STATE EQUATIONS FOR LINEAR NETWORK MODELS CONTAINING NULLATORS AND NORATORS

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Several circuits are known [1, 2] suitable to model two-ports with given parameters using networks containing nullators, norators, and impedances. These models can be used also in the case of two-ports with extreme parameters, such as ideal transformers, negative impedance converters, gyrators, current- or voltage-controlled current or voltage sources. For the analysis of the steady state of networks containing such models, calculation methods are available [3, 4, 5, 6]. To introduce a further method, a method will be presented for writing the state equation of the above mentioned models.

Two-port model with nullators and norators

The nullator is a two-pole, both current and voltage of which are zero. Symbol indicated in Fig. 1a. will be applied. There is no restriction with regard to current and voltage of the norator. Symbol is shown in Fig. 1b. A network analysis problem can be unique if a relationship exists between currents and voltages of its two-poles. The nullator involves two restrictions, the norator no one. Therefore, to obtain a unique solution of the equations, the network should contain as many nullators as norators.

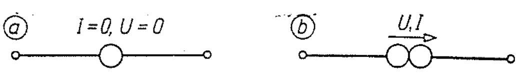

Fig. 1

Connecting a nullator and a norator according to Fig. 2 produces a nullor. The nullor is a two-port with the primary side connected to a nullator, and the secondary side to a norator. The nullor can be regarded as the model of the operational amplifier.

A general procedure is known for producing the network model containing nullators, norators and impedances of the two-port defined by impedance, admittance, or hybrid parameters [2]. Omitting a detailed discussion, a possible model of controlled sources and of a two-port defined by impedance, admittance or hybrid parameters are given in Figs 3 and 4, respectively.

Let us consider in the following the state equations of a network consisting of impedances, nullators, and norators.

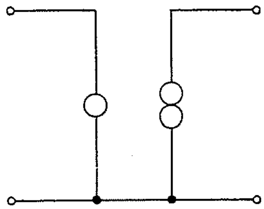

Fig. 2

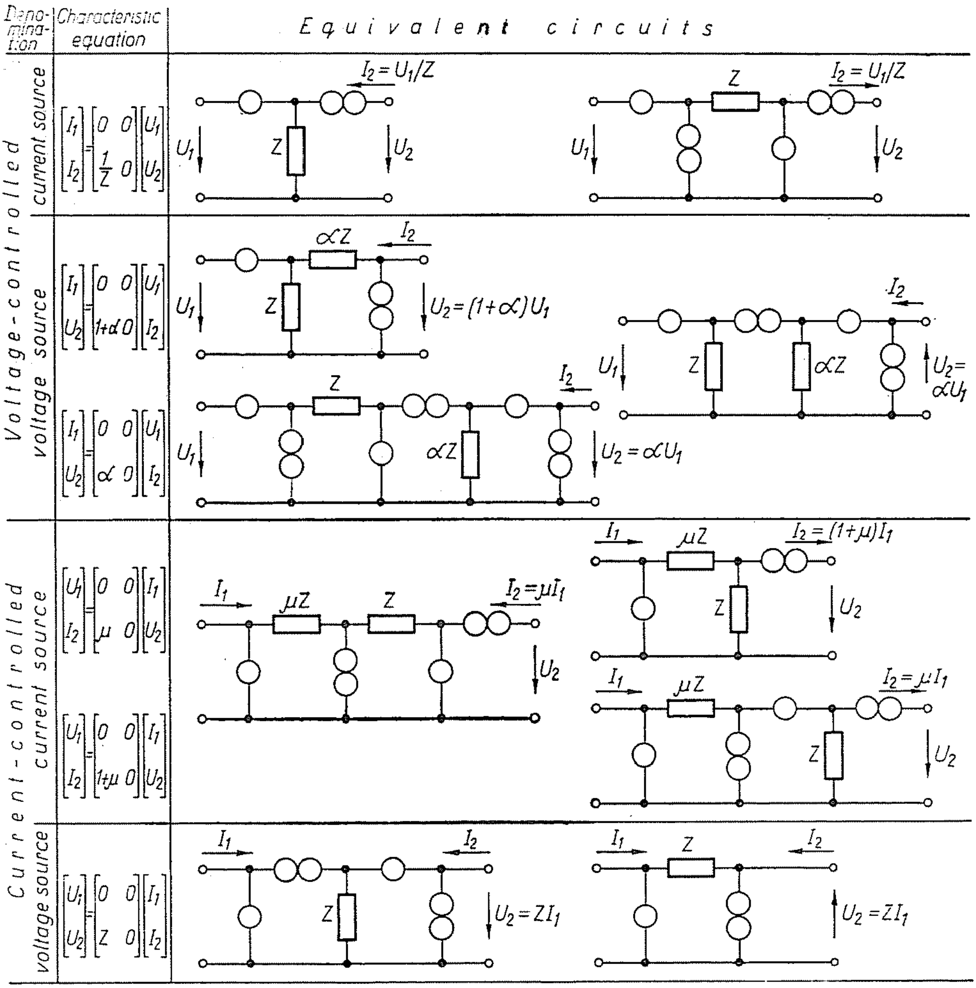

Fig. 3

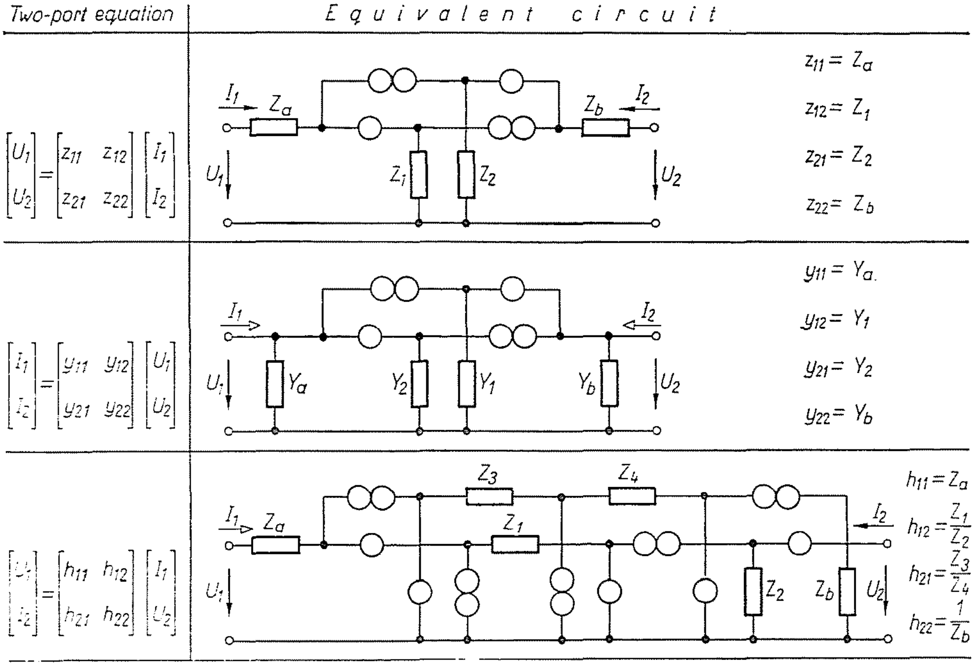

Fig. 4

Selection of state variables

State vector $\mathbf{x}$ of the network is a column matrix with state variables of the network as elements permitting to determine all the currents and voltages of the network. The state vector satisfies the

$$\dot{\mathbf{x}} = \boldsymbol{A}\mathbf{x} + \boldsymbol{B}\mathbf{z} \tag{1}$$

state equation where $\mathbf{z}$ is the column matrix of the time-dependent exciting currents, exciting voltages, $\boldsymbol{A}$ and $\boldsymbol{B}$ depend on the characteristics of the passive elements and the structure of the network, in case of a linear invariant network they are independent of exciting signals and of time.

The state variable of electric networks may be the charge of capacitors and the flux of coils. In linear networks it is advisable to choose the voltage of capacitors and the current of coils as state variables.

If the network contains a loop consisting exclusively of capacitors, voltage sources, and nullators (capacitive loop), then voltage of one capacitor in the loop can be expressed by that of the other elements of the loop, thus the voltage of this capacitor is no state variable.

If the network contains a cut-set consisting exclusively of inductances, current sources, and nullators (inductive cut-set), then the current of one of the inductances in the cut-set can be expressed by that of the other branches the cut-set, accordingly, the current of this inductances is no state variable.

It should be mentioned that extreme parameter two-ports may result in "hidden" capacitive loops, inductive cut-sets. These can be shown by using the method described in [7].

There are as many state variables as the total number of capacitors and inductances in the network less the number of capacitive loops and inductive cut-sets.

Kirchhoff's equations of the network

For writing the equations of the network, each voltage source, current source, resistor, inductance, capacitor, nullator, norator is considered as a separate branch. For our calculations a tree of the graph of the network is chosen, in which a twig corresponds to each voltage source, short circuit, nullator, and capacitor, while a link corresponds to each current source, break, norator and inductance. From among non-zero and non-infinite resistances, those connected in series with a capacitor are chosen as links, those connected in parallel with inductances as twigs, while the others should be chosen as twigs or links in such a way that twigs listed in the preceding, together with twigs corresponding to resistors, should form a tree of the graph of the network. If there are neither capacitive loops, nor inductive cut-sets in the network, grouping of branches can be performed according to the preceding. Accordingly, branches of the network have eight categories:

1. Links containing current source, numbering b_1;
2. Links containing norator, numbering b_2;
3. Links containing inductance, numbering b_3;
4. Links containing finite conductance, numbering b_4;
5. Twigs containing finite resistance, numbering b_5;
6. Twigs containing capacitor, numbering b_6;
7. Twigs containing nullator, numbering b_7;
8. Twigs containing voltage source, numbering b_8

Nullators and norators are equal in number, accordingly $b_2 = b_7$. The individual branches will be numbered in such a way that branches in Group 1 have 1, 2, ... , b_1, those in Group 2 $b_1 + 1$, $b_1 + 2$, . . . , $b_1 + b_2$, and so on, in the order of groups. Loops generated by links get the same number as the corresponding links, and cut-sets are numbered in the order of the generating twigs.

Independent loop equations of the network can be written in the form

$$\boldsymbol{B}\mathbf{u} = \mathbf{0} \tag{2}$$

where $\boldsymbol{B}$ is the loop matrix of the network and $\mathbf{u}$ is the column matrix of branch voltages. Partitioning $\boldsymbol{B}$ and $\mathbf{u}$ according to the eight groups of branches:

$$\begin{array}{c} \\ b_1 \\ b_2 \\ b_3 \\ b_4 \end{array} \begin{array}{c} \begin{array}{cccccccc} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & b_8 \end{array} \\ \left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & \boldsymbol{F}_{11} & \boldsymbol{F}_{12} & \boldsymbol{F}_{13} & \boldsymbol{F}_{14} \\ 0 & 1 & 0 & 0 & \boldsymbol{F}_{21} & \boldsymbol{F}_{22} & \boldsymbol{F}_{23} & \boldsymbol{F}_{24} \\ 0 & 0 & 1 & 0 & \boldsymbol{F}_{31} & \boldsymbol{F}_{32} & \boldsymbol{F}_{33} & \boldsymbol{F}_{34} \\ 0 & 0 & 0 & 1 & \boldsymbol{F}_{41} & \boldsymbol{F}_{42} & \boldsymbol{F}_{43} & \boldsymbol{F}_{44} \end{array}\right] \end{array} \left[\begin{array}{c} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{u}_3 \\ \mathbf{u}_4 \\ \mathbf{u}_5 \\ \mathbf{u}_6 \\ \mathbf{0} \\ \mathbf{u}_8 \end{array}\right] = \mathbf{0} \tag{3}$$

The numbers of columns and rows of each matrix block are indicated atop and at the left side, respectively.

Considering that $\mathbf{u}_8 = \mathbf{u}_g$ is the column matrix formed of source voltages of voltage sources, (3) can be written as the following four matrix equations:

$$\mathbf{u}_1 + \boldsymbol{F}_{11}\mathbf{u}_5 + \boldsymbol{F}_{12}\mathbf{u}_6 + \boldsymbol{F}_{14}\mathbf{u}_g = \mathbf{0} \tag{4}$$

$$\mathbf{u}_2 + \boldsymbol{F}_{21}\mathbf{u}_5 + \boldsymbol{F}_{22}\mathbf{u}_6 + \boldsymbol{F}_{24}\mathbf{u}_g = \mathbf{0} \tag{5}$$

$$\mathbf{u}_3 + \boldsymbol{F}_{31}\mathbf{u}_5 + \boldsymbol{F}_{32}\mathbf{u}_6 + \boldsymbol{F}_{34}\mathbf{u}_g = \mathbf{0} \tag{6}$$

$$\mathbf{u}_4 + \boldsymbol{F}_{41}\mathbf{u}_5 + \boldsymbol{F}_{42}\mathbf{u}_6 + \boldsymbol{F}_{44}\mathbf{u}_g = \mathbf{0}\,. \tag{7}$$

Cut-set equations can be written in the form

$$\underset{\sim}{Q}\mathbf{i} = \mathbf{0} \tag{8}$$

where $\underset{\sim}{Q}$ is the cut-set matrix of the network and $\mathbf{i}$ the column matrix formed of branch currents. Partitioning these according to the eight groups of branches, with the previous numbering of branches, loops, and cut-sets:

$$\boldsymbol{B} = [\mathit{1} \quad \boldsymbol{F}]\,; \qquad \underset{\sim}{Q} = [-\boldsymbol{F}^{+} \quad \mathit{1}]\,, \tag{9}$$

where $\boldsymbol{F}^{+}$ is transpose of $\boldsymbol{F}$, and $\mathit{1}$ the unit matrix. (8) can be written as follows:

$$\begin{array}{c} \\ b_5 \\ b_6 \\ b_7 \\ b_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & b_8 \end{array} \\ \left[\begin{array}{cccccccc} -\boldsymbol{F}_{11}^{+} & -\boldsymbol{F}_{21}^{+} & -\boldsymbol{F}_{31}^{+} & -\boldsymbol{F}_{41}^{+} & 1 & 0 & 0 & 0 \\ -\boldsymbol{F}_{12}^{+} & -\boldsymbol{F}_{22}^{+} & -\boldsymbol{F}_{32}^{+} & -\boldsymbol{F}_{42}^{+} & 0 & 1 & 0 & 0 \\ -\boldsymbol{F}_{13}^{+} & -\boldsymbol{F}_{23}^{+} & -\boldsymbol{F}_{33}^{+} & -\boldsymbol{F}_{43}^{+} & 0 & 0 & 1 & 0 \\ -\boldsymbol{F}_{14}^{+} & -\boldsymbol{F}_{24}^{+} & -\boldsymbol{F}_{34}^{+} & -\boldsymbol{F}_{44}^{+} & 0 & 0 & 0 & 1 \end{array}\right] \end{array} \left[\begin{array}{c} \mathbf{i}_1 \\ \mathbf{i}_2 \\ \mathbf{i}_3 \\ \mathbf{i}_4 \\ \mathbf{i}_5 \\ \mathbf{i}_6 \\ \mathbf{0} \\ \mathbf{i}_8 \end{array}\right] = \mathbf{0} \tag{10}$$

where $\mathbf{i}_1 = \mathbf{i}_g$ the column matrix of the source currents of current sources. (10) yields the following equations:

$$-F_{11}^{+}\mathbf{i}_g - F_{21}^{+}\mathbf{i}_2 - F_{31}^{+}\mathbf{i}_3 - F_{41}^{+}\mathbf{i}_4 + \mathbf{i}_5 = \mathbf{0} \tag{11}$$

$$-F_{12}^{+}\mathbf{i}_g - F_{22}^{+}\mathbf{i}_2 - F_{32}^{+}\mathbf{i}_3 - F_{42}^{+}\mathbf{i}_4 + \mathbf{i}_6 = \mathbf{0} \tag{12}$$

$$-F_{13}^{+}\mathbf{i}_g - F_{23}^{+}\mathbf{i}_2 - F_{33}^{+}\mathbf{i}_3 - F_{43}^{+}\mathbf{i}_4 = \mathbf{0} \tag{13}$$

$$-F_{14}^{+}\mathbf{i}_g - F_{24}^{+}\mathbf{i}_2 - F_{34}^{+}\mathbf{i}_3 - F_{44}^{+}\mathbf{i}_4 + \mathbf{i}_8 = \mathbf{0} \tag{14}$$

(4) to (7), and (11) to (14) are the Kirchhoff equations of the network where $\mathbf{i}_3$ and $\mathbf{u}_6$, $\mathbf{u}_8 = \mathbf{u}_g$ and $\mathbf{i}_1 = \mathbf{i}_g$ are the column matrices of state variables, source currents, and source voltages, respectively.

The state equation

To write the state equation, response signals other than state variables have to be eliminated from the above equations. To this aim the known relationship between currents and voltages of branches in each group will be utilized. Thus

$$\mathbf{u}_3 = L\dot{\mathbf{i}}_3 , \tag{15}$$

where L contains the coefficients of self-induction of branches in Group 3 in the main diagonal, and the coefficients of mutual induction between branches of Group 3 outside the main diagonal.

$$\mathbf{i}_4 = G\mathbf{u}_4 \tag{16}$$

G being the diagonal matrix formed of the conductances of branches in Group 4.

$$\mathbf{u}_5 = R\mathbf{i}_5 \tag{17}$$

R being the diagonal matrix formed of the resistances of branches in Group 5.

$$\mathbf{i}_6 = C\dot{\mathbf{u}}_6 \tag{18}$$

C being the diagonal matrix formed of the capacitances of capacitors.

Since the number of nullators and norators is equal, F_{23} is quadratic and, if it is not singular, the norator currents can be expressed from (13):

$$\mathbf{i}_2 = -F_{23}^{+-1}F_{13}^{+}\mathbf{i}_g - F_{23}^{+-1}F_{33}^{+}\mathbf{i}_3 - F_{23}^{+-1}F_{43}^{+}\mathbf{i}_4 \tag{19}$$

Substituted into (11) and (12):

$$(\boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{33} - \boldsymbol{F}^{+}_{31})\,\mathbf{i}_3 + (\boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43} - \boldsymbol{F}^{+}_{41})\,\mathbf{i}_4 + \mathbf{i}_5 = = (\boldsymbol{F}^{+}_{11} - \boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{13})\,\mathbf{i}_g \tag{20}$$

$$(\boldsymbol{F}^{+}_{22}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{33} - \boldsymbol{F}^{+}_{32})\,\mathbf{i}_3 + (\boldsymbol{F}^{+}_{22}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43} - \boldsymbol{F}^{+}_{42})\,\mathbf{i}_4 + \mathbf{i}_6 = (\boldsymbol{F}^{+}_{12} - \boldsymbol{F}^{+}_{22}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{13})\,\mathbf{i}_g \tag{21}$$

From (7), using (16), we obtain

$$\mathbf{i}_4 = -\boldsymbol{G}\boldsymbol{F}_{41}\mathbf{u}_5 - \boldsymbol{G}\boldsymbol{F}_{42}\mathbf{u}_6 - \boldsymbol{G}\boldsymbol{F}_{44}\mathbf{u}_g \tag{22}$$

From (20), taking (17) into consideration:

$$\mathbf{u}_5 = \boldsymbol{R}(\boldsymbol{F}^{+}_{11} - \boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{13})\,\mathbf{i}_g + \boldsymbol{R}(\boldsymbol{F}^{+}_{31} - \boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{33})\,\mathbf{i}_3 + + \boldsymbol{R}(\boldsymbol{F}^{+}_{41} - \boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43})\,\mathbf{i}_4 \tag{23}$$

Substituting into (22):

$$\mathbf{i}_4 = [\boldsymbol{1} + \boldsymbol{G}\boldsymbol{F}_{41}\boldsymbol{R}(\boldsymbol{F}^{+}_{41} - \boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43})]^{-1}\,[\boldsymbol{G}\boldsymbol{F}_{41}\boldsymbol{R}(\boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{33} - \boldsymbol{F}^{+}_{31})\,\mathbf{i}_3 - - \boldsymbol{G}\boldsymbol{F}_{42}\mathbf{u}_6 + \boldsymbol{G}\boldsymbol{F}_{41}\boldsymbol{R}(\boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{13} - \boldsymbol{F}^{+}_{11})\,\mathbf{i}_g - \boldsymbol{G}\boldsymbol{F}_{44}\mathbf{u}_g] \tag{24}$$

Substituting (22) into (23):

$$\mathbf{u}_5 = [\boldsymbol{1} + \boldsymbol{R}(\boldsymbol{F}^{+}_{41} - \boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43})\boldsymbol{G}\boldsymbol{F}_{41}]^{-1}\,[\boldsymbol{R}(\boldsymbol{F}^{+}_{31} - \boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{33})\,\mathbf{i}_3 + + \boldsymbol{R}(\boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43} - \boldsymbol{F}^{+}_{41})\boldsymbol{G}\boldsymbol{F}_{42}\mathbf{u}_6 + \boldsymbol{R}(\boldsymbol{F}^{+}_{11} - \boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{13})\,\mathbf{i}_g + + \boldsymbol{R}(\boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43} - \boldsymbol{F}^{+}_{41})\,\boldsymbol{G}\boldsymbol{F}_{44}\mathbf{u}_g] \tag{25}$$

From (6), substituting (15) and (25), further from (21), taking (24) and (18) into consideration, the state equation of the network is found to be:

$$\begin{bmatrix} \dot{\mathbf{i}}_3 \\ \dot{\mathbf{u}}_6 \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C}^{-1} \end{bmatrix} \left\{ \begin{bmatrix} \boldsymbol{D}_{11} & \boldsymbol{D}_{12} \\ \boldsymbol{D}_{21} & \boldsymbol{D}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{i}_3 \\ \mathbf{u}_6 \end{bmatrix} + \begin{bmatrix} \boldsymbol{E}_{11} & \boldsymbol{E}_{12} \\ \boldsymbol{E}_{21} & \boldsymbol{E}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{i}_g \\ \mathbf{u}_g \end{bmatrix} \right\} \tag{26}$$

where

$$\boldsymbol{D}_{11} = \boldsymbol{F}_{31}[\boldsymbol{1} + \boldsymbol{R}(\boldsymbol{F}^{+}_{41} - \boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43})\,\boldsymbol{G}\boldsymbol{F}_{41}]^{-1}\boldsymbol{R}(\boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{33} - \boldsymbol{F}^{+}_{31}) \tag{27}$$

$$\boldsymbol{D}_{12} = \boldsymbol{F}_{31}[\boldsymbol{1} + \boldsymbol{R}(\boldsymbol{F}^{+}_{41} - \boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43})\,\boldsymbol{G}\boldsymbol{F}_{41}]^{-1}\boldsymbol{R}(\boldsymbol{F}^{+}_{41} - \boldsymbol{F}^{+}_{23}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43})\,\boldsymbol{G}\boldsymbol{F}_{42} - \boldsymbol{F}_{32} \tag{28}$$

$$\boldsymbol{D}_{21} = \boldsymbol{F}^{+}_{32} - \boldsymbol{F}^{+}_{22}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{33} + (\boldsymbol{F}^{+}_{42} - \boldsymbol{F}^{+}_{22}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43})\,[\boldsymbol{1} + \boldsymbol{G}\boldsymbol{F}_{41}\boldsymbol{R}(\boldsymbol{F}^{+}_{41} - - \boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43})]^{-1}\,\boldsymbol{G}\boldsymbol{F}_{41}\boldsymbol{R}(\boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{33} - \boldsymbol{F}^{+}_{31}) \tag{29}$$

$$\boldsymbol{D}_{22} = (\boldsymbol{F}^{+}_{22}\boldsymbol{F}^{+-1}_{23}\boldsymbol{F}^{+}_{43} - \boldsymbol{F}^{+}_{42})\,[\boldsymbol{1} + \boldsymbol{G}\boldsymbol{F}_{41}\boldsymbol{R}(\boldsymbol{F}^{+}_{41} - \boldsymbol{F}^{+}_{21}\boldsymbol{F}^{+-1}_{23}\mathbf{F}^{+}_{43})]^{-1}\,\boldsymbol{G}\boldsymbol{F}_{42} \tag{30}$$

$$\boldsymbol{E}_{11} = \boldsymbol{F}_{31}[\mathit{1} + \boldsymbol{R}(\boldsymbol{F}_{41}^{\div} - \boldsymbol{F}_{21}^{\div}\boldsymbol{F}_{23}^{\div\,-1}\boldsymbol{F}_{43}^{\div})\boldsymbol{G}\boldsymbol{F}_{41}]^{-1}\boldsymbol{R}(\boldsymbol{F}_{21}^{\div}\boldsymbol{F}_{23}^{\div\,-1}\boldsymbol{F}_{13}^{\div} - \boldsymbol{F}_{11}^{\div}) \tag{31}$$

$$\boldsymbol{E}_{12} = \boldsymbol{F}_{31}[\mathit{1} + \boldsymbol{R}(\boldsymbol{F}_{41}^{\div} - \boldsymbol{F}_{21}^{\div}\boldsymbol{F}_{23}^{\div\,-1}\boldsymbol{F}_{43}^{\div})\boldsymbol{G}\boldsymbol{F}_{41}]^{-1}\boldsymbol{R}(\boldsymbol{F}_{41}^{\div} - \boldsymbol{F}_{21}^{\div}\boldsymbol{F}_{23}^{\div\,-1}\boldsymbol{F}_{43}^{\div})\boldsymbol{G}\boldsymbol{F}_{44} - \boldsymbol{F}_{34} \tag{32}$$

$$\boldsymbol{E}_{21} = \boldsymbol{F}_{21}^{\div} - \boldsymbol{F}_{22}^{\div}\boldsymbol{F}_{23}^{\div\,-1}\boldsymbol{F}_{13}^{\div} + (\boldsymbol{F}_{42}^{\div} - \boldsymbol{F}_{22}^{\div}\boldsymbol{F}_{23}^{\div\,-1}\boldsymbol{F}_{43}^{\div})\,[\mathit{1} + \boldsymbol{G}\boldsymbol{F}_{41}\boldsymbol{R}(\boldsymbol{F}_{41}^{\div} - \boldsymbol{F}_{21}^{\div}\boldsymbol{F}_{23}^{\div\,-1}\boldsymbol{F}_{43}^{\div})]^{-1}\,\boldsymbol{G}\boldsymbol{F}_{41}\boldsymbol{R}(\boldsymbol{F}_{21}^{\div}\boldsymbol{F}_{23}^{\div\,-1}\boldsymbol{F}_{13}^{\div} - \boldsymbol{F}_{11}^{\div}) \tag{33}$$

$$\boldsymbol{E}_{22} = (\boldsymbol{F}_{22}^{\div}\boldsymbol{F}_{23}^{\div\,-1}\boldsymbol{F}_{43}^{\div} - \boldsymbol{F}_{42}^{\div})\,[\mathit{1} + \boldsymbol{G}\boldsymbol{F}_{41}\boldsymbol{R}(\boldsymbol{F}_{41}^{\div} - \boldsymbol{F}_{21}^{\div}\boldsymbol{F}_{23}^{\div\,-1}\boldsymbol{F}_{43}^{\div})]^{-1}\,\boldsymbol{G}\boldsymbol{F}_{44} \tag{34}$$

In the knowledge of state variables, (24) yields $\mathbf{i}_4$, (25) yields $\mathbf{u}_5$, in turn on the basis of (20) $\mathbf{i}_5$, on the basis of (4) $\mathbf{u}_1$, (5) $\mathbf{u}_2$, (6) $\mathbf{u}_3$, (7) $\mathbf{u}_4$, (19) $\mathbf{i}_2$, (12) $\mathbf{i}_6$ and $\mathbf{i}_8$ can be calculated from (14).

Example

The low-frequency amplifier shown in Fig. 5 can be modelled from the aspect of periodic signals, by the circuit shown in Fig. 6, if among the hybrid parameters of the transistor $h_{12} \approx 0$. The equivalent circuit of the amplifier containing nullators and norators, illustrated in Fig. 7, is used for writing the state equation, using notation:

$$R_0 = R_g + h_{11}\,.$$

The graph of the equivalent circuit in Fig. 7 is illustrated in Fig. 8a. Upon classifying branches we find that no one belongs to Groups 1 and 3. Branches 1, 2 are to be placed into Group 2, branch 9 into Group 6, branches 10, 11

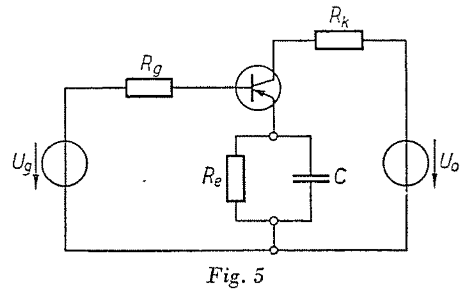

Fig. 5

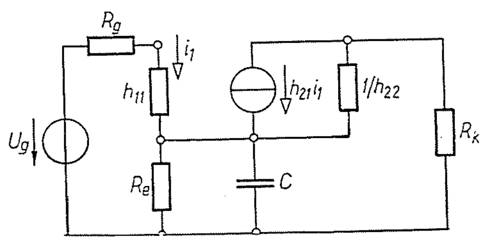

Fig. 6

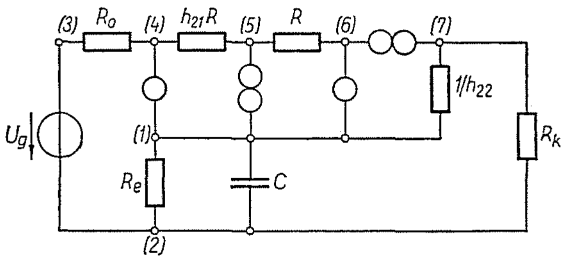

Fig. 7

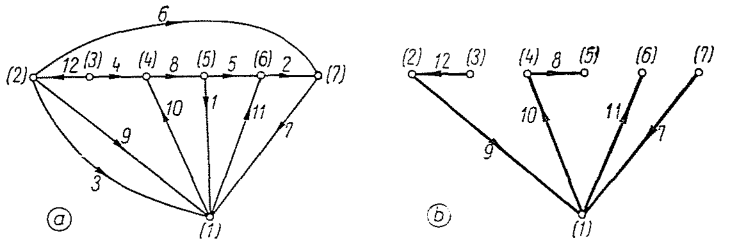

Fig. 8

into Group 7, branch 12 into Group 8, and among the resistors, branch 3 belongs to Group 4. Branches of Group 6, 7 and 8 should be completed to a tree by means of branches containing resistors. Choose branches 7 and 8 as twigs (Fig. 8b), this means that these are branches of Group 5, while branches 4, 5, and 6 those of Group 4. The matrix of the fundamental loop system generated by the tree chosen in this way:

$$\boldsymbol{B} = \left[\begin{array}{cc|cccc|cc|c|cc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & 0 & 0 \end{array}\right]$$

that is,

$$\boldsymbol{F}_{11} = \boldsymbol{0}\,; \quad \boldsymbol{F}_{12} = \boldsymbol{0}\,; \quad \boldsymbol{F}_{13} = \boldsymbol{0}\,; \quad \boldsymbol{F}_{14} = \boldsymbol{0}\,;$$

$$\boldsymbol{F}_{21} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad \boldsymbol{F}_{22} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad \boldsymbol{F}_{23} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad \boldsymbol{F}_{24} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\boldsymbol{F}_{31} = \boldsymbol{0}\,; \quad \boldsymbol{F}_{32} = \boldsymbol{0}\,; \quad \boldsymbol{F}_{33} = \boldsymbol{0}\,; \quad \boldsymbol{F}_{34} = \boldsymbol{0}\,;$$

$$\boldsymbol{F}_{41} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad \boldsymbol{F}_{42} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ -1 \end{bmatrix}; \quad \boldsymbol{F}_{43} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & -1 \\ 0 & 0 \end{bmatrix}; \quad \boldsymbol{F}_{44} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

Further matrices describing the network:

$$G = \left\langle \frac{1}{R_e} \quad \frac{1}{R_0} \quad \frac{1}{R} \quad \frac{1}{R_k} \right\rangle$$

$$R = \left\langle \frac{1}{h_{22}} \quad h_{21}R \right\rangle$$

$$C = C$$

Hence, on the basis of relationships (27) to (34):

$D_{11} = 0$; $D_{12} = 0$; $D_{21} = 0$;

$$D_{22} = [1 \quad 1 \quad 0 \quad 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & h_{21} & 1 & 0 \\ 0 & 0 & \frac{1}{h_{22}R_k} & 1+\frac{1}{h_{22}R_k} \end{bmatrix} \begin{bmatrix} -\frac{1}{R_e} \\ -\frac{1}{R_0} \\ 0 \\ -\frac{1}{R_k} \end{bmatrix} =$$

$$= -\left(\frac{1}{R_e} + \frac{1}{R_0}\,\frac{1+h_{22}R_k+h_{21}}{1+h_{22}R_k} + \frac{h_{22}}{1+h_{22}R_k}\right);$$

$E_{11} = 0$; $E_{12} = 0$; $E_{21} = 0$;

$$E_{22} = [1 \; 1 \; 0 \; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -h_{21} & 1 & 0 \\ 0 & \frac{h_{21}}{1+h_{22}R_k} & -\frac{1}{1+h_{22}R_k} & \frac{h_{22}R_k}{1+h_{22}R_k} \end{bmatrix} \begin{bmatrix} 0 \\ -\frac{1}{R_0} \\ 0 \\ 0 \end{bmatrix} =$$

$$= -\frac{1}{R_0}\,\frac{1+h_{22}R_k+h_{21}}{1+h_{22}R_k},$$

Yielding the state equation of the network on the basis of (26):

$$\dot{u}_6 = \dot{u}_9 = -\frac{1}{C}\left(\frac{1}{R_e} + \frac{1}{R_0}\,\frac{1+h_{22}R_k+h_{21}}{1+h_{22}R_k} + \frac{h_{22}}{1+h_{22}R_k}\right) u_9 - $$
$$- \frac{1}{CR_0}\,\frac{1+h_{22}R_k+h_{21}}{1+h_{22}R_k}\,u_g \,.$$

The solution can be written in the knowledge of $u_g(t)$.

Summary

Two-ports with given parameters can be modelled by means of networks containing resistances, inductances, capacitances, nullators and norators, even in the case of extreme parameters. The paper presents a graph theory method for writing the state equation of networks containing such models.

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