CIRCUIT MODELS OF DIRECT CURRENT MACHINES

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In electric rotating machines, electric and mechanical process take place. Circuit models readily lend themselves for examining electrical process [1], [2], [3], [4], [5], [6]. The ways of applying linear circuit models for the simultaneous description of mechanical and electrical phenomena encountered in direct current machines will be presented. Similar goals are set in the revelant chapter of [7], in this case, however, the models may also contain nonlinear circuit elements.

1. Equations of the direct current machines with two coils

There is one coil each on the stator and the rotor of d. c. machines with two coils. The commutator coil on the rotor is known to produce a magnetic flux ψ of constant direction independently of the rotation of the rotor. This direction is parallel to the line connecting the brushes, i. e. the brush axis. The direction of the stator coil flux is called direction d, the direction perpendicular to this, generally coincident with the direction of the brush axis, is the direction q. The coils, with their currents and fluxes, are shown in Fig. 1. Quantities relating to the stator and the rotor are designated by subscripts s, and r, respectively.

From Maxwell's 2nd equation for the curve l_s passing along the stator coil, we obtain:

$$\oint_{l_s} \overline{E}\,\overline{\mathrm{d}l} = -u_s + R_s i_s = -\int_{A_s} \frac{\partial \overline{B}}{\partial t}\,\overline{\mathrm{d}A}_s = -\frac{\partial \psi_s}{\partial t}, \qquad (1)$$

that is, in the case of constant permeability,

$$u_s = R_s i_s + L_s \frac{\mathrm{d}i_s}{\mathrm{d}t}. \qquad (2)$$

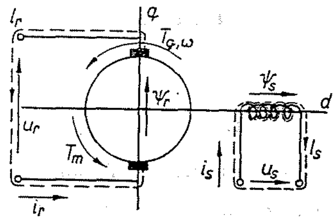

Fig. 1

R_s denotes the resistance of the stator coil, L_s its coefficient of self-induction. In the case of the rotor coil, Maxwell's 2nd equation has to be completed with the term for the movement of the rotor [8]:

$$\oint_{l_r} \overline{E}\,\overline{\mathrm{d}l} = \int_{A_r} \frac{\partial \overline{B}}{\partial t}\,\overline{\mathrm{d}A_r} + \oint_{l_r} (\bar{v}\times\overline{B})\,\overline{\mathrm{d}l}\,, \tag{3}$$

where l_r denotes the curve passing along the rotor coil, and $\bar{v}$ is the circumferential velocity of the rotor.

The following relationships can be written for the individual members:

$$\oint_{l_r} \overline{E}\,\overline{\mathrm{d}l} = -u_r + i_r R_r\,, \tag{4}$$

$$\int_{A_r} \frac{\partial \overline{B}}{\partial t}\,\overline{\mathrm{d}A_r} = \frac{\partial \psi_r}{\partial t} = L_r \frac{\mathrm{d}i_r}{\mathrm{d}t}\,, \tag{5}$$

$$\oint_{l_r} (\bar{v}\times\overline{B})\,\overline{\mathrm{d}l} = B\omega \frac{D_k}{2} l N_r 2\,, \tag{6}$$

where R_r is the resistance of the rotor coil, L_r the coefficient of its self- inductance, D_k the diameter of the rotor, ω its angular velocity, l its length, N_r the number of windings in the rotor coil, and B the air-gap induction.

Relationship (5) was obtained by considering the rotor coil as being at rest. Eq. (6) refers to the rotor during rotation since it represents the movement-induced voltage. Here an induction of radial direction, having a sinusoidal distribution in each pole was supposed to be generated in the air-gap.

Since in the case of constant permeability,

$$B\,D_k\,l = \frac{\psi_s}{N_s} = \frac{L_s\,i_s}{N_s}\,, \tag{7}$$

thus

$$\oint_{l_r} (\bar{v}\times\bar{B})\,\overline{\mathrm{d}l} = \omega L_{rs}\,i_s\,, \tag{8}$$

where L_{rs} is the coefficient of mutual inductance of the stator and coils, if the coupling factor of the two coils is $k = 1$.

From Eqs (3) through (8) obviously,

$$u_r = i_r\,R_r + L_r\,\frac{\mathrm{d}i_r}{\mathrm{d}t} - \omega\,L_{rs}\,i_s\,. \tag{9}$$

The factor $-L_{rs}$ in Eq. (9) is also called the coefficient of motional inductance [2], [4], [9]. Introduce for this designation

$$M_{rs} = -L_{rs}. \tag{10}$$

Voltage equations (2) and (9) written in matrix form:

$$\begin{bmatrix} u_s \\ u_r \end{bmatrix} = \left\{ \begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix} + \omega \begin{bmatrix} 0 & 0 \\ M_{rs} & 0 \end{bmatrix} \right\} \begin{bmatrix} i_s \\ i_r \end{bmatrix} + \begin{bmatrix} L_s & 0 \\ 0 & L_r \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} i_s \\ i_r \end{bmatrix}. \tag{11}$$

In a more compact form of writing:

$$\mathsf{u} = (\mathbf{R} + \omega\mathbf{M})\,\mathsf{i} + \mathbf{L}\,\frac{\mathrm{d}}{\mathrm{d}t}\,\mathsf{i}\,. \tag{12}$$

From a comparison with (11), the meaning of matrices in (12) is evident. Eq. (12) is, however, valid also in the case of machines with more than two coils, if the involved matrices are interpreted according to the number of coils.

For the complete characterization of the d. c. machine, also an equation is required, for describing the movement of the rotor, such as:

$$T_g = J\,\frac{\mathrm{d}\omega}{\mathrm{d}t} + K\omega - T_m\,, \tag{13}$$

where J denotes the moment of inertia of the rotor, K the viscous friction coefficient, T_m the mechanical torque, T_g the electrical torque, and the reference directions according to Fig. 1 are used. T_g can be determined on the basis of output conditions of the machine [4], [9]. From (12), the momentary output is found to be

$$p(t) = \mathsf{i}^*\mathsf{u} = \mathsf{i}^*\mathbf{R}\mathsf{i} + \mathsf{i}^*\mathbf{L}\frac{\mathrm{d}}{\mathrm{d}t}\mathsf{i} + \omega\mathsf{i}^*\mathbf{M}\mathsf{i}\,, \tag{14}$$

where $p_m = \omega\mathsf{i}^*\mathbf{M}\mathsf{i}$ denotes mechanical output.
Thus

$$T_g = \frac{p_m}{\omega} = \mathsf{i}^*\mathbf{M}\mathsf{i}\,. \tag{15}$$

Substituting (15) into (13),

$$\mathsf{i}^*\mathbf{M}\mathsf{i} = J\frac{\mathrm{d}\omega}{\mathrm{d}t} + K\omega - T_m\,. \tag{16}$$

(*denotes transposing.)

2. Conditions of application of the linear network model

On the basis of relationships (12) and (16) characterising the d. c. machine, following statements can be made on the application of the network model.

1. Eqs (12) and (16) form a system of non-linear differential equations, non-linearity being due to the product of two variables ($\omega\mathsf{i}$ and $\mathsf{i}^*\mathbf{M}\mathsf{i}$). Therefore the mechanical behaviour of the machine cannot be simulated by a linear electric network, so as to be completely general. A linear network model can only be applied for the examination of the electric machine if its equations can be linearized. This model is likely to be a good approximation in case of small-amplitude oscillations, or if one of the variables causing non-linearity can be regarded as a constant [1], [4], [10], [11].

2. The examined equations also contain non-electric quantites as variables. In order to obtain an electric network model simulating the whole of the d. c. machine mechanical quantities are substituted by electric quantities. Equations obtained in this way permit to construct an electric network supplying the required model. In a similar way, network models are constructed by Meisel for d. c. machines [7], by Seely for electromechanical transducers [12], and also in electroacoustics, [13], [14], models constructed along the above lines are used. Mechanical quantities are substituted on the basis of analogy between electric networks and certain mechanical systems. In the analogy, two electric networks, dual to each other, can be made to correspond to the mechanical system [7], [13], [15]. Analogous quantities are compiled in the following table:

Mechanical quantity		Electric I (Inverse analogous)		Electric II (Direct analogous)	
x α	displacement angular displacement	q	charge	Φ	flux
v ω	velocity angular velocity	i	current	u	voltage
F T	force torque	u	voltage	i	current
K	viscous friction coefficient	R	resistance	G	conductivity
m J	mass moment of inertia	L	inductance	C	capacitance
C_m	flexibility constant	C	capacitance	L	inductivity
$\frac{1}{2}mv^2$ $\frac{1}{2}J\omega^2$	kinetic energy rotation energy	$\frac{1}{2}Li^2$	magnetic energy	$\frac{1}{2}Cu^2$	electric energy
Fx $T\alpha$	mechanical work	uq	work in electric field	Φi	work in magnetic field

It should be noted that the analogy is not merely a formal similarity, since Eqs (12) and (16) can be deduced uniformly on the basis of Lagrange's dynamic equations of the second kind [1], [7], [12], [16], [17].

3. The network model of the two-coil d. c. machine

Substituting mechanical quantities in the equations of the two-coil d. c. machine by electric quantities according to analogy I, then from (11) and (16):

$$u_s = R_s i_s + L_s \frac{di_s}{dt},$$

$$u_r = R_r i_r + L_r \frac{di_r}{dt} + M_{rs} i_\omega i_s, \tag{17}$$

$$M_{rs} i_r i_s = L_J \frac{di_\omega}{dt} + R_K i_\omega - u_{Tm}.$$

Subscripts of the substituted electric quantities indicate the corresponding mechanical quantities. For linearizing the equations let us suppose that deviations referred to some steady state are examined. The value of variables in the steady state is indicated by a block letter, deviations from this by a dash above. Thus, from Eq. (17) we obtain:

$$
\begin{aligned}
U_s + \bar{u}_s &= R_s(\bar{i}_s + I_s) + L_s \frac{d\bar{i}_s}{dt}, \\
U_r + \bar{u}_r &= R_r(\bar{i}_r + I_r) + L_r \frac{d\bar{i}_r}{dt} + M_{rs}(I_\omega + \bar{i}_\omega)(I_s + i_s), \\
M_{rs}(I_r + \bar{i}_r)(I_s + \bar{i}_s) &= L_J \frac{d\bar{i}_\omega}{dt} + R_K(I_\omega + \bar{i}_\omega) - (U_{Tm} + \bar{u}_{Tm}).
\end{aligned}
\tag{18}
$$

Since system of equations (17) is valid also for the values characterizing the steady state, by neglecting changes of the second order:

$$
\begin{aligned}
\bar{u}_s &= R_s \bar{i}_s + L_s \frac{d\bar{i}_s}{dt}, \\
\bar{u}_r &= R_r \bar{i}_r + L_r \frac{d\bar{i}_r}{dt} + M_{rs} I_\omega \bar{i}_s + M_{rs} I_s \bar{i}_\omega, \\
M_{rs} I_r \bar{i}_s + M_{rs} I_s \bar{i}_r &= L_J \frac{d\bar{i}_\omega}{dt} + R_K \bar{i}_\omega - \bar{u}_{Tm},
\end{aligned}
\tag{19}
$$

where, on the basis of (18),

$$
\begin{aligned}
I_s &= \frac{U_s}{R_s}, \\
I_r &= \frac{U_r R_K R_s - U_{Tm} U_s M_{rs}}{M_{rs}^2 U_S^2 + R_r R_K R_S^2} R_s, \\
I_\omega &= \frac{U_r U_s M_{rs} - U_{Tm} R_r R_s}{M_{rs}^2 U_S^2 + R_r R_K R_S^2} R_s.
\end{aligned}
\tag{20}
$$

Arranging Eq. (19) and writing it in the matrix form:

$$
\begin{bmatrix} \bar{u}_s \\ \bar{u}_r \\ \bar{u}_{Tm} \end{bmatrix} N =
\begin{bmatrix} R_s & 0 & 0 \\ M_{rs} I_\omega & R_r & M_{rs} I_s \\ -M_{rs} I_r & -M_{rs} I_s & R_K \end{bmatrix}
\begin{bmatrix} \bar{i}_s \\ \bar{i}_r \\ \bar{i}_\omega \end{bmatrix} +
\begin{bmatrix} L_s & 0 & 0 \\ 0 & L_r & 0 \\ 0 & 0 & L_J \end{bmatrix}
\begin{bmatrix} \frac{d\bar{i}_s}{dt} \\ \frac{d\bar{i}_r}{dt} \\ \frac{d\bar{i}_\omega}{dt} \end{bmatrix}
\tag{21}
$$

These are the equations of the two-coil d. c. machine for the linear case. Eq. (21) can be written also in the form

$$
\bar{\mathrm{u}} = \bar{\mathbf{R}}\,\bar{\mathrm{i}} + \bar{\mathbf{L}} \frac{d}{dt} \bar{\mathrm{i}}, \tag{22}
$$

where $\bar{\mathbf{R}}$ is the electromechanical resistance matrix, $\bar{\mathbf{L}}$ the electromechanical inductance matrix. The networks corresponding to Eqs (21) and (22) contain resistances, controlled sources respectively, and, since $\bar{\mathbf{L}}$ is diagonal, decoup-

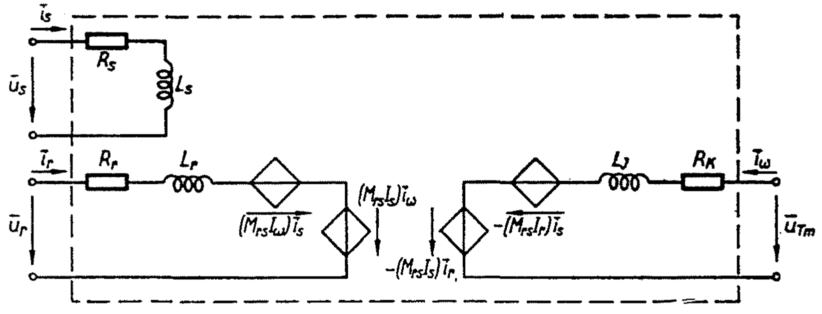

Fig. 2

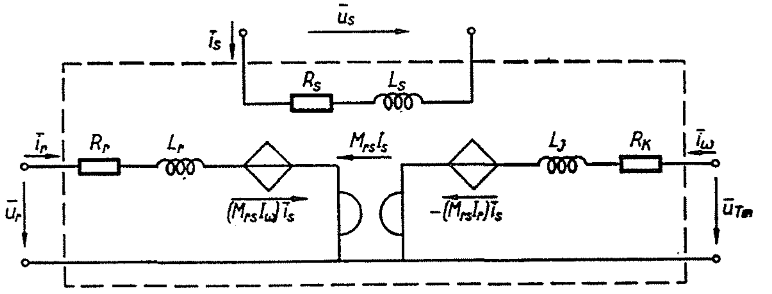

Fig. 3

ling inductivities. A three-port network corresponding to (21) is shown in Fig. 2. In this case matrix $\bar{\mathbf{R}}$ is realized in such a way that the elements along the main diagonal are taken into consideration by resistances connected in series with the corresponding port, while the other elements by controlled sources [18]. It is evident that various but equivalent networks correspond to (21) and (22), respectively, depending on the realization of $\bar{\mathbf{R}}$. A variety different from the one given in Fig. 2, is shown in Fig. 3. Here the individual controlled sources of the network shown in Fig. 2 were substituted by gyrators. The justification is evident upon decomposing matrix $\bar{\mathbf{R}}$:

$$\bar{\mathbf{R}} = \begin{bmatrix} R_s & 0 & 0 \\ M_{rs} I_\omega & R_r & 0 \\ -M_{rs} I_r & 0 & R_K \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & M_{rs} I_s \\ 0 & -M_{rs} I_s & 0 \end{bmatrix},$$

where the second member is the impedance matrix of a gyrator connected to the port having a current $\bar{i}_r$ and $\bar{i}_\omega$ [19]. The gyrator acts as an energy transformer, expressing the electromechanical energy transformation in the d. c. machine. This process can be better followed if conditions are examined under condition $i_S =$ constant. In this case Eqs (17) are linear, consequently the network model is not valid for changes alone. The respective network can be obtained either directly on the basis of the equations, or by e. g. transforming the network shown in Fig. 3. accordingly, as shown in Fig. 4. Connecting resistance R_m to voltage u_{Tm}, and the voltage sources to the other two ports, results in the model of the d. c. motor with constant excitation current. If the rotor circuit is closed by resistance R_t, and a current or voltage source is connected to the mechanical side, the model corresponding to the externally excited generator is obtained. In both cases, the gyrator provides for energy exchange between the rotor circuit and the mechanical circuit.

Models differing somewhat from those given above are obtained if mechanical quantities are substituted by electric quantitites according to

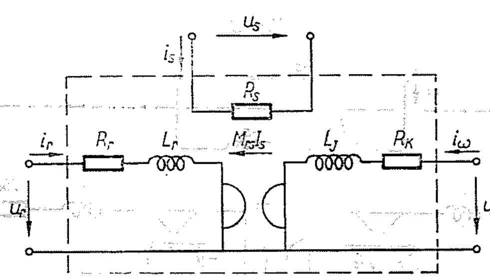

Fig. 4

analogy II. In this case, in Eqs (11) and (16), u_ω is written for ω, and i_{Tm} for T_m. For the case of small changes, after linearization, we obtain:

$$\bar{u}_s = R_s \bar{i}_s + L_s \frac{d\bar{i}_s}{dt},$$

$$\bar{u}_r = R_r \bar{i}_r + L_r \frac{d\bar{i}_r}{dt} + M_{rs} U_\omega \bar{i}_s + M_{rs} I_s \bar{u}_\omega, \tag{23}$$

$$\bar{i}_{Tm} = G_K \bar{u}_\omega + C_J \frac{d\bar{u}_\omega}{dt} - M_{rs} I_r \bar{i}_s - M_{rs} I_s \bar{i}_r.$$

These correspond to the hybrid-parameter equations of the three-port. The last equation in (19) is the dual of the last equation in (23), therefore, taking the dual of the mechanical circuit in the network shown in Fig. 2, one of the networks corresponding to (23) is obtained. Calculations are advantageous by keeping low the number of controlled sources, therefore is (23) applied to determine a network similar to that in Fig. 3.

Decompose the hybrid matrix formed of the coefficients

$$\overline{\mathbf{H}}_1 = \begin{bmatrix} R_s & 0 & 0 \\ M_{rs} U_\omega & R_s & M_{rs} I_s \\ -M_{rs} I_r & -M_{rs} I_s & G_K \end{bmatrix}$$

to the sum of two members:

$$\overline{\mathbf{H}}_1 = \begin{bmatrix} R_s & 0 & 0 \\ M_{rs} U_\omega & R_r & 0 \\ -M_{rs} I_r & 0 & G_K \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & M_{rs} I_s \\ 0 & -M_{rs} I_s & 0 \end{bmatrix}.$$

The first member in the decomposition can be realized, similarly as in the foregoing, by resistances and controlled sources, while the second member by an ideal transformer inserted between the rotor circuit and the mechanical circuit. This is, namely, the hybrid matrix of an ideal transformer with the turns ratio $M_{rs} I_s : 1$ [19], [20].

The matrix

$$\overline{\mathbf{H}}_2 = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_r & 0 \\ 0 & 0 & C_J \end{bmatrix}$$

is realized by series inductivity, and parallel capacity, with suitable ports, resulting in the network shown in Fig. 5.

In the case where $i_s = I_s$ is constant, we obtain the network shown in Fig. 6. The role of the transformer is here similar to that of the gyrator above

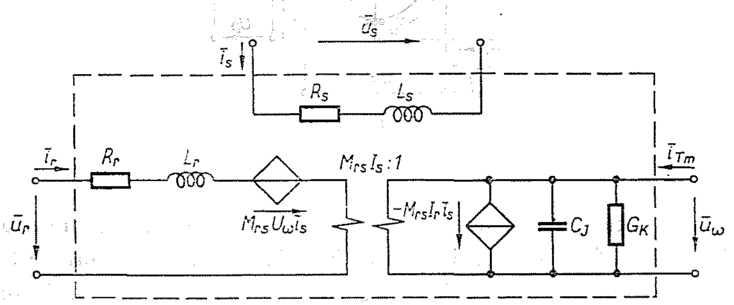

Fig. 5

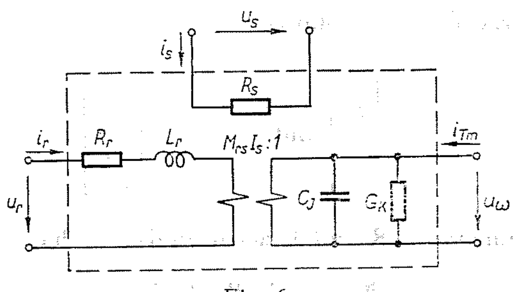

Fig. 6

i. e., it is simulating energy transfer between the rotor circuit and the mechanical circuit. It should be noted that Meisel [7] has introduced an identical model for d. c. motors, further that a special form of the model in Fig. 6. is used also for the examination of d. c. motors [5], [10], [11]. Provided the model is to be realized rather than to be used in calculations alone, it is more advantageous to take the network given in Fig. 3. as basis, since the gyrator and the controlled source are simpler to realize than is the ideal transformer.

4. The primitiv d. c. machine and its network model

The primitiv d. c. machine is a model helping to examine various d. c. machines. Let us consider here the four-coil primitiv machine (Fig. 7), the most widespread of all [2], [3], [4]. This machine has two stator coils and two

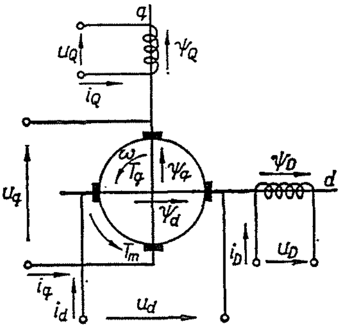

Fig. 7

rotor coils with axes perpendicular to each other. The arrangement of the coils and the direction of axes is indicated in Fig. 7. Quantities relative to the individual coils bear subscripts indicating the direction of the axis of the coil, using capitals for a coil on the stator, and minuscules for the rotor coils. The equations of the primitive d. c. machine are obtained from Eqs (12) and (16) involving determination of matrices **R**, **M**, and **L**. Currents and voltages are arranged in matrix form as follows:

$$\mathsf{i} = \begin{bmatrix} i_D \\ i_Q \\ i_d \\ i_q \end{bmatrix} \text{ and } \qquad \mathsf{u} = \begin{bmatrix} u_D \\ u_Q \\ u_d \\ u_q \end{bmatrix}. \tag{24}$$

Arranging the elements of R and L accordingly, we have

$$\mathbf{R} = < R_D \, R_Q \, R_d \, R_q >$$

and

$$\mathbf{L} = \begin{bmatrix} L_D & 0 & L_{Dd} & 0 \\ 0 & L_Q & 0 & L_{Qq} \\ L_{Dd} & 0 & L_d & 0 \\ 0 & L_{Qq} & 0 & L_q \end{bmatrix}, \tag{25}$$

obvious from Fig. 7. Motional inductances arise, similarly as in the case of the two-coil machine, only in the case of coils with axes perpendicular to each other. Thus, neglecting the details, we obtain for $\mathbf{M}$: [3], [4, [9], [11],

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & L_{Qd} & 0 & L_{dq} \\ -L_{Dq} & 0 & -L_{qd} & 0 \end{bmatrix}. \tag{26}$$

Since the rotor of the d. c. machine is of symmetrical construction, while the stator is not:

$$L_{Qd} = L_{Qq},\ L_{Dq} = L_{Dd},\ L_{dq} = L_q,\ L_{qd} = L_d\,.$$

Thus

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & L_{Qq} & 0 & L_q \\ -L_{Dd} & 0 & -L_d & 0 \end{bmatrix}. \tag{27}$$

Substituting electrical for mechanical quantities, according to analogy I, linearizing the obtained equations to consider small deviations from the steady state, and arranging results in the relationship:

$$\begin{bmatrix} \bar{u}_D \\ \bar{u}_Q \\ \bar{u}_d \\ \bar{u}_q \\ \bar{u}_{Tm} \end{bmatrix} = \begin{bmatrix} L_D & 0 & L_{Dd} & 0 & 0 \\ 0 & L_Q & 0 & L_{Qq} & 0 \\ L_{Dd} & 0 & L_D & 0 & 0 \\ 0 & L_{Qq} & 0 & L_q & 0 \\ 0 & 0 & 0 & 0 & L_J \end{bmatrix} \frac{\mathrm{d}}{\mathrm{dt}} \begin{bmatrix} \bar{i}_D \\ \bar{i}_Q \\ \bar{i}_d \\ \bar{i}_q \\ \bar{i}_\omega \end{bmatrix} + \tag{28}$$

$$+ \begin{bmatrix} R_D & 0 & 0 & 0 & 0 \\ 0 & R_Q & 0 & 0 & 0 \\ 0 & L_{Qq}I_\omega & R_d & L_q I_\omega & L_q I_q + L_{Qq} I_Q \\ -I_\omega L_{Dd} & 0 & -I_\omega L_d & R_q & -L_d I_d - L_{Dd} I_D \\ L_{Dd} I_q & -L_{Qq} I_d & (L_d - L_q) I_q - L_{Qq} I_Q & (L_d - L_q) I_d + L_{Dd} I_D & R_K \end{bmatrix} \begin{bmatrix} \bar{i}_D \\ \bar{i}_Q \\ \bar{i}_d \\ \bar{i}_q \\ \bar{i}_\omega \end{bmatrix}.$$

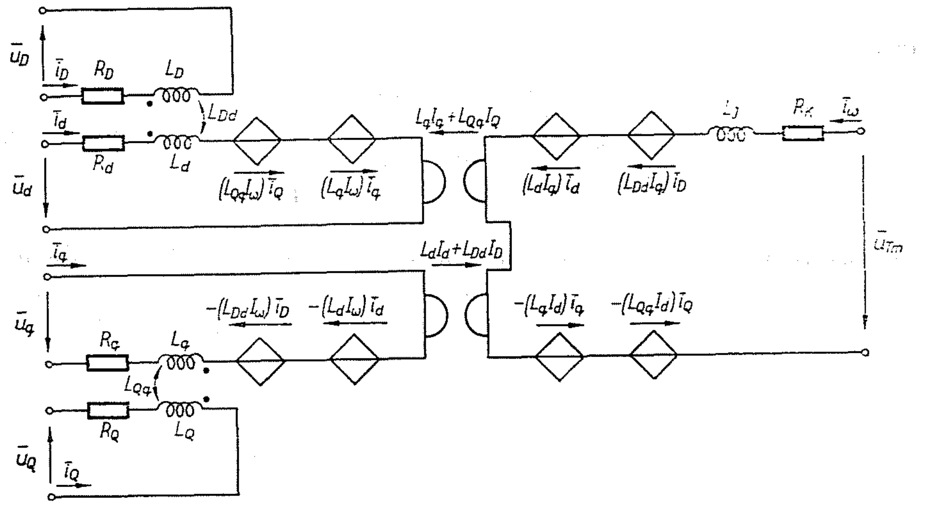

Fig. 8

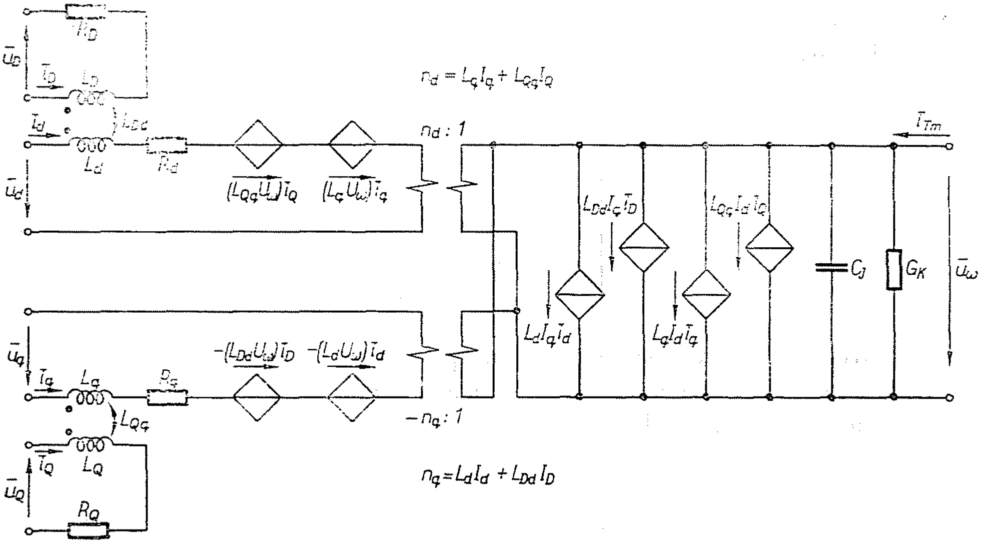

Fig. 9

Accordingly, the network model of the primitive machine can be determined in a procedure similar to that for the two-coil machine. Fig. 8 shows the network, the special case of which is the network shown in Fig. 3.

There is no difficulty of determining the network model in the case where electric quantities are made to correspond to the mechanical quantities according to analogy II. Without writing the equations for this case, a relationship completely identical in form with (28), is obtained only the quantities characterizing the mechanical circuit should be replaced by their duals, permitting to determine the equivalent circuit (Fig. 9).

5. Problems

The primary objective of the present paper is to reduce the examination of d. c. machines to network calculation methods. Therefore the described network models will be applied for two simple problems.

The first problem is to examine the change of armature current and of angular velocity of a motor with constant exciting current, if the armature circuit is connected to a direct voltage. The machine imposes a moment of inertia J_t and a friction K_t upon the motor. Under the condition i_s = constant, e. g. the model shown in Fig. 6 was seen to be adequate. In consequence of the load, the port corresponding to the mechanical quantities should be closed by an element of conductivity G_{Kt} and capacity C_{Jt} resulting in the network shown in Fig. 10.

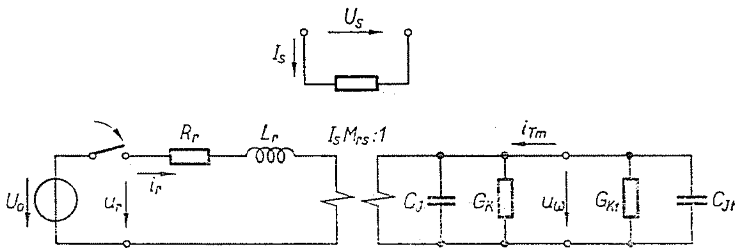

Fig. 10

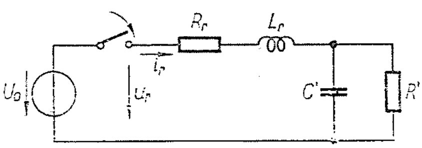

Fig. 11

By transforming the mechanical circuit to the rotor circuit [19] the network shown in Fig. 11 is obtained where

$$C' = \frac{C_J + C_{Jt}}{(M_{rs} I_s)^2} \qquad \text{and} \qquad R' = \frac{(M_{rs} I_s)^2}{G_K + G_{Kt}} .$$

The Laplace transform of i_r:

$$I_r(s) = \frac{U_0}{s} \frac{1}{R_r + sL_r + \dfrac{R'}{1 + sR'C'}} =$$

$$= \frac{U_0}{s} \frac{1 + sR'C'}{R_r + R' + s(L_r + R_r R'C') + s^2 L_r R'C'} .$$

To simplify calculations with parameters, let us assume that friction is negligible, i. e. $R' = \infty$. In this case

$$I_r(s) = U_0 \frac{C'}{1 + sR_rC' + s^2L_rC'}$$

the inverse transform being:

$$i_r(t) = 1(t) \frac{U_0}{R_r} \frac{e^{-\alpha t} - e^{-\beta t}}{(\beta - \alpha) T_r}$$

where

$$T_r = \frac{L_r}{R_r}, \qquad T_m = R_rC' = \frac{R_r (C_J + C_{Jt})}{(M_{rs} I_s)^2}$$

$$\alpha = \frac{1}{T_r}\left(0.5 - \sqrt{0.25 - \frac{T_r}{T_m}}\right), \qquad \beta = \frac{1}{T_r}\left(0.5 + \sqrt{0.25 - \frac{T_r}{T_m}}\right).$$

The change of angular velocity is given by u_ω, what is found, on the basis of the relationship for the currents of the ideal transformer, to be

$$u_\omega(t) = \frac{M_{rs} I_s}{C_J + C_{Jt}} \int_0^t i_r(\tau)\, d\tau = \frac{U_0}{\beta - \alpha} \frac{1(t)}{M_{rs} I_s} (\alpha e^{-\beta t} - \beta e^{-\alpha t} + \beta - \alpha) .$$

It should be noted that the solution of the problem is found in books dealing with d. c. machines [5], [11], leading to results identical with those given above. The equivalent circuit corresponding to Fig. 11 is also in usage.

In the following application problem an amplidyn is examined. Let us determine the correlation between the load circuit of the amplidyn and the voltage, the so-called external characteristics, from the network model for the steady state! On the stator of the amplidyn the control coil is mounted, while the commutator holds two brush couples with axes perpendicular to each other, one of which is short circuited, while the load is connected to the other. The second circuit includes also the compensating coil to counterbalance armature reaction [2], [5], [11]. The model of the amplidyn can accordingly

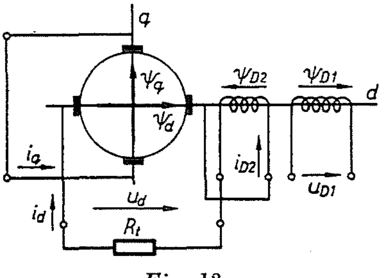

Fig. 12

be obtained from the primitive machine by omitting one of the stator coils, and mounting the compensating coil in common axis with the other one (Fig. 12). By a corresponding transformation, the network model of the amplidyn can be obtained from the network model of the primitive machine. The network model of the amplidyn for the steady state (Fig. 13) has been plotted according to Fig. 8. Here subscripts v, c, and t refer to the control coil, to the compensating coil, and to quantities characterizing the load circuit, respectively, futher

$$r_d = L_q I_q,$$

and

$$r_q = L_{D1d} I_{D1} - L_{D2d} I_{D2} + L_d I_d = L_{vd} I_v - (L_d - L_{cd}) I_t$$

the value of gyrator resistances.

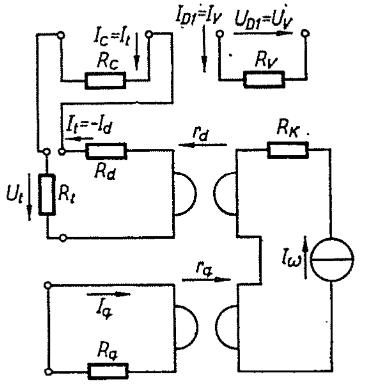

Fig. 13

The required relationship can be obtained from Kirchhoff's equation written for coil d:

$$U_t = L_q I_q I_\omega - I_t (R_d + R_c) .$$

Eliminating I_q:

$$U_t = \frac{L_q}{R_q} \frac{L_{vd}}{R_v} I_\omega^2 U_v - \frac{L_q}{R_q} I_\omega^2 L_d \left(1 - \frac{L_{cd}}{L_d} + R_d + R_c\right) I_t .$$

Suitable selection of the inductivity of the compensating coil (L_{cd}) leads to:

$$\frac{L_{cd}}{L_d} = 1 + R_d + R_c$$

in this case, the voltage appearing at the load side is

$$U_t = \frac{L_q L_{vd}}{R_q R_v} I_\omega^2 U_v ,$$

irrespective of the value of the loading current.

6. Conclusion

The models introduced above are advantageous by being suited for the joint examination of mechanical and electric processes taking place in d. c. machines, accessible only for network calculation methods. Thus the known processes and methods for the calculating electric networks can be used also for various d. c. machines, further, for electrically or mechanically coupled machine aggregates. A further possibility to be mentioned consists in the application of the presented models also for single phase commutator machines, by introducing the complex way of writing usual in the calculation of networks with sinusoidal current, and — linearizing by sections — also if the condition on the linearization of the equations is omitted (Chapter III.).

Summary

Linear circuit models lend themselves for the joint description of mechanical and electric phenomena in d. c. machines. To describe the whole of the d. c. machine by electric network model, mechanical quantities are substituted by electric quantities, based on the analogy between electric networks and certain mechanical systems. The network part describing the mechanical behaviour of the d. c. machine is coupled by a gyrator or ideal transformer to the network part describing the electric behaviour of the rotor. Beyond the linear network model of two-coil d. c. machines, also network models of primitive d. c. machines are described, helping to examine various d. c. machines, and electrically or mechanically coupled machine groups by means of mere network calculation methods.

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