MODELLING CONTROL LOOPS IN THE TIME DOMAIN

By

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1. General remarks

The problem of modelling can be defined as follows:

Given a physical system S. Find or construct another physical system M which substitutes for S in some sense. M is termed as the model of the system S. It is important to note that the behaviour of M does not correspond to that of S in every respect. Thus a real physical system may have several models, which substitute for different properties of the system.

Of course, it is often not needed to construct the model as a real physical system, it is sufficient to describe the substituted properties of S by some mathematical function or algorithm.

In this paper a modelling procedure is presented for single variable open control loops. The structure of the model should be as simple as possible. In this case the number of parameters to be determined is small, and due to this the computing time of modelling is likewise small. This is a reasonable advantage in real time applications.

The structure of an adequate model depends on the nature of the system. To demonstrate the modelling procedure, let us take into account those systems which occur most frequently in process control. The open loops of such control systems are proportional or integrating, their step response is a time function, mostly positive definite monotonic or increasing damped oscillation. Although these systems may be nonlinear, after linearization their frequency characteristic exists. A peculiarity of the Bode plot is that the amplitude as well as the phase plot tend to decrease. This statement does not exclude a short increasing part of these plots, the phase plot, however, intersects the value $\varphi = -\pi$ only once. This means that in the following no conditionally stable system will be taken into consideration.

By synthetizing control loops from such systems a 1 to 1.5 decade portion of the frequency characteristic plays a decisive role on the transient response of the closed loop. This portion lies around that frequency ω_c for which the phase angle is $-\pi$. This is the middle frequency portion (M. F. P.) of the frequency characteristic (Fig. 1).

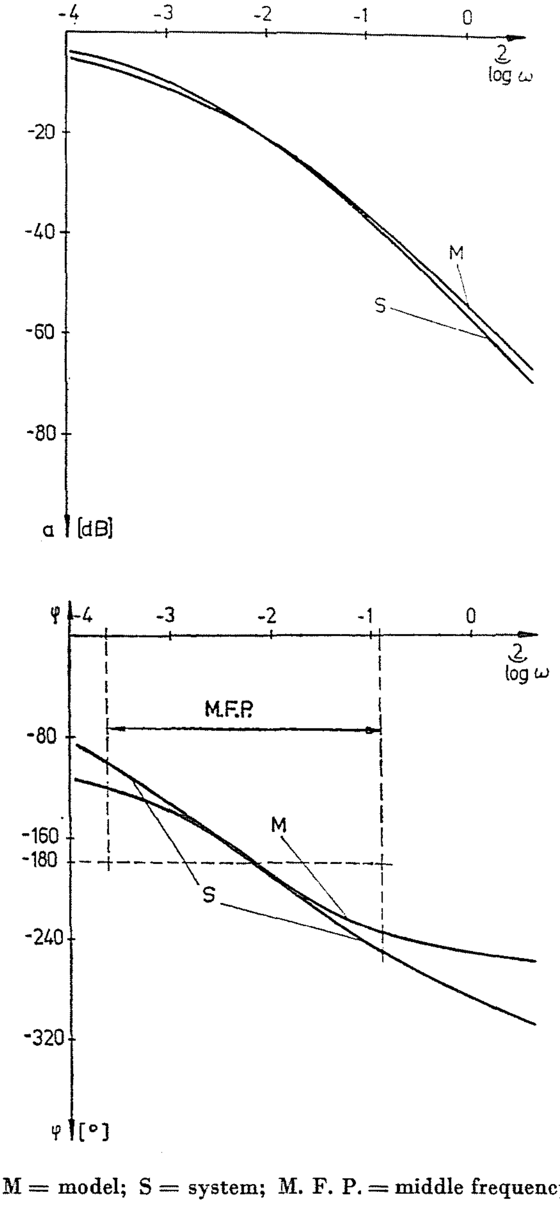


Fig. 1. M = model; S = system; M. F. P. = middle frequency portion

Transfer function of the model $W_m(s) = \frac{0.0373}{s(1 + 2 \cdot 0.71 \cdot 4.6s + 4.6^2 s^2)}$

Transfer function of the system $W_s(s) = \frac{1 + 10s}{(1 + 16s)(1 + 8s)^2 (1 + 4s)(1 + s)}$

It appears practical to choose a model, the frequency characteristic of which can be matched with the characteristic of the system at least in this region. This means that the slope of the amplitude plot of the model has to vary at least between −20 dB/decade and −60 dB/decade and, accordingly, the phase plot between $-\pi/2$ and $3\pi/2$. The simplest structure which fulfils this requirement is

$$W_m(s) = \frac{A_m}{T_m s(T_m^2 s^2 + 2\zeta_m T_m s + 1)} \tag{1}$$

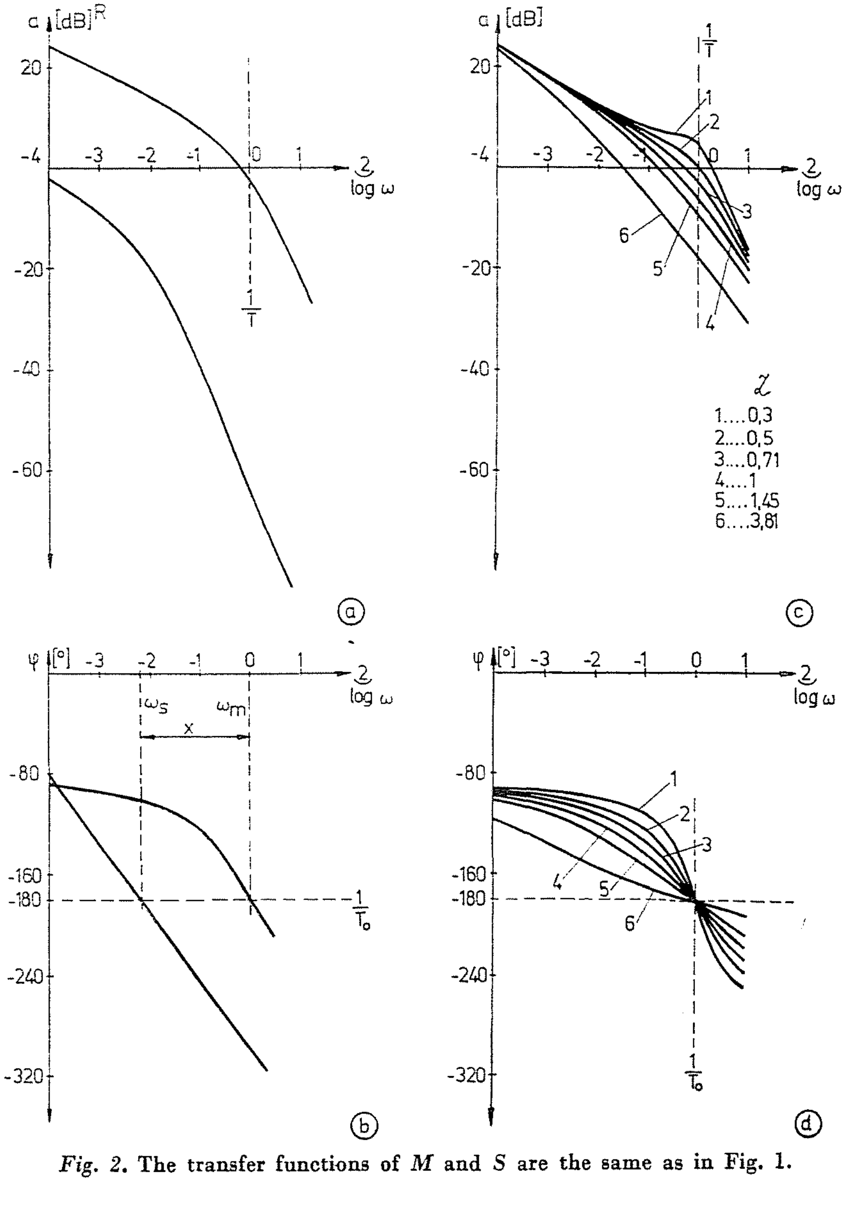


Fig. 2. The transfer functions of M and S are the same as in Fig. 1.

The middle frequency portion of the Bode plot of such a model can be matched well with that of the system if not more than three poles of the latter lie close to each other. (If this condition does not hold, then the phase plot may be so steep that only a less simple model can be used. An example will be presented in Sec. 4.) The simple model structure according to Eq. (1) has the advantage that the shape of its Bode diagram depends only on the damping fector ζ_m. The gain A_m affects only the vertical position of the amplitude

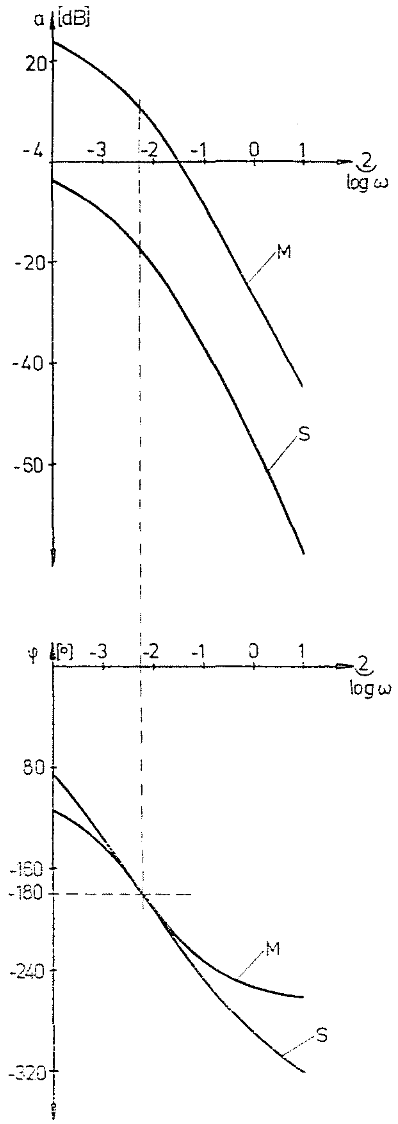


Fig. 3. The transfer functions of M and S are the same as in Fig. 1.

plot of the Bode diagram, and has no effect on the phase plot. Changing the time constant T_m, only the horizontal position of the amplitude and phase plots are shifted, but their shape is not modified at all.

Fig. 2/d demonstrates that the slope of the phase plot of such a model structure is approximately constant in the vicinity of $1/T_m$, and its value increases if the damping factor ζ_m decreases.

Each of the three model parameters can be determined in three separated steps.

First, the damping factor ζ_m is selected so that the average slope of the phase function of the system be equal to that of the model in the middle frequency region (Fig. 2/b). Thus the phase plot of the system is approximately parallel to the phase plot of the model in this region. By proper selection of T_m it is, therefore, possible to fit the two phase plots close to each other (Fig. 3).

To determine the third model parameter A_m, a comparison of the amplitude plots of the system and the model, respectively, is needed (Fig. 2/a).

The question arises: in what respect can the system be replaced by such a model. However, the question will not be answered here. A similar modelling procedure will be presented directly in the time domain with the help of the rise function [1], and discussed will be the validity of the procedure.

2. Modelling procedure in the time domain, using the simplest model

In [1] the first-order rise function of a linear single-input — single-output system is defined as

$$\alpha(t) = \frac{t \cdot \mathrm{d}v/\mathrm{d}t}{v},$$

where $v = v(t)$ is the step response of the system.

Suppose that the first-order rise function $\alpha_s(t)$ of a system is given. For this system we try to choose a model having a transfer function according to Eq. (1). The modelling is based upon the fitting of the rise function $\alpha_m(t)$ of the model to that of the system $\alpha_s(t)$ in a suitable time domain.

In Fig. 4 the rise functions of the model are plotted. Two properties of the rise functions are to be observed:

1° The value $\alpha_m(t) = 2$ is reached approximately at $t \approx 2.6\ T_m$.

2° Using the logarithmic scale for the time variable, the middle portions of the rise functions are approximately linear and their slope increases when the damping factor ζ_m decreases.

Fig. 5 demonstrates how the average slope of the middle portion of the rise function m_m depends on the damping factor ζ_m. m_m is the slope of the

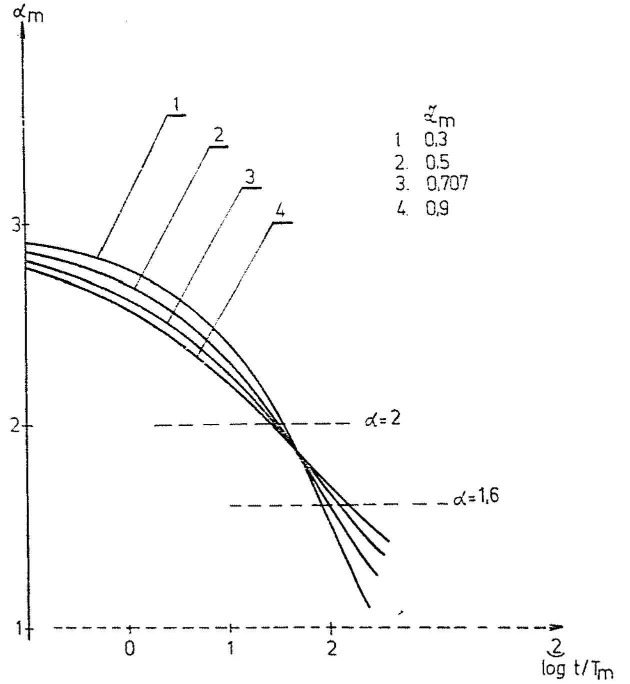


Fig. 4. $W_m(s) = \dfrac{A_m}{s(1 + 2\zeta_m T_m s + T_m^2 s^2)}$

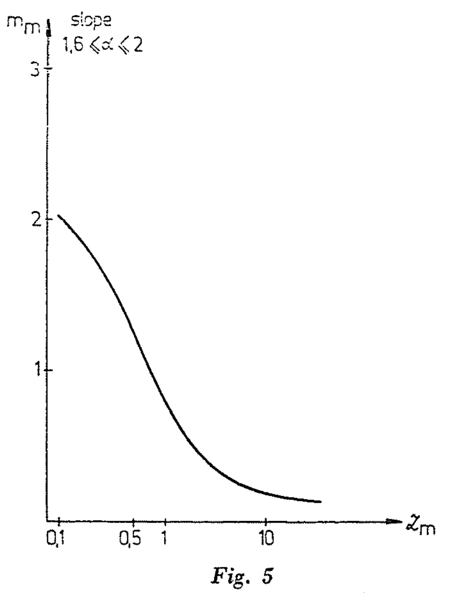


Fig. 5

chord between the values $\alpha = 1.6$ and $\alpha = 2$. These values have proved to be adequate for modelling a wide class of control loops.

Remark the main advantage of the model according to Eq. (1): namely m depends only on ζ_m. If T_m changes, then the plot of the rise function is only shifted horizontally without changing its shape.

Let $t_{(x)}$ denote that moment in which the value of the first-order rise function is exactly $\alpha(t_{(x)}) = x$. It is obvious that the ratio $t_{(x)}/T_m$ depends only on ζ_m.

For instance, the relation

$$\frac{t_{(1,8)}}{T_m} = f(\zeta_m) \tag{2}$$

is demonstrated in Fig. 6. If the functions

$$m_m = m_m(\xi_m) \tag{3}$$

and

$$\frac{t_{(1,8)}}{T_m} = f(\zeta_m) \tag{4}$$

are stored numerically in a digital computer, then the two parameters of the model can be computed very fast. For this reason one has to know the middle portion of the rise function of the system to be modelled (Fig. 7). Then the average slope in this region m_s of the rise function (i.e. the slope

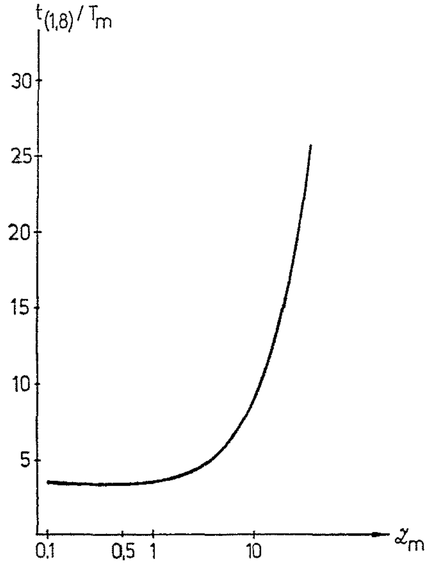


Fig. 6

of the chord intersecting the rise function at the values $\alpha_s = 1.6$ and $\alpha_s = 2$) will be calculated

$$m_s = \frac{\Delta\alpha}{\Delta \ln t} = \frac{0{,}4}{\ln\,[t_{(2)}/t_{(1,6)}]} \tag{5}$$

The damping factor of the model must be chosen so that the average slope of the rise function of the model be equal to that of the system:

$$m_m = m_s$$

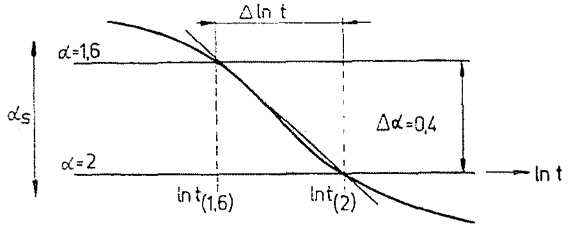


Fig. 7

Thus the damping factor can be determined by the function demonstrated in Fig 5.

The next step is the determination of T_m. According to Eq. (2) and the precalculated function $f(\zeta_m)$ (see Fig. 6)

$$T_m = \frac{t_{(1,8)}}{f(\zeta_m)} \tag{6}$$

This procedure fits the rise function of the system to that of the model in the interval where both rise functions satisfy the inequality

$$2 \geq \alpha(t) \geq 1.6$$

Remember the following property of the rise function (Ref. [1])

$$\alpha = \frac{\mathrm{d} \ln v(t)}{\mathrm{d} \ln t}$$

where $v(t)$ is the step response. Consequently

$$\frac{\mathrm{d} \ln v_m(t)}{\mathrm{d} \ln t} \approx \frac{\mathrm{d} \ln v_s(t)}{\mathrm{d} \ln t}\,; \quad t_{(1,6)} \geq t \geq t_{(2)} \tag{7}$$

where v_m and v_s are the step responses of the model and of the system, respectively.

By integrating Eq. (7) from $t_{(1,6)}$ to $t_{(2)}$ with respect to $\ln t$ we get

$$\ln v_m(t) - \ln v_m(t_{(1,6)}) \approx \ln v_s(t) - \ln v_s(t_{(1,6)}) + c$$

Thus

$$v_s(t) \approx k v_m(t) \tag{9}$$

where

$$k = \frac{v_s(t_{(1,6)})}{e^c v_m(t_{(1,6)})}$$

Eq. (9) expresses the fact that the step response of the system differs from that of the model only by a constant factor. Therefore, by appropriate choice of the model gain A_m, the step response of the model can approximately be fitted to the step response of the system. In order to do this, a single value of the step response of the model has to be made equal to that of the system. Tests have shown that setting

$$v_m(t_{(1,8)}) = v_s(t_{(1,8)}) \tag{10}$$

gives good results.

It is therefore suitable to precalculate the values of $v_m(t_{(1,8)})$. But by fixed ζ_m every value of v_m is proportional to the gain A_m, and thus the ratio $v_m(t_{1,8)})/A_m$ depends only on ζ_m. Making use of the precalculated function

$$\frac{v_m(t_{(1,8)})}{A_m} = g(\zeta_m) \tag{11}$$

which is plotted in Fig. 8, we can find the gain A_m:

$$A_m = \frac{g(\zeta_m)}{v_m(t_{(1,8)})} = \frac{g(\zeta_m)}{v_s(t_{(1,8)})} \tag{12}$$

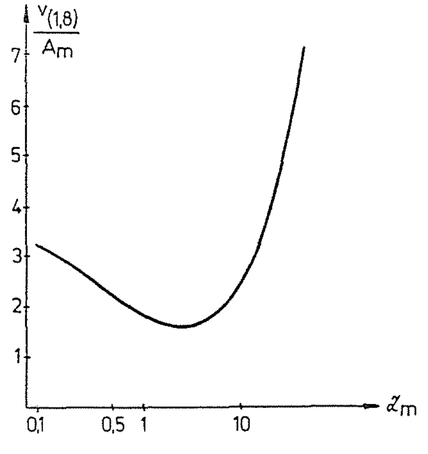


Fig. 8

Let us summarize the modelling procedure:

Three data of the system to be modelled are needed

1° the average slope of its rise function m_s in the interval $1.6 \leq \alpha_s(t) \leq 2$,

2° the time $t_{(1\cdot 8)}$ at which the value of the step response of the system is $\alpha_s = 1.8$,

3° the value of the step response of the system at this moment $v_{s(1,8)} = v_s(t_{(1,8)})$.

The three constants (ζ_m, T_m, A_m) of the model of the structure according to Eq. (1) can be obtained in three separated steps. In each step one of the three precalculated functions demonstrated in Figs 5, 6 and 8 is needed.

1° Using Eq. (3) and setting $m_m = m_s$, the damping factor ζ_m can be obtained (Fig. 5).

2° Next, T_m will be calculated by Eq. (6) (Fig. 6).

3° Finally, the gain A_m will be obtained from Eq. (12) (Fig. 8).

One advantage of this modelling procedure is, that only that part of the rise function of the model is needed for which $\alpha_s > 1.6$.

Of course, the time function of the model does not substitute for the end part of the transient of the system. Nevertheless, tests have shown that selecting the simple model structure according to Eq. (1) and fitting its rise function and step response, in the time domain where $2 \geq \alpha(t) \geq 1.6$ holds, an adequate model can be constructed for a wide class of system structures.

For other systems a more general model structure is needed and one has to fit the rise functions through a wider time interval. However, the constants of the model can be determined by a similar procedure. The use of a more general model will be discussed in Sec. 4.

3. The validity of the simplest model

It is to be noted that the signal transfer properties of the model differ from those of the system. It is important to choose some numerical "figure of merit" characterizing the goodness of the model.

The main object of the modelling procedure is to analyze and synthetize the transient of the closed loop containing the system. In this respect the estimation of the model could be based upon the comparison of the transient behaviour of the closed loops built up from the model and system, respectively.

The comparison can be done by several usual criteria (maximum overshoot, settling time, various integral criteria, etc.). These criteria are not equivalent.

Therefore, as a first step, we shall use a fairly simple method to estimate the "goodness" of modelling: it is evident that by changing the gain of the system, the gain of its model changes proportionally. Let the gain of the system be set so that the closed loop of the system be on the limit of stability (this is the "critical gain" of the closed loop system). Of course, the model of this system will not be on the limit of stability, except if the structure of the system is exactly the same as the structure of the model.

In general, the gain of the model has to be multiplied by a factor c to get the closed loop on the stability limit. Obviously, it is a necessary (but not sufficient) condition of the goodness of a model that the value of c be near unity.

Therefore, c is a possible estimate of the goodness of the modelling. To test the modelling procedure, a general structure of systems has been modelled. The transfer function of the system was

$$W_s(s) = \frac{K_{kr} \prod_{k=0}^{2} (1 + s\tau_k)}{s^i \prod_{j=1}^{5} (1 + sT_j)} \qquad i = 0 \text{ or } 1 \qquad (13/a)$$

One pair of the time constants T_j might be conjugate complex.
In this case

$$W_s(s) = \frac{K_{kr} \prod_{k=0}^{2} (1 + s\tau_k)}{s^i(1 + 2\zeta Ts + T^2s^2) \prod_{j=1}^{3} (1 + sT_j)} \qquad (13/b)$$

The constants of these systems have been varied widely. The gain was set to its critical value, by which the closed loop of the system was on the limit of stability. Approximately 300 tests were carried out with various structures and system constants.

The parameters of their model of structure

$$W_m(s) = \frac{A_m}{s(1 + 2\zeta_m T_m s + T_m^2 s^2)}$$

were evaluated. In addition, the value of c was determined.

The data of some tests are tabulated in Table 1 for demonstration. As a first guess we exclude from the models considered "good" those for which the inequality

$$0.8 \leq c \leq 1.2 \qquad (14)$$

does not hold.

This means that we do not accept a model of a system being on the limit of stability if the gain of the model differs from its own critical value by more than 20%.

Table I

K_{kr}	T_1	T_2	T_3	T_4	T_5	τ_1	τ_2	i	A_m	ζ_m	T_m	A_{mkr}	$m_m = m_s$	$C = \frac{A_{mkr}}{A_m}$
36.13	1	1	16	—	—	—	—	0	1.81	0.86	0.92	1.87	0.79	1.03
4	1	1	1	1	—	—	—	0	0.58	0.20	0.95	0.42	1.76	0.71+
13.93	1	1	2	16	—	—	—	0	0.53	0.49	2.01	0.49	1.20	0.92
2.89	1	1	1	1	1	—	—	0	0.30	0.01	0.46	0.04	2.42	0.15+
12.19	1	1	1	16	16	—	—	0	0.37	0.60	4.80	0.25	1.04	0.99
0.44	1	4	8	8	16	—	—	0	0.07	0.19	8.15	0.05	1.79	0.65+
0.15	1	2	8	8	—	—	—	1	0.14	0.73	11.04	0.13	0.90	0.93
0.10	1	2	4	8	8	—	—	1	0.09	0.62	14.28	0.09	1.02	0.91
6.63	1	1	1	1	—	0.5	—	0	1.37	0.53	0.67	1.58	1.14	1.16
9.45	1	1	1	1	4	0.5	2	0	1.35	0.59	0.78	1.51	1.06	1.12
0.49	1	1	1	1	1	0.5	—	0	0.42	0.53	2.89	0.37	1.13	0.86

K_{kr}	T_1	T_2	T_3	T		τ_1	τ_2	i	A_m	ζ_m	T_m	A_{mkr}	$m_m = m_s$	$C = \frac{A_{mkr}}{A_m}$
20.70	8	16	—	1	0.5	—	—	0	0.62	0.74	2.48	0.60	0.89	0.97
1.96	8	8	—	16	0.5	—	—	0	0.03	0.09	11.40	2.02	2.07	0.49+
11.62	8	8	—	1	0.71	—	—	0	0.47	0.54	2.42	0.45	1.12	0.95
0.42	2	4	—	1	0.5	—	—	1	0.39	0.73	3.98	0.37	0.90	0.95
0.065	1	4	—	16	0.71	—	—	1	0.058	0.51	20.13	0.050	1.17	0.87

Of course, further investigations are needed to decide whether these models are really "good". Putting this investigation off, we presume that models satisfying condition (14) are "good". But this condition has no practical sense, because the critical gain of the system to be modelled is not known, and thus the value of c cannot be determined.

Since the modelling procedure is done in the time domain, practical significance comes to such a typical performance index which can be determined directly from some time function of the system. The average slope m_s of the rise function of the system in the region $1.6 \leq \alpha_s \leq 2$ (which is given in Eq. (5)) was found to be such an appropriate index. Remember that the damping factor ζ_m of the model has been determined from m_s with the aid of the precalculated function given in Fig. 5.

In Table 1 the values of m_s are tabulated, too. On the strength of the tests it has been recognized that the value of c generally satisfies inequality (14) if the slope does not exceed 1.2—1.25:

$$m_s = \frac{\Delta\alpha}{\Delta \ln t} < 1.2 \div 1.25 \tag{15}$$

Since the existence of the above condition can be established easily from the time function of the system, the applicability of a model of a structure according to Eq. (1) will be decided through the value of m_s.

The goodness of the modelling procedure was proved by the following consideration: Three term (PID) controllers were selected to various systems, and the transient response of the closed loop was plotted by an analogue computer. The constants of the controller were set to make the response to a step change of the reference input satisfactory (in our case the rise time should be a minimum, and the first overshoot 10%). The same controller was applied in the closed control loop of the model, and its transient response was compared with that of the system.

Some examples are shown in Figs 9—16. On diagram *a)* the open loop step responses of the system (S) and model (M) are plotted. The gain of the system was set on its critical value. On diagram *b)* three curves are plotted. S and M denote the step response of the closed loop of the system and that of the model, respectively. *C* is the step response of the controller used to compensate the control loop of the system as well as that of the model.

From these plots it can be concluded that the initial parts of the transients, including the first half period, run close to each other. Of course, the end part of the transients may differ, mainly if the system is a proportional one (containing no integrating element). This is obvious, because the model itself is integrating.

Let us summarize the main points of our investigations. The time interval where the value of the rise function of a system lies between approximately 2 and 1.6 designates an important part of the step response. Namely, this part has the main influence on the rise time as well as on the first overshoot of the transient of the closed control loop. Thus it is reasonable to term it as the "decisive time interval" of the step response. The decisive time interval is analogous to the middle frequency portion of the frequency response, i.e. this is the portion which influences the above-mentioned features of the transient response of the closed loop. Therefore, if a model is chosen in the way that its step response be close to the step response of the system in the decisive time interval, then the initial part of the transient response of the closed loop can be synthetized by means of this model.

It is to be noted that the model of structure (1) is applicable only if the average slope of the rise function of the system in the decisive time interval is not greater than 1.2—1.25. In this case the damping factor of this most simple model is not less than $\zeta_m = 0.45-0.5$. Else ζ_m would be smaller, but a strongly oscillating model cannot stand for a system having no oscillating properties. Thus, in this case a more general structure is needed. If the model of structure (1) is applicable, then the proportional gain and derivative (rate) time of the control loop can be determined with its aid. However, this model cannot be used for setting the integration time.

A method of setting the controller parameters and the extension of the model structure, when the simplest one — according to Eq. (1) — is not applicable, will be discussed later.

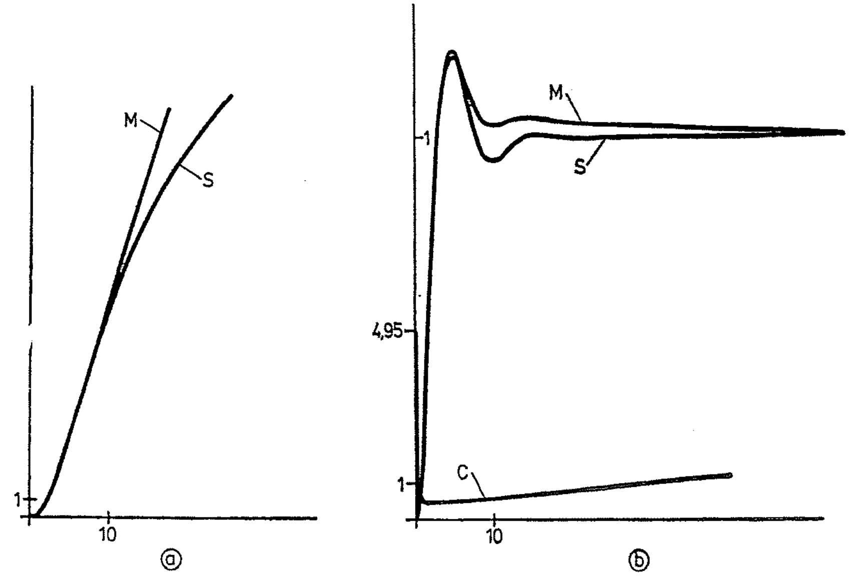


Fig. 9. $W_m = \dfrac{1.32}{s(1 + 2 \cdot 0.86 \cdot 1.25s + 1.25^2 s^2)}$;

$$W_s = \frac{28.69}{(1 + s)(1 + 2s)(1 + 16s)};$$

$$W_c = 0.45 \left(1 + \frac{1}{25s} + \frac{s \cdot 1}{1 + 0.1s}\right)$$

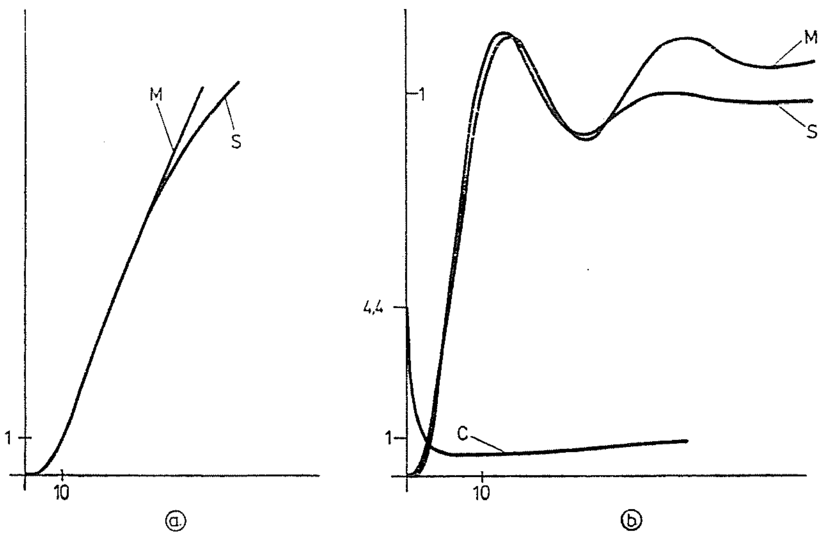


Fig. 10. $W_m = \dfrac{1.77}{s(1 + 2 \cdot 0.7 \cdot 0.72s + 0.72^2 s^2)}$;

$$W_s = \frac{19.61(1 + 4s)(1 + 12s)}{(1 + s)(1 + s)(1 + 2s)(1 + 8s)(1 + 16a)};$$

$$W_c = 0.55 \left(1 + \frac{1}{8s} + \frac{s \cdot 1.5}{1 + 0.15s}\right)$$

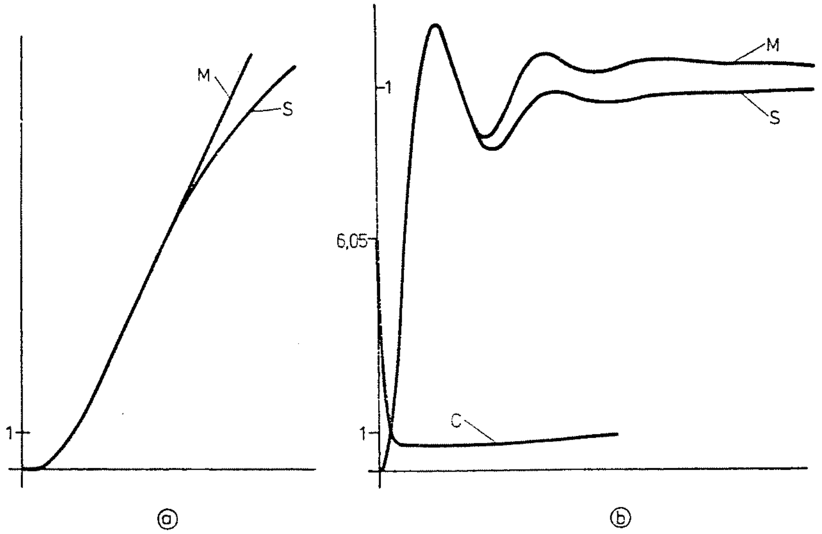


Fig. 11. $$W_m = \frac{2.01}{s(1 + 2 \cdot 0.68 \cdot 0.62s + 0.62^2 s^2)};$$
$$W_s = \frac{15.23(1 + 2s)(1 + 6s)}{(1 + s)(1 + s)(1 + s)(1 + 4s)(1 + 8s)};$$
$$W_c = 0.55\left(1 + \frac{1}{9s} + \frac{s \cdot 1.25}{1 + 0.125s}\right)$$

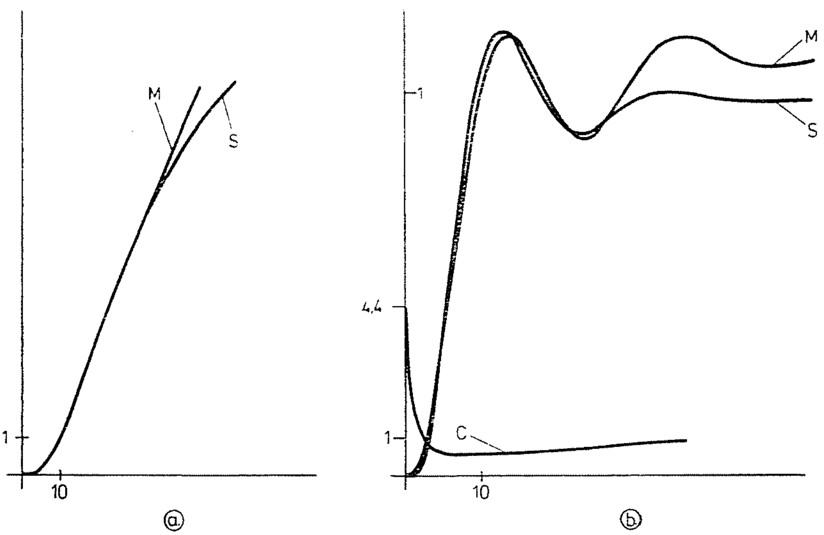
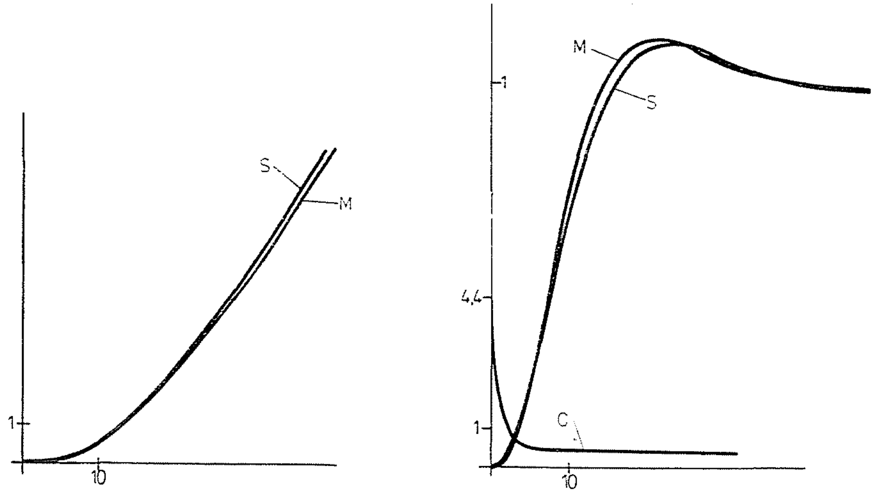


Fig. 12. $$W_m = \frac{0.25}{s(1 + 2 \cdot 0.6 \cdot 4.8s + 4.8^2 s^2)};$$
$$W_s = \frac{12.19}{(1 + s)(1 + s)(1 + s)(1 + 16s)(1 + 16s)};$$
$$W_c = 0.4\left(1 + \frac{1}{25s} + \frac{s \cdot 14}{1 + 1.4s}\right)$$

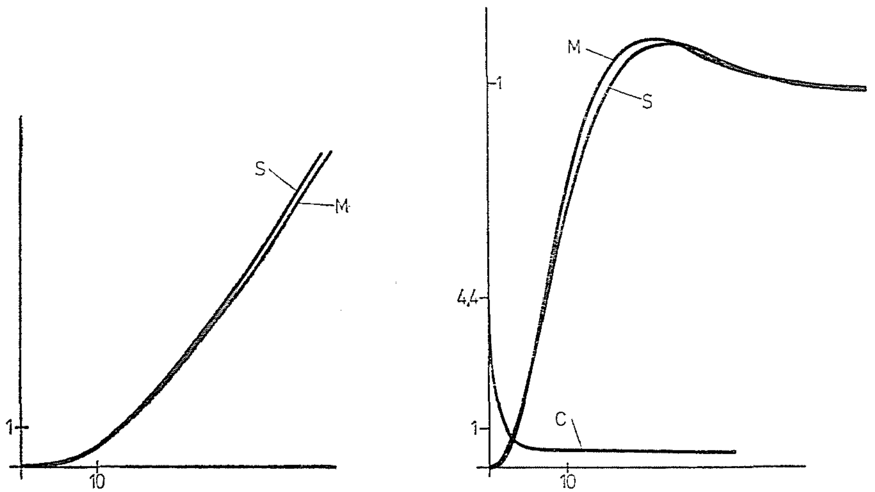


a b

Fig. 13. $W_m = \dfrac{0.36}{s(1 + 2 \cdot 1.28 \cdot 7.2\, s + 7.2^2 s^2)}$;

$$W_s = \frac{0.37}{s(1 + s)(1 + 2s)(1 + 16s)};$$

$$W_c = 0.4\left(1 + \frac{s \cdot 12}{1 + 1.2s}\right)$$



a b

Fig. 14. $W_m = \dfrac{0.82}{s(1 + 2 \cdot 0.67 \cdot 1.8s + 1.8^2 s^2)}$;

$$W_s = \frac{0.89}{s(1+s)(1 + s)(1 + s)};$$

$$W_c = 0.4\left(1 + \frac{s \cdot 1.33}{s + 0.133s}\right)$$

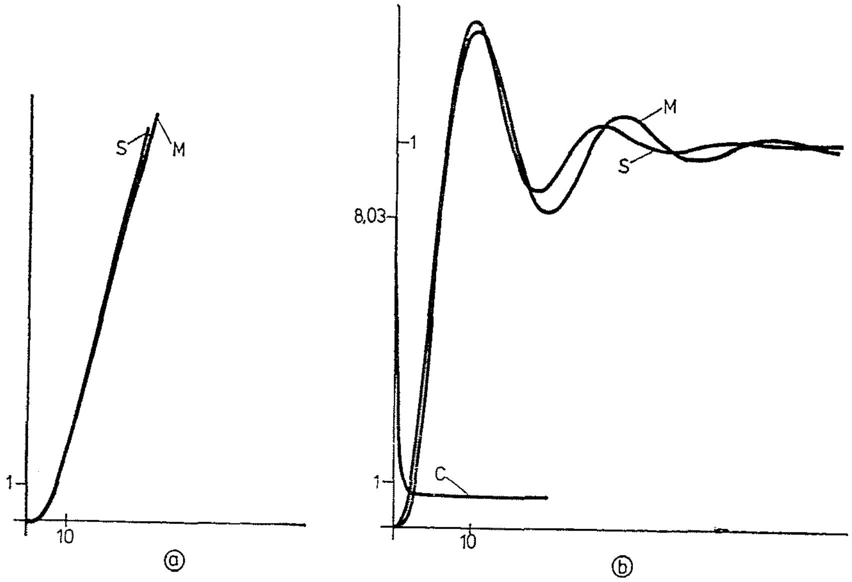


Fig. 15. $W_m = \frac{0.38}{s(1 + 2 \cdot 0.71 \cdot 4.01s + 4.01^2 s^2)}$;

$$W_s = \frac{0.42}{s(1+s)(1+s)(1+s)(1+4s)};$$

$$W_c = 0.73\left(1 + \frac{s \cdot 4.3}{1 + 0.43s}\right)$$

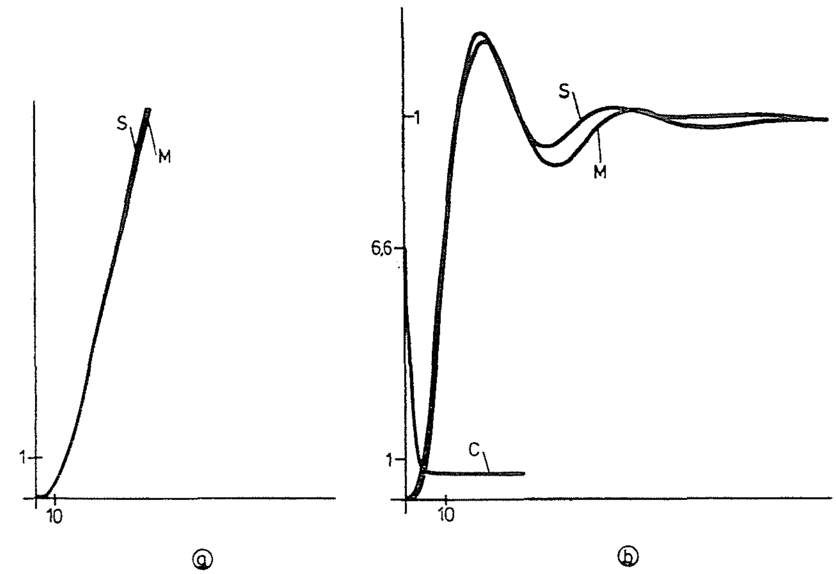


Fig. 16. $W_m = \frac{0.21}{s(1 + 2 \cdot 0.78 \cdot 7.79s + 7.79^2 s^2)}$;

$$W_s = \frac{0.22}{s(1+s)(1+s)(1+4s)(1+8s)}; \qquad W_c = 0.6\left(1 + \frac{s \cdot 10}{1 + 1 \cdot s}\right)$$

4. A more general model

If inequality (15) does not hold, then the time delay of the system is of such a character that the control loop cannot easily be compensated with a PID controller. Therefore, a more general model is needed. In this case the following model structure turned out to be appropriate:

$$W_m(s) = \frac{A_m}{s(1 + s\beta T_m)(1 + 2\zeta_m T_m s + T_m^2 s^2)} \tag{16}$$

The rise function of such a model can be steep enough without choosing a very small damping factor.

The shape of the rise function is influenced by two parameters (β and ζ_m). By changing T_m, only the position of the rise function is shifted horizontally. The parameters of this model can be determined by the same method as that used before.

The only difference is that the shape of the rise function of the simpler model depends only on the single parameter ζ_m. Thus, the average slope of the rise function of the model in the domain $2 \geq \alpha \geq 1.6$ could be related to ζ_m by a single variable function (Fig. 4).

To determine both parameters (ζ_m and β) of the more general model, two slope values are required. Therefore, instead of a single variable function, a vector-vector function

$$\overline{m} = \overline{m}(\overline{p}) \tag{17}$$

is to be precalculated, where

$$\overline{p} = (\beta, \zeta_m) \tag{18}$$

is the parameter vector, and

$$\overline{m} = (m_1, m_2) \tag{19}$$

the vector of two average slopes.

From numerical investigations it appeared advisable to choose

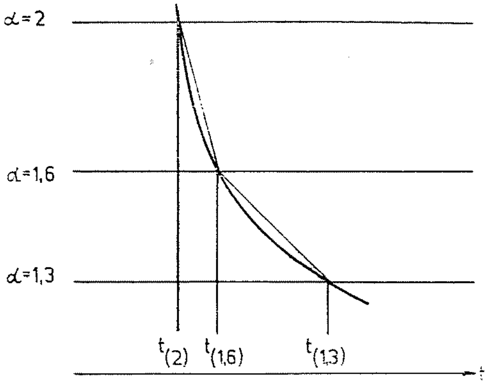
$$m_1 = \frac{0{,}4}{\ln\left[t_{(2)}/t_{(1,6)}\right]} \tag{20}$$

and

$$m_2 = \frac{0{,}3}{\ln\left[t_{(1,6)}/t_{(1,3)}\right]} \tag{21}$$

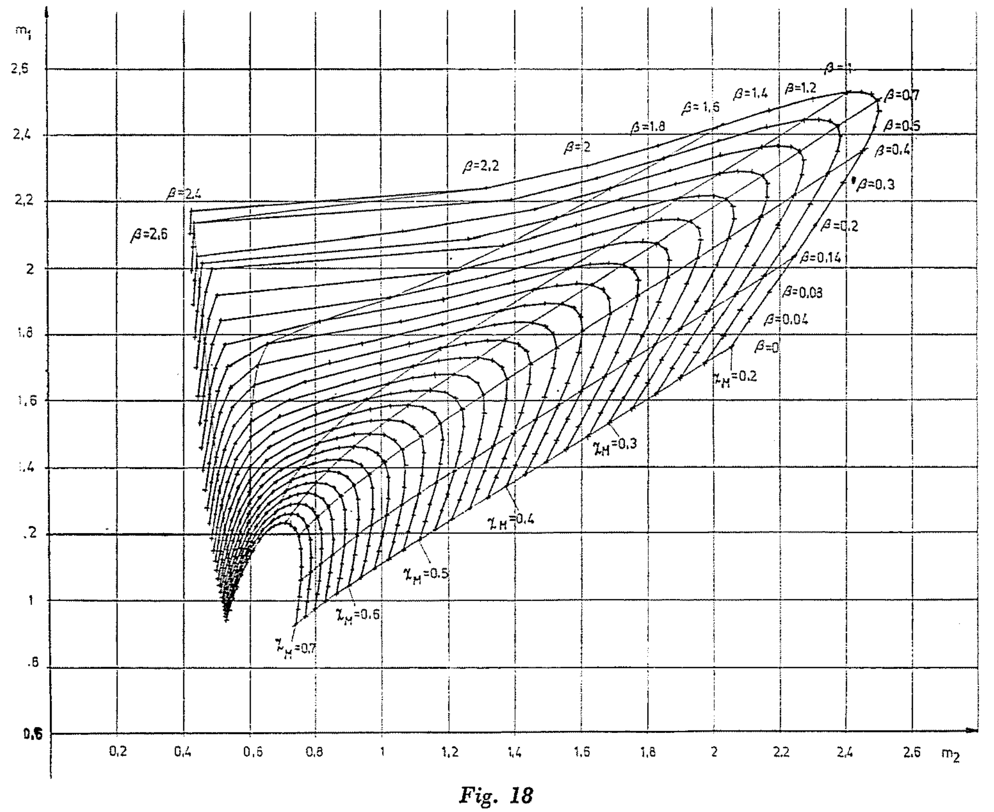
Thus m_1 and m_2 are the chords of the rise function between the values $\alpha = 2 \ldots 1.6$ and $\alpha = 1.6 \ldots 1.3$, respectively (Fig. 17).

The vector-vector function (18) has been precalculated and tabulated. The parameters have been varied between the limits $0.2 \leq \zeta_m \leq 0.7$ and



α = 2
α=1,6
α=1,3
t(2)
t(1,6)
t(1,3)
t

Fig. 17



m1
m2
β=1
β=1,2
β=1,4
β=1,6
β=1,8
β=2
β=2,2
β=2,4
β=2,6
β=0,7
β=0,5
β=0,4
β=0,3
β=0,2
β=0,14
β=0,08
β=0,04
β=0
τM=0,2
τM=0,3
τM=0,4
τM=0,5
τM=0,6
τM=0,7

Fig. 18

$0 \leq \beta \leq 2.6$. The function $\overline{m} = \overline{m}(\overline{p})$ is presented in Fig. 18. The precalculated points are connected by straight lines.

With the help of the precalculated function an individual parameter vector (β, ζ_m) can be related to every value of the vector (m_1, m_2) of the average slopes. Only a linear interpolation is needed on the basis of three precalculated values of the function lying in the environment of this vector.

The steps of the modelling procedure are as follows:

i) Measuring or calculating the rise function of the system, the average slopes m_1 and m_2 according to Fig. 17, have to be determined.

ii) Regarding m_1 and m_2 as the corresponding characteristics of the model itself, from the precalculated and tabulated function $\overline{m} = \overline{m}(\overline{p})$, the parameters β and ζ_m of the model are calculated, using linear interpolation.

iii) It is evident that for any α the ratio $t_{(\alpha)}/T_m$ depends only on β and ζ_m. (Remember that $t_{(\alpha)}$ was previously defined by the following relation: $\alpha = \alpha(t_{(\alpha)})$. Selecting for α an appropriate value — in our case $\alpha = 1.6$ turned out to be such — the function

$$\frac{t_{(1,6)}}{T_m} = f_1(\beta, \zeta_m) \tag{22}$$

has been precalculated. Then we have to determine $t_{(1,6)}$ from the rise function of the system and substitute it into Eq. (22). Thus

$$T_m = \frac{t_{(1,6)}}{f_1(\beta, \zeta_m)} \tag{23}$$

This results that the point $\alpha = 1.6$ of the rise function curve of the system coincides with that of the model. But in the previous steps the slopes m_1 and m_2 of the rise function of the system and the model, respectively, have been made equal. The chords having the slope m_1 and m_2 intersect each other just at the point $\alpha = 1.6$ (Fig. 17). Therefore, the points $\alpha = 1.3$ and $\alpha = 2$ of the rise functions of the system and the model coincide as well. The coincidence of three points of the rise function makes it certain that the rise function of the model fits to that of the system in a wider region.

iv) In the above region the step response of the model having an arbitrary gain A_m differs from the step response of the system by a constant factor:

$$v_m \approx c v_3 \qquad t_{(2)} \leq t \leq t_{(1,3)} \tag{24}$$

A_m is to be determined so that $c = 1$. Taking into consideration that the ratio $v_m(t_{(\alpha)})/A_m$ depends only on β and ζ_m, it is reasonable to precalculate and tabulate the function.

$$\frac{v_m(t_{(\alpha)})}{A_m} = \varphi_{(\alpha)}(\beta, \zeta_m) \tag{25}$$

Setting $v_3(t_{(\alpha)}) = v_m(t_{(\alpha)})\ A_m$ can be calculated from this function.

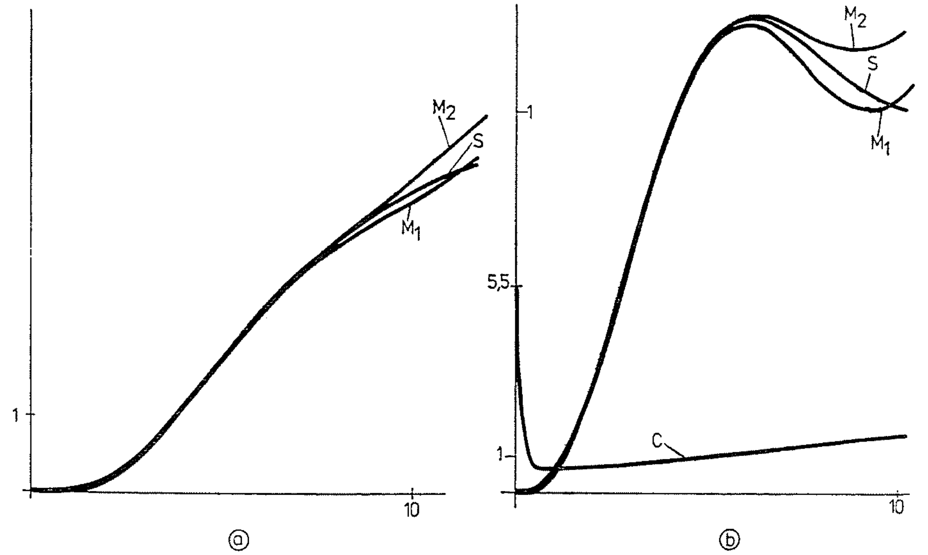


Fig. 19. $W_{m1} = \dfrac{0.39}{s(1 + 2 \cdot 0.18 \cdot 1.48s + 1.48^2s^2)}$;

$$W_{m2} = \frac{0.46}{s(1 + 0.27s)(1 + 2 \cdot 0.31 \cdot 1.33s + 1.33^2s^2)};$$

$$W_s = \frac{5.02}{(1 + 2s)(1 + 4s)(1 + 2 \cdot 0.7s + s^2)};$$

$$W_c = 0.5\left(1 + \frac{1}{4.55s} + \frac{s \cdot 1.45}{1 + 0.14s}\right)$$

The goodness of this model can be tested by the method used previously. Table 2 presents some illustrative examples. Systems have been modelled by the model-structure according to Eq. (16). The slope m_1 defined by Eq. (20) does not satisfy inequality (15), therefore the simplest model structure according to Eq. (1) cannot be applied. However, for a comparison, the factors c_1 related to this improper model have been calculated. It can be seen that their values are far from unity. Using the more general model structure according to Eq. (16), a much better figure of merit (expressed by the factor c_2) has been achieved. This model gives good results even for those systems which are difficult to compensate (for instance, a system having five equal poles). Among the examples there are two systems having the same

Table 2

$W_E(s)$	m_1	A_m	A_{mKE}	T_m	β	ζ_m	c_1	c_2
$\frac{2{,}885}{(1+s)^5}$	2.42	0.391	0.397	1.072	0.597	0.209	0.15	1.013
$\frac{4.052}{(1+s)^4(1+4s)}$	2.08	0.354	0.364	1.46	0.637	0.289	0.35	1.028
$\frac{10{,}698}{(1+s)^4(1+16s)}$	1.52	0.423	0.419	1.868	0.538	0.466	0.75	0.991
$\frac{5{,}784}{(1+s)^3(1+4s)(1+16s)}$	1.65	0.268	0.265	2.55	0.479	0.376	0.73	0.988
$\frac{4.5}{(1+s+s^2)(1+2s)(1+4s)}$	2.13	0.451	0.423	1.189	0.454	0.271	0.33	0.938
$\frac{5.021}{(1+1.41s+s^2)(1+2s)(1+4s)}$	1.79	0.463	0.439	1.382	0.27	0.319	0.63	0.948
$\frac{0.75}{s(1+s+s^2)(1+s)}$	1.58	0.75	0.75	1.003	1.051	0.5	0.63	1.0
$\frac{0.21}{s(1+4s+16s^2)(1+s)}$	1.44	0.212	0.216	3.944	1.073	0.515	0.79	1.017
$\frac{0.186}{s(1+4s+16s)^2(1+s)^2}$	1.67	0.188	0.192	3.786	2.746	0.46	0.61	1.021

structure as the model (Nos 7 and 8). It is remarkable that the method reproduces the system parameters as the corresponding parameters of the model with high accuracy.

In Fig. 19 the step responses of a system and those of their models are compared. S denotes the step response of the system M_1, and M_2 those of the models according to Eq. (1) and Eq. (16), respectively. It can be recognized that the M_2-s fit better to S. Of course, one may choose other model structures, too, but the modelling procedure can be done in a similar manner.

Summary

A modelling procedure is presented for linear single input-single output systems. The modelling is based on the step response of the system, which is expressed by the "rise function", a new function in the time domain containing the full information of the system dynamics.

The model has a fairly simple structure. The modelling is carried out by fitting the rise function of the system to that of the model in an appropriate time domain, which has the greatest influence on the transient response of the closed control loop.

For testing the procedure, a comparison is made between the transients of the closed control loops of the systems and those of their models compensated by the same controller.

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