# DETERMINATION OF THE STEADY-STATE TEMPERATURE DISTRIBUTION OF TRANSFORMER WINDINGS BY THE HEAT FLUX NETWORK METHOD

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## Introduction

Our previous study dealt with the determination of the steady-state temperature field of oil-cooled transformers with disc-type winding. The method of distributed parameters was used in the calculations [1] which — with its numerous advantages — has some shortcomings as well. First of all, the compromises in connection with the consideration of additional effects deriving from the stray flux and of the uneven distribution of the heat sources are to be mentioned.

This study will present the application of the calculation with the heat flux network model.

The principle of network methods is based on the substantial analogy between transport processes.

Network models (electrical networks, heat flux networks, mass flow networks) offer a good possibility of simulating the transport processes taking place in complex structural systems.

The heat flux model is constructed by means of "lumping" the thermal system, then characterizing the discrete parts with concentrated parameters and coupling the parts in accordance with their interactions.

The heat flux network model can be succesfully applied to the calculation of the steady-state and transient warming of electrical rotating machines, switching devices and electronic units [2. . .5]. The method will be applied below to the examination of the steady-state warming of oil-cooled transformers with disc-type windings.

## Main factors influencing the temperature field

The internal temperature distribution of coils is affected by numerous factors which can be grouped as:

a) constructional features of the coil (geometry and material properties),

b) thermal boundary conditions of the coil (flow and heat transfer conditions of the oil),

*c)* intensity and distribution of the heat sources inside the coil.

These factors are not independent of each other and of the thermal processes taking place in other parts of the transformer. Accordingly, a sufficiently accurate determination of the temperature distribution in the coil discs and coil columns requires the examination of the whole transformer as a thermal system [6, 7].

Our examinations were restricted to one single coil column. The effects of other parts of the transformer on the coil coumn colnsidered were determined on the basis of earlier investigations [6] and design practice [8].

## Model assumptions

The assumptions applied for the heat flux network model are as follows:

1. The coil discs constituting the coil column consist of circularly symmetrical concentrical rings.
2. The outer and inner circumferences of the coil discs (the length of the individual rings) are practically equal.
3. In the horizontal oil ducts separating the coil discs, the vertical temperature gradient can be neglected (between the individual discs there is practically no heat flux).
4. The thermal resistance of the electrical conducting material in the coil disc is negligible as compared with that of the heat-insulating layer.
5. The heat conduction coefficient of the electrical insulating material is independent of temperature.
6. The thermal and flow conditions of the oil ducts over and below the coil discs are identical.
7. The thermal interaction of oil and coil-disc in the horizontal oil ducts is, based on earlier investigations [6], considered as turbulent heat conduction.

   The steady-state conduction heat flux in the horizontal oil ducts was calculated by means of the relationship

$$\Phi_h = A_V \varepsilon \lambda \frac{\Delta t}{\delta} \tag{1}$$

where

$\Phi_h$ = conduction heat flux

$A_V$ = the conduction areas perpendiuclar to the heat flux

$\varepsilon$ = turbulence factor of the heat conduction coefficient

$\lambda$ = heat conduction coefficient of the oil in rest,
$\delta$ = conduction length in the direction of the heat flux
$\Delta t$ = temperature difference along the conduction length.

8. With given layer thickness and disc width, the turbulence factor is dependent on the mean surface heat flux density alone [6].
9. The flow paths of turbulent heat conduction are considered identical with the oil sections corresponding in size to the position of the disc turns (Fig. 1).
10. The coil disc is "lumped" by turn pairs and each turn pair is described with concentrated characteristics assuming two-directional heat conduction.

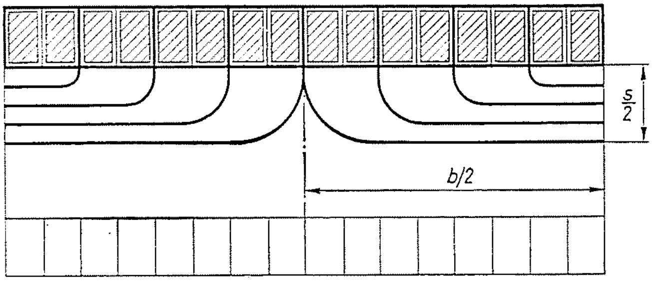


*Fig. 1.* Block scheme of the coil disc, paths of turbulent heat conduction (b: disc width, s: thickness of the horizontal oil duct)

## The heat flux network model

Thermal resistances ($R_f + R_v$) corresponding to the flow paths of the horizontal oil ducts between the coil discs are determined on the basis of the model assumptions 6...9 (and Fig. 1) coupled to the surfaces of the turn pairs (Fig. 2). The points marked with 0 in the Figure are the connecting points of horizontal and vertical oil ducts, while the points marked 9...16 correspond to the lower surfaces of the turn pairs.

It is in these points that the horizontal and vertical thermal resistances $R_v$ and $R_z$ are joined (Fig. 3). The network, as follows from the foregoing, is symmetrical about the points marked A. This condition permits to simplify the network in the well-known manner.

Points A correspond to the conductors of the turn pairs with the assumed identical temperature. It is in these points that the source flux generators deriving from the internal heat source and considered concentrated, are acting. Their fluxes are marked $\Phi_1 \ldots \Phi_8$.

The heat fluxes of the sources are not of equal value. Their differences are due to the local deviations of the stray flux and to the temperature dependence of the D.C. resistance of the conductors.

The heat flux network of the coil discs according to Fig. 3 can be reduced to the network in Fig. 4.

The whole heat flux dissipated in the coil disc passes from the network into the oil ducts of temperature $t_j$ and $t_b$, resp. (voltage generators attached to the point marked 0).

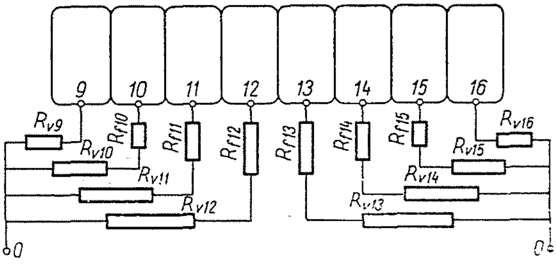


*Fig. 2.* Resistance network representing the mixed (turbulent) heat conduction of the horizontal oil duct

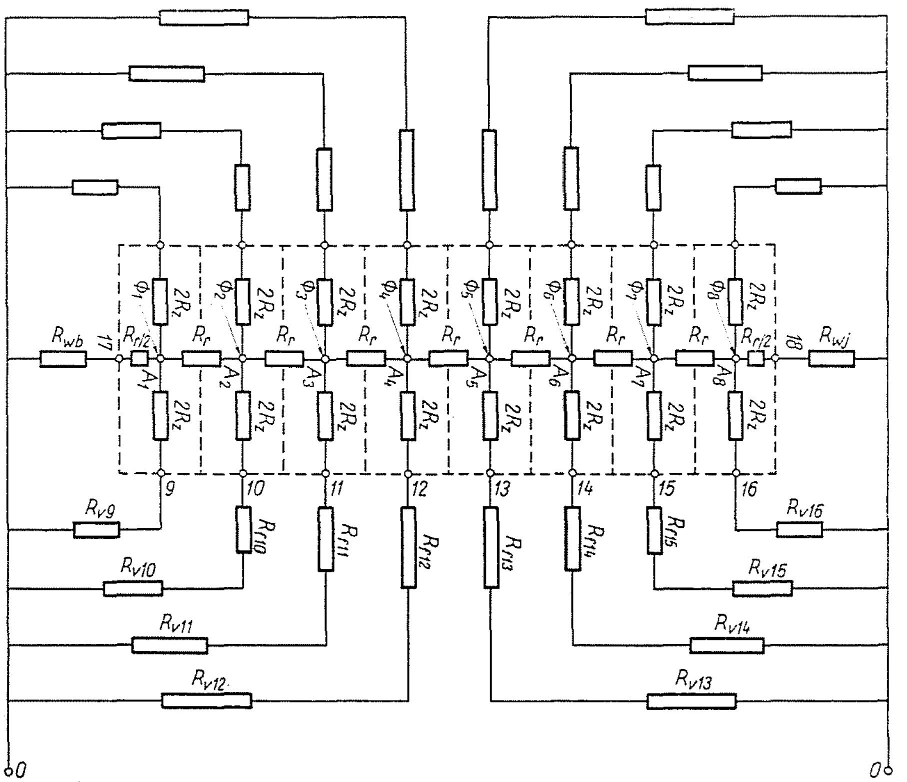


*Fig. 3.* Heat flux network model of a coil disc for steady state ($\Phi$: source heat fluxes)

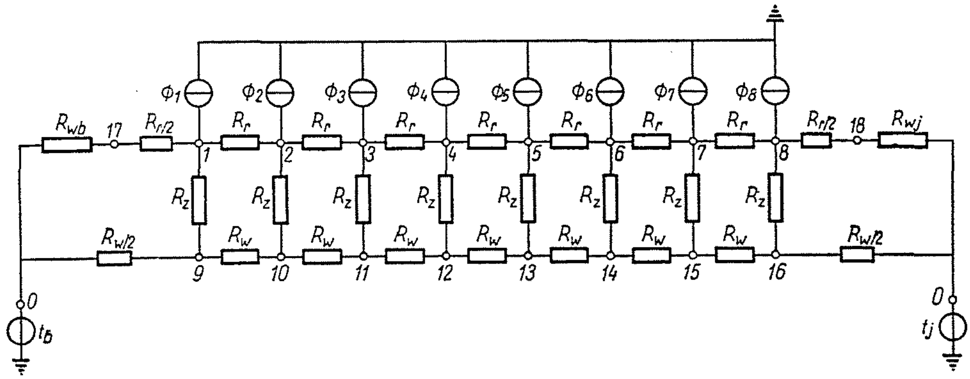


*Fig. 4.* Reduced heat flux network model of a coil disc for steady state

The thermal resistances constituting the network are interpreted as follows:

$R_r$: resistance to the radial heat conduction between two adjacent turn pairs of the disc coil (for the two outer turn pairs a resistance $R_r/2$ is taken into account).

$R_z$: resultant vertical thermal resistance (between the points $A$ and the horizontal disc surfaces).

$R_w$: resistance to the turbulent heat conduction in the horizontal oil duct (after reduction of the resistance chain shown in Figs 2 and 3). On both edges of the disc coil the value $R_w/2$ has to be taken into account.

$R_{wj}$ and $R_{wb}$: heat convection resistances on the surfaces of the right-side and left-side outer turn pairs of the disc ($R_w = 1/\alpha A_0$).

The heat flux network model of the whole coil column can be constructed by coupling the network models of the particular coil discs. Coupling is accomplished through coupling resistances $R_c$ (Fig. 5).

*Coupling resistances* $R_c$ are essentially formal resistors. They are subject to a temperature change equal to that arising in the oil rising in the lateral channel upon a heat flux arriving from a disc. For instance, in the left-side oil duct, in the height of the $i$-th disc, the mean temperature $t_{bi}$ of the mixed oil (i.e. the "ambient" temperature of the $i$-th disc) is:

$$t_{bi} = t_{b(i-1)} + \frac{(\sum \Phi_b)_{i-1}}{(\Phi_m c_0)_b} \tag{2}$$

where

$(\sum \Phi_b)_{i-1}$ is the full heat flux passing from the $(i$-1)-th coil disc into the left-side oil duct.

$(\Phi_m)_b$ is the mass flow of the oil in the left-side oil duct, and

$c_0$ is the specific heat of oil.

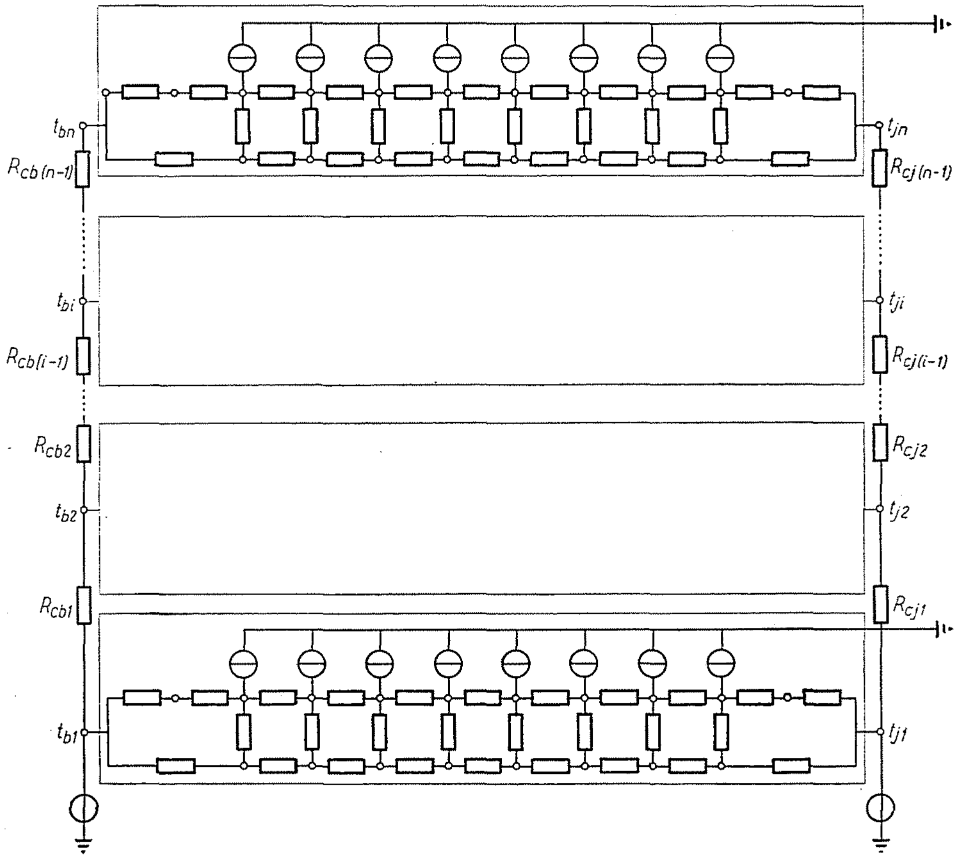


*Fig. 5.* Heat flux network model of a whole coil column

It can be concluded from Eq. (2), that the role of the formal resistance $R_c$ is fulfilled by the reciprocal of the heat capacity flux $\Phi_c = \Phi_m c_0$ of the oil ($R_c = 1/\Phi_m c_0$).

## Equation system of the heat flux network model

The calculation aims primarily at determining the temperature field of the coil column examined. The calculation of the phenomena of flow and heat transfer taking place in the oil ducts requires knowledge of the distribution of heat flux density on the surface of the coil disc.

The model of the coil column according to Figs 4 and 5 is a linear network, that may be calculated by means of the nodal (Kirchhoff I) and the loop (Kirchhoff II) equations. The source-heat fluxes are treated as branch currents.

Nodal equations are written with the following sign conventions:

a) assumed flux directions: in the horizontal resistances from left to right, in the vertical resistances downwards,

b) the flux flowing into the nodes has a positive sign, that flowing out has a negative sign.

In the equations the subscripts of the branch current correspond to the numerals of the nodes bounding the branches. For the node $i = 1\ldots18$ of the heat flux network of the coil disc (see Fig. 4) there are 18 node equations. For $i$ nodes and for $j = 1\ldots n$ branches in each node:

$$\sum_{j=1}^{n} \Phi_{ij} = 0, \quad i = 1\ldots18. \tag{3}$$

Instead of applying the loop equations mechanically, the conduction equations will be written branch by branch. According to the topology of the network (Fig. 4) there are 28 branch-fluxes to be determined. Generally:

$$\Phi_{\substack{i,k \\ i\neq k}} = \frac{t_i - t_k}{R}, \tag{4}$$

where $i$ and $k$ refer to nodes bounding the branch, $R$ is the heat conduction resistance of the branch ($R_r$, $R_z$ and $R_w$, to the sense).

Substituting the heat fluxes of Eqs (3) into the corresponding Eq (4), then performing the prescribed operations and arranging the sources and the absorbing heat fluxes to the right side of the equation, an inhomogeneous linear equation system will be obtained.

The *source heat fluxes* $\Phi_i (i = 1\ldots8)$ consist of two parts:

$$\Phi_e = \Phi_0 + \Phi_j \tag{5}$$

where the D.C. loss $\Phi_e$ is proportional to the resistance of the current conductor of the coil, and this resistance is nearly linearly proportional to the temperature. Thus it can be written:

$$\Phi_e = \Phi_{e0}[1 + \beta(t - t_0)] \tag{6}$$

where $\beta$ is the temperature coefficient of the conductor, and subscript 0 refers to the reference temperature (e.g. 0 °C).

The eddy current loss $\Phi_j$ caused by stray fluxes is considered to be inversely proportional to the direct current resistance:

$$\Phi_j = \Phi_{j0}[1 - \beta(t - t_0)]. \tag{7}$$

After substitution into (5), based on (6) and (7), the heat flux $\Phi_i$ ($i = 1 \ldots 8$) can be expressed as

$$\Phi_i = \Phi_{ie0}[1 + \beta(t - t_0)] + \Phi_{ij0}[1 - \beta(t - t_0)]. \tag{8}$$

After reducing the inhomogeneous linear equation system in the way that the heat source correction terms come to the left side, it can be written in the form

$$\bar{\Phi}_0 = \bar{\bar{M}} \cdot \bar{t} \tag{9}$$

where

$\bar{\Phi}_0$: formal heat flux vector

$\bar{t}$: formal temperature vector

$\bar{\bar{M}}$: corrected heat conduction matrix.

To produce the temperature distribution in the *whole coil column*, equation system (9) will be solved for each coil disc, and the solutions coupled by means of the coupling resistors $R_c$ described in connection with (2). The process of calculation will be as follows:

1. Taking as reference the temperature of the oil entering from the radiator into the coil space, determine the temperature distribution in the bottom coil disc of the coil column.
2. Knowing the temperature distribution in the bottom disc, determine the value of heat flux density on the lateral surface of the coil disc.
3. Using (2), determine the temperature of the oil in the next coil disc.
4. The procedure will be repeated in compliance with the number of the coil discs.

If in resistor $R_w$ the value of the heat transfer coefficient ($\alpha$) vs. height is considered to be constant, then the corrected heat conduction matrix $\bar{\bar{M}}$ is identical for each disc, and thus the calculation requires the inverse matrix $\bar{\bar{M}}^{-1}$ to be determined only once. For the $n$-th disc:

$$\bar{t}_n = \bar{\bar{M}}^{-1} \cdot \bar{\Phi}_{0n}. \tag{10}$$

If the change of the heat transfer coefficient with the place has to be taken into account, the value of matrix $\bar{\bar{M}}$ will be different for each disc, whereby the computer time increases.

## Applications

The described method of heat flux network can be applied to any coil column of any transformer. Omitting the computer algorithm and program, the calculation results of a 16-turn low-voltage coil column (100 discs) of a 40 MVA transformer is presented for illustration.

Fig. 6 shows the temperature distribution in the bottom and top coil discs of the coil column. In accordance with earlier test results [6] the curves are seen to flatten near the lateral gaps and to exhibit inflexion points.

Fig. 7 shows the distribution of the heat flux density on the surface of the top (hottest) coil disc.

Calculation results were used to examine in detail how the hot spot excess temperature of the disc $(t_h - t_{0IN})$ and the mean excess temperature $(t_{tk} - t_{0IN})$ of the coil disc are influenced by:

a) the insulation spacers reducing the free surface;

b) the value of the coefficient of turbulent heat conduction $\varepsilon$;

c) the lateral heat transfer coefficient $\alpha$.

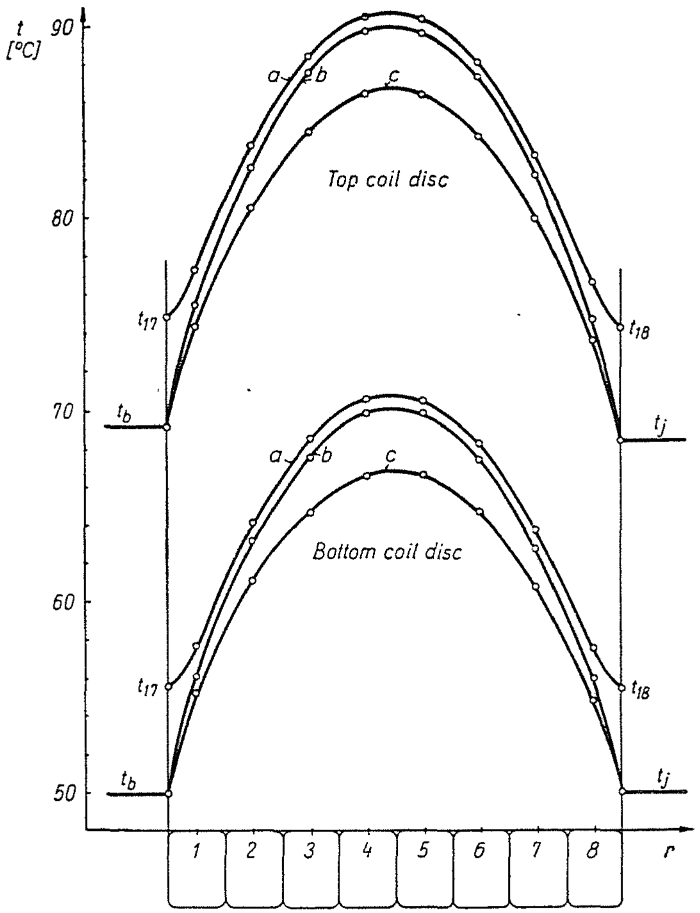


*Fig. 6.* Temperature distribution in the bottom and top coil discs of a coil column, based on the heat flux network model in Figs. 4 and 5. (a: temperature of the copper conductor; b: temperature of the outer surface of the paper insulation; c: mid oil temperature)

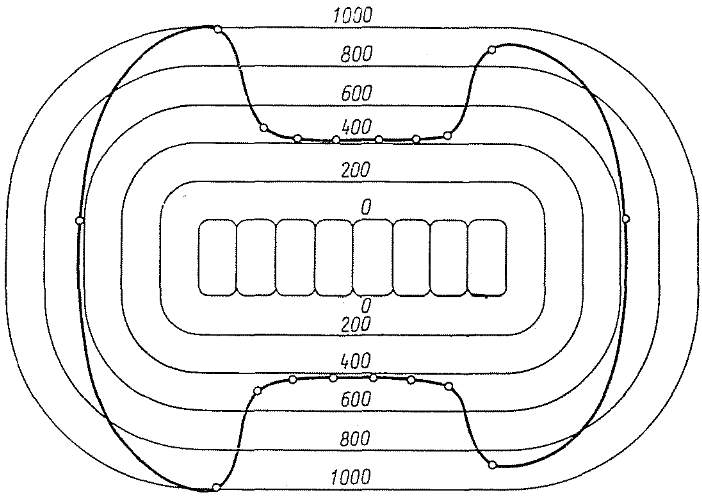


*Fig. 7.* Distribution of heat flux density on the surface of the hottest coil disc

To estimate their effect, the relative transfer coefficient (the change percentage of the calculated characteristic for a change by 1% of a parameter) will be applied. The computation results are contained in Table 1.

| | Excess temperature of the hot spot $t_i - t_{0IN}$ | Mean excess temperature $t_{ik} - t_{0IN}$ |
|---|---|---|
| Free disc surface: F | —0.585 | —0.69 |
| Coefficient of turbulent heat conduction: $\varepsilon$ | —0.42 | —0.51 |
| Heat transfer coefficient on the lateral surface of the disc: $\alpha$ | —0.063 | —0.061 |

Among the parameters under test, the free disc surface is seen from the table to have the strongest effect. Therefore, the effect of the spacers to reduce the heat transfer surface must be determined carefully.

Also the mixed (turbulent) heat conduction coefficient $\varepsilon$ has a considerable effect. Its value can be estimated experimentally [6].

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## Summary

The heat flux network model of the coil columns of oil-cooled transformers with disc-type windings can be constructed by coupling the heat flux network models of the individual coil discs. The equation of the network can be written with consideration of the temperature dependence of the D.C. loss and of the eddy current losses caused by stray flux. Based on the computer solution of the equation system, the temperature field and the distribution of the superficial heat flux density can be determined with an accuracy satisfying the practical requirements.


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