Z_{Ts}^* & O^* & R_k \end{bmatrix} \begin{bmatrix} i_s' \\ i_r' \\ I_\Omega \end{bmatrix} \tag{67}$$

Transformation currents and voltages of the stator into the system of "phase" co-ordinates:

$$\begin{bmatrix} u_s \\ u_r' \\ u_{Tm} \end{bmatrix} = \begin{bmatrix} \mathbf{C} & 0 & 0 \\ 0 & 1 & 0 \\ 0^* & 0^* & 1 \end{bmatrix} \begin{bmatrix} u_s' \\ u_r' \\ u_{Tm} \end{bmatrix}, \tag{68}$$

$$\begin{bmatrix} i_s \\ i_r' \\ \mathbf{I}_\Omega \end{bmatrix} = \begin{bmatrix} \mathbf{C} & 0 & 0 \\ 0 & 1 & 0 \\ 0^* & 0^* & 1 \end{bmatrix} \begin{bmatrix} i_s' \\ i_r' \\ I_\Omega \end{bmatrix} \tag{69}$$

where **C** is the transformation matrix according to (34) for $\omega_k = 0$. Substituting (68) and (69) into (67):

$$\begin{bmatrix} u_s \\ u'_r \\ u_{Tm} \end{bmatrix} = \begin{bmatrix} \mathbf{C}\mathbf{Z}_{ss}\mathbf{C}^* & \mathbf{C}\mathbf{Z}_{sr} & \mathit{0} \\ \mathbf{Z}_{rs}\mathbf{C}^* & \mathbf{Z}_{rr} & \mathit{0} \\ \mathbf{Z}^*_{Ts}\mathbf{C}^* & \mathit{0}^* & R_K \end{bmatrix} \begin{bmatrix} i_s \\ i'_r \\ I_\Omega \end{bmatrix}, \tag{70}$$

where

$$\mathbf{C}\mathbf{Z}_{ss}\mathbf{C}^* = \mathbf{R}_s + \mathbf{L}_s \frac{\mathrm{d}}{\mathrm{d}t}, \tag{71}$$

$$\mathbf{C}\mathbf{Z}_{sr} = \sqrt{\frac{3}{2}} L_{sr} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & -\sqrt{\frac{3}{2}} & 0 \\ -\frac{1}{2} & -\sqrt{\frac{3}{2}} & 0 \end{bmatrix},$$

$$\mathbf{Z}_{rs}\mathbf{C}^* = \sqrt{\frac{3}{2}} L_{sr} \begin{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} & -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t} + \frac{3}{2}\Omega & -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t} - \frac{3}{2}\Omega \\ -\Omega & \frac{3}{2}\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{2}\Omega & -\frac{3}{2}\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{2}\Omega \\ 0 & 0 & 0 \end{bmatrix} \tag{72}$$

$$\mathbf{Z}^*_{Ts}\mathbf{C}^* = \sqrt{\frac{3}{2}} L_{sr} \left[i_{r\beta} \quad -\frac{1}{2} i_{r\beta} - \sqrt{\frac{3}{2}} i_{r\alpha} \quad -\frac{1}{2} i_{r\beta} + \sqrt{\frac{3}{2}} i_{r\alpha} \right].$$

The above matrix products were obtained by means of relationships introduced in (58).

Relationships obtained by transformation show that the correlaion between currents and voltages of the stator can be modeled by a coil system connected according to basic equation (11), the stator and rotor are connected by a coupling which can be described by an inductive, controlled source, while the mechanical circuit and the electric circuits are connected by a nonlinear coupling. The zero-order circuit is seen — similarly to the above — not be to coupled with the other circuits in the case of the rotor, thus it is open to examination independent.

For the sake of intelligibility stator coils in Fig. 9, are shown in star connection and substituted by uncoupled coils [14] while the mutual induction coefficients of stator and rotor coils are determined by the relationship

$$L_k = \sqrt{\frac{3}{2}} L_{sr} \cos \gamma, \tag{73}$$

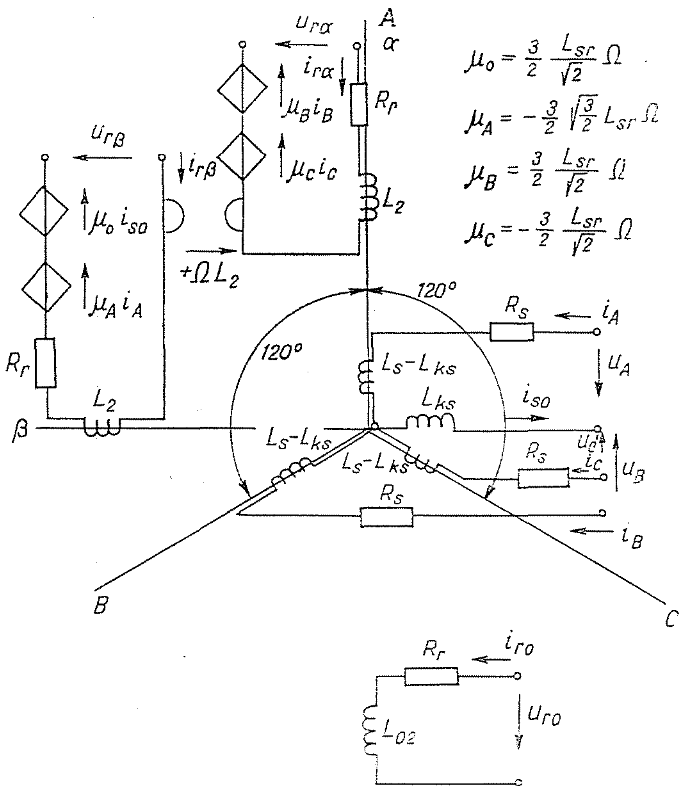


Fig. 9

where γ is the angle included between axes of the examined coils. It deserves mentioning that a relationship essentially identical to (70) was given by Tshaban for asynchronous machines, using the transformation elaborated in [15].

Models partially different from those given above can be constructed for the asynchronous machine by applying dynamical analogy so that voltage u_ω corresponds to angular velocity, and current i_{Tm} to moment. In this case the linearized form of the transformed equations of the asynchronous machine is the Eq. (73) (see p. 177).

In Eq. (73), mechanical parameters are replaced by conductance and capacity:

$$G_K = \frac{K}{p_0^2} \quad \text{and} \quad C_J = \frac{J}{p_0^2}, \tag{74}$$

respectively. Fig. 10 shows a network corresponding to (73) differing from that in Fig. 5, by the ideal transformer applied for coupling the mechanical circuit and the rotor circuits, in place of gyrator. Zero-order circuits are not indicated in Fig. 10, since these can be examined in themselves.

$$
\begin{bmatrix} \bar{u}_{su} \\ \bar{u}_{sv} \\ \bar{u}_{s0} \\ \bar{u}_{ru} \\ \bar{u}_{rv} \\ \bar{u}_{r0} \\ \bar{i}_{Tm} \end{bmatrix}
=
\begin{bmatrix}
R_s+L_1\frac{\mathrm{d}}{\mathrm{d}t} & -\omega_k L_1 & 0 & M_{12}\frac{\mathrm{d}}{\mathrm{d}t} & -\omega_k M_{12} & 0 & 0 \\
\omega_k L_1 & R_s+L_1\frac{\mathrm{d}}{\mathrm{d}t} & 0 & \omega_k M_{12} & M_{12}\frac{\mathrm{d}}{\mathrm{d}t} & 0 & 0 \\
0 & 0 & R_s+L_{01}\frac{\mathrm{d}}{\mathrm{d}t} & 0 & 0 & 0 & 0 \\
M_{12}\frac{\mathrm{d}}{\mathrm{d}t} & -(\omega_k-\Omega)M_{12} & 0 & R_r+L_2\frac{\mathrm{d}}{\mathrm{d}t} & -(\omega_k-\Omega)L_2 & 0 & M_{12}I_{sv}+L_2 I_{rv} \\
(\omega_k-\Omega)M_{12} & M_{12}\frac{\mathrm{d}}{\mathrm{d}t} & 0 & (\omega_k-\Omega)L_2 & R_r+L_2\frac{\mathrm{d}}{\mathrm{d}t} & 0 & M_{12}I_{su}-M_{12}I_{ru} \\
0 & 0 & 0 & 0 & 0 & R_r+L_{02}\frac{\mathrm{d}}{\mathrm{d}t} & 0 \\
M_{12}I_{rv} & -M_{12}I_{ru} & 0 & -M_{12}I_{sv} & M_{12}I_{su} & 0 & G_K+C_J\frac{\mathrm{d}}{\mathrm{d}t}
\end{bmatrix}
\begin{bmatrix} \bar{i}_{su} \\ \bar{i}_{sv} \\ \bar{i}_{so} \\ \bar{i}_{ru} \\ \bar{i}_{rv} \\ \bar{i}_{ro} \\ \bar{u}_{\omega} \end{bmatrix}
\tag{73}
$$

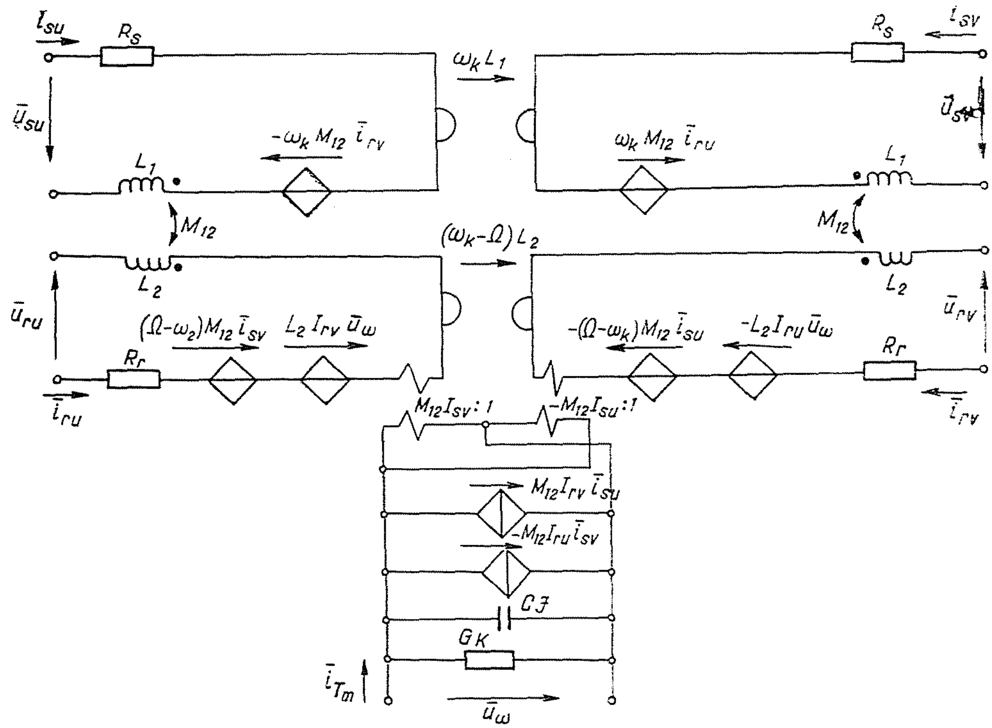


Fig. 10

Applications

Application of models given in the preceding chapter, reduce examination of asynchronous machines to the analysis of electric networks containing, in addition to passive two-poles, also two-ports. For the calculation of similar networks, suitable methods are described in [16], [17], [18], Since the description of these methods and their application would go considerably beyond the scope of the present paper, the use of the given models will be demonstrated on two simple examples, which can be discussed also in other ways.

1. Examine the possibility of transforming the model in Fig. 7, for steady state and a symmetrical three-phase voltage system supplying the stator. Obviously on account of symmetric excitation and machine symmetry, currents arising in the network of Fig. 7 form a symmetrical two-phase system, hence, complex time functions are:

$$i_{s\alpha} = I_s e^{j\omega_0 t}, \qquad i_{s\beta} = I_s e^{j\left(\omega_0 t - \frac{\pi}{2}\right)} = -j i_{s\alpha}$$
$$i_{r\alpha} = I_r e^{j\omega_0 t}, \qquad i_{r\beta} = I_r e^{j\left(\omega_0 t - \frac{\pi}{2}\right)} = -j i_{r\alpha} \tag{75}$$

Considering (75), it is sufficient to examine e.g. the components α, and the respective circuits. Transforming the circuit in Fig. 7 accordingly, and considering (75), we obtain the network shown in Fig. 11/a, with a non-reciprocal coupling between stator and rotor circuits. Fig. 11/a took into consideration that the rotor winding of the asynchronous machine is short-circuited, hence $u_{r\alpha} = 0$. Expressing parameters of rotor circuit elements by slip:

$$s = \frac{\omega_0 - \Omega}{\omega_0} \,. \tag{76}$$

We obtain:

$$\begin{aligned} (\omega_0 - \Omega)L_2 &= s\omega_0 L_2 \\ (\omega_0 - \Omega)M_{12} &= s\omega_0 M_{12} \,. \end{aligned} \tag{77}$$

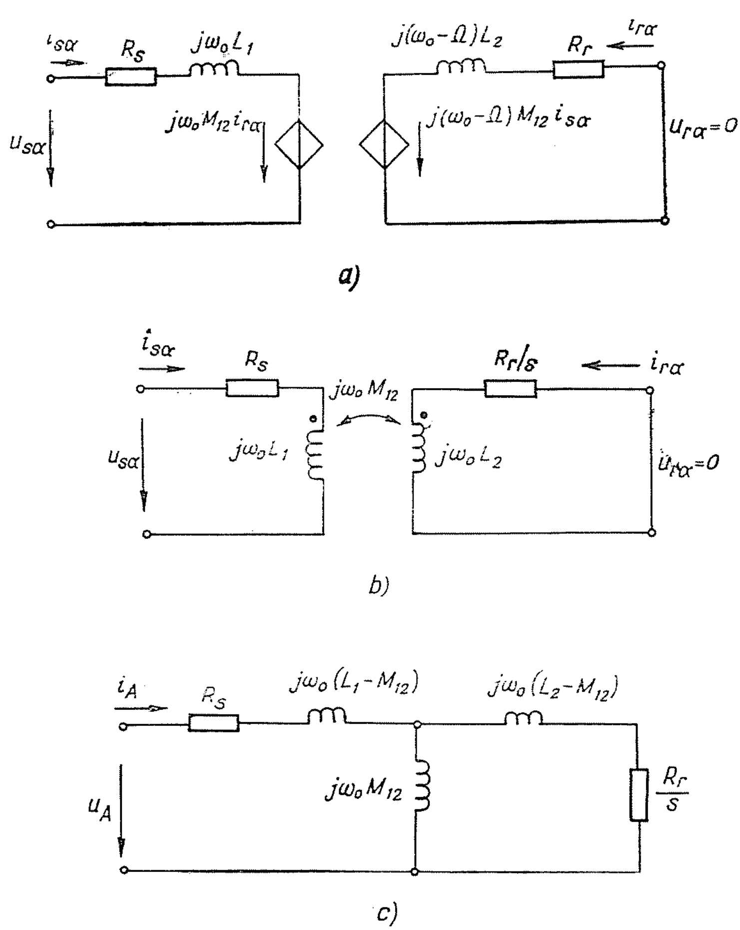


Fig. 11

Current in the rotor circuit

$$I_{r\alpha} = \frac{js\omega_0 M_{12} I_{s\alpha}}{R_r + js\omega_0 L_2} . \tag{78}$$

Dividing both numerator and denominator by the slip yields:

$$I_{\alpha} = \frac{j\omega_0 M_{12} I_{s\alpha}}{R_r/s + j\omega_0 L_2} . \tag{79}$$

That is, changing rotor circuit parameters according to relationship (79), current remains unaltered. By such a modification, the coupling of stator and rotor circuits becomes reciprocal, hence it can be modelled by coupled coils according to Fig. 11/b. Introducing the equivalent T circuit for coupled circuits [14], and since for $\omega_k = 0$, $i_0 = 0$, and $u_0 = 0$, transformation according to (34) yields:

$$i_A = i_{s\alpha} , \qquad u_A = u_{s\alpha} . \tag{80}$$

The network shown in Fig. 11/c is obtained giving directly the phase current of the stator of the asynchronous machine. It should be noted that the circuit in Fig. 11/c is identical to the single-phase equivalent circuit of the asynchronous machine.

2. As a second example, determine currents arising in starting an asynchronous motor, with an impedance Z inserted in series with the phase coil in one phase. Assuming the three-phase network supplying the motor to be symmetrical and stator coils to be star connected. The problem is advisable solved by using the model in Fig. 9. where impedance Z can be directly inserted. The network used in further calculations is shown in Fig. 12, where Z has been inserted into phase B and steady state, as well as motionless rotor, i.e. $\omega = 0$ assumed. Unknown phase currents have been determined by the method of loop currents, using the independent loop system in Fig. 12. Thus the equations for loop currents are:

$$\begin{bmatrix} 2Z_s + Z & Z_s + Z & \frac{3}{2} Z_m & \frac{\sqrt{3}}{2} Z_m \\ Z_s + Z & 2Z_s + Z & 0 & \sqrt{3} Z_m \\ \frac{3}{2} Z_m & 0 & Z_r & 0 \\ \frac{\sqrt{3}}{2} Z_m & \sqrt{3} Z_m & 0 & Z_r \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} = \begin{bmatrix} U_{AB} \\ -U_{BC} \\ 0 \\ 0 \end{bmatrix} , \tag{81}$$

where

$$Z_s = R_s + j\omega_0 L_1 ,$$

$$Z_m = j\omega_0 \sqrt{\frac{3}{2}} L_{sr} , \tag{82}$$

$$Z_r = R_r + j\omega_0 L_2 ,$$

taking into consideration that the coefficient of mutual induction of rotor and stator can be determined by (72). By solving Eqs (81) for loop currents, — omitting derivations — we obtained:

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} = \frac{1}{Z_{be}(3Z_{be}+2Z)} \begin{bmatrix} 2Z_{be}+Z & -Z_{be}-Z \\ -Z_{be}-Z & 2Z_{be}+Z \\ -\frac{3}{2} Y_r Z_m (2Z_{be}+Z) & \frac{3}{2} Y_r Z_m (2Z_{be}+Z) \\ \frac{\sqrt{3}}{2} Y_r Z_m Z & -\frac{\sqrt{3}}{2} Y_r Z_m (3Z_{be}+Z) \end{bmatrix} \begin{bmatrix} U_{AB} \\ -U_{BC} \end{bmatrix} \tag{83}$$

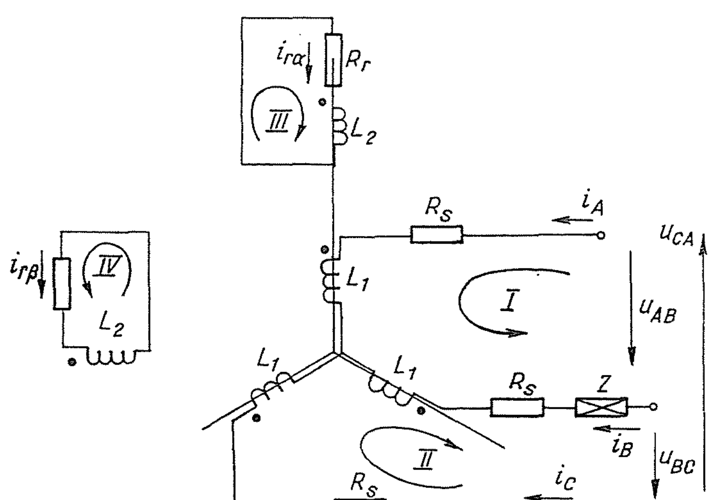


Fig. 12

with notations:

$$Y_r = \frac{1}{Z_r} \qquad Z_{be} = Z_s - 1{,}5\ Y_r Z_m^2 \tag{84}$$

Accordingly, the complex effective values of stator phase currents and of rotor currents are, respectively:

$$\begin{aligned}
I_A = J_1 &= \frac{U_{AB} - U_{CA}}{3Z_{be} + 2Z} - \frac{U_{CA} Z}{Z_{be}(3Z_{be} + 2Z)}, \\
I_B &= -J_1 - J_2 = \frac{U_{BC} - U_{AB}}{3Z_{be} + 2Z}, \\
I_C = J_2 &= \frac{U_{CA} - U_{BC}}{3Z_{be} + 2Z} + \frac{U_{CA} Z}{Z_{be}(3Z_{be} + 2Z)}, \\
I_{r\alpha} = J_3 &= -\frac{Y_r Z_m}{3Z_{be} + 2Z}\left(3U_{AB} + \frac{3}{2} U_{BC} - \frac{3Z}{2Z_{be}} U_{CA}\right), \\
I_{r\beta} = J_4 &= -\frac{Y_r Z_m}{3Z_{be} + 2Z}\left(-3\frac{\sqrt{3}}{2} U_{BC} + \frac{\sqrt{3}}{2}\frac{Z}{Z_{be}} U_{CA}\right),
\end{aligned} \tag{85}$$

permitting all the necessary calculations.

Summary

The presented models are characterized by their suitability for the simultaneous examination of mechanical and electromagnetic processes in asynchronous machines, under the discussed conditions. Models refer to asynchronous machines of symmetrical structure alone but are likely to be generalized for asymmetrical cases with the exception of the one shown in Fig. 8. Calculations based on these models correspond to the analysis of electric networks consisting of two-poles and of two-ports containing controlled sources. An other than (u,v,o) co-ordinate transformation may lead to somewhat different models for the asynchronous machine. The transient or steady state of systems consisting of several asynchronous machines and two-poles can be examined by means of the electric network formed by suitably connecting the given models, the use of a computer needing, however, because of the involved extensive calculations.

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