PERFORMANCE OF THREE-PHASE SQUIRREL-CAGE INDUCTION MOTORS WITH ROTOR ASYMMETRIES

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1. Introduction

Squirrel-cage induction machines are the most commonly used electrical machines, they represent a great part of the electrical machines produced by the manufacturing industry all over the world. The main reasons of general application are reliability and simple technology. Squirrel-cage rotors for high performance machines are manufactured with copper rotor bars and end rings, while machines of lower performance are being manufactured with die-casting technologies. Manufacturing die-casted rotors raises several technological problems, as rotors must be free from impurities. Proper die-casting methods assume properly gated moulds, holding pressure, pressure on the die-casting material and proper heating processes. Tests of the rotors of squirrel cage induction motors have shown asymmetries in the rotor circuit of these machines, which in case of die-casted rotors are due to technological difficulties. During operation, the copper rotor bars and end rings of squirrel cage motors may break as a consequence of the improper technology or very heavy operation conditions.

As asymmetry of the rotor circuit is very detrimental to the performance of the machine, it is important to study its effect.

GOERGES [1] was the first to study the operation of three-phase induction motors with asymmetrical rotor circuit. Since then, many investigators have studied the effect of rotor asymmetries in case of slip-ring induction motors, including unbalanced external impedances and unbalanced connections. Only few papers deal, however, with the problem of three-phase squirrel-cage induction motors with rotor asymmetries.

A theoretical investigation of a squirrel cage induction motor was described in [2] for a case where the rings had cuts 360 electrical degrees apart, and cuts in the front ring were displaced against those in the back ring by 180 electrical degrees. An approximative calculation of current distribution was made. In [3] the current distribution of squirrel cage was determined with one rotor bar broken, but calculation was carried out for a model where the rotor was slotless and has a thin current sheet on its surface. The pulsating torques of machines having symmetries d, q are calculated in [4], and also a d, q equivalent circuit can be found in [5] and [6]. Application of these equivalent circuits becomes problematic when the elements of the network have to be calculated for a three-phase squirrel-cage induction motor with rotor asymmetries. Also an equivalent circuit has been derived in [7], but the equivalent circuit is erroneous as the end rings are omitted and the different bar impedances of the same rotor have been neglected.

2. Stating the problem, assumptions

The symmetrical component theory will be applied to derive an equivalent circuit for the case of squirrel cage machines with rotor asymmetry. Applicability of the model in case of rotor bars with different impedances will be presented for bars beside each other, or symmetrically spaced. With this assumption also the effect of some impurities in the rotor bars can be considered. It is shown how the model leads to the case of large asymmetries, such as the breaks of rotor bars. Anyhow the model can be applied very effectively in cases of rotor asymmetries where the fault is concentrated around an axis of the rotor circuit, or around two mutually perpendicular axes.

The following assumptions are being made:

1. The stator is symmetrical, balanced three-phase voltages of fundamental frequency are imposed upon them.
2. The air gap is constant.
3. The electromagnetic fields are sinusoidally distributed in space.
4. Saturation and eddy currents are neglected, although saturated values can be considered in the equivalent circuit.
5. The impedances are independent of currents.
6. In the equivalent circuit, all rotor quantities are referred to the stator.
7. The circumferential currents flowing in the rotor (as a consequence of imperfect bar iron insulation) are negligible.

3. Deriving the equivalent circuit

Let the rotor of the machine consist of m phases. In general, the symmetrical component rotor currents are:

$$I' = \mathbf{Y}' \cdot \mathbf{U}' \tag{1}$$

where

$$I' = \begin{bmatrix} I'_0 \\ I'_1 \\ \vdots \\ I'_{m-1} \end{bmatrix} \quad u' = \begin{bmatrix} U'_0 \\ U'_1 \\ \vdots \\ U'_{m-1} \end{bmatrix} \quad \mathbf{Y}' = \begin{bmatrix} Y'_0 & Y'_{m-1} Y'_{m-2} & \dots & Y'_1 \\ Y'_1 & Y'_0 \quad Y'_{m-1} & \dots & Y'_2 \\ \vdots & & & \\ Y'_{m-1} & Y'_{m-2} Y'_{m-3} & \dots & Y'_0 \end{bmatrix} \tag{1a}$$

I' is the column vector of the symmetrical component rotor currents, $\mathbf{Y}'$ is the symmetrical component admittance matrix, and U' is the column vector of symmetrical component rotor voltages. The transformed parameters are obtained from the phase variables by using

$$\mathbf{T} = \frac{1}{m}\begin{bmatrix} 1 & 1 & \ldots\ldots & 1 \\ 1 & \hat{\varepsilon}_m & \ldots\ldots & \hat{\varepsilon}_m^{(m-\prime)} \\ \vdots & & & \\ 1 & \hat{\varepsilon}_m^{(m-\prime)} & \ldots\ldots & \hat{\varepsilon}_m^{(m-1)\,(m-1)} \end{bmatrix} (\varepsilon_m = e^{j2\Pi/m}). \tag{2}$$

If the end rings are symmetrical, and only the positive and negative $(m-1)$ sequence rotor currents are considered, then:

$$\begin{bmatrix} I_1' \\ I_{m-1}' \end{bmatrix} = \begin{bmatrix} Y_0' & Y_2' \\ Y_{m-2}' & Y_0' \end{bmatrix} \begin{bmatrix} U_1' \\ U_{m-1} \end{bmatrix}. \tag{3}$$

The neglect of all other sequence components is permitted, since the stator coils are sinusoidally distributed, therefore all the rotor sequence component currents with the exception of the 1 and $m-1$ sequence ones, will produce magnetic fields with pole numbers different from that of the stators. (For the sake of simplicity in the following a two-pole machine will be considered, but the theory is easy to apply to machines with more pole numbers.) As a criterion for the allowable degree of asymmetry, the value of the torque dip existing because of the rotor asymmetry can be taken, and as the average torque in case of a cylindrical machine depends only from the 1 and $m-1$ sequence rotor currents, with the assumptions made above the determined average torque will absolutely be correct.

Considering (3) no equivalent circuit coupling the machine's positive and negative sequence equivalent circuits can directly be derived, but a specially chosen rotor co-ordinate system can be involved to show that $Y_2' = Y_{m-2}'$. In the following, this will be proved.

The symmetrical component admittances expressed in terms of the phase co-ordinates, derived from the modal matrix of a m-phase cyclical symmetrical admittance matrix, are:

$$Y_i' = \frac{1}{m}\sum_{k=1}^{m} Y_k\, \varepsilon_m^{(k-1)i} \quad i = 0, \ldots m-1. \tag{4}$$

If a single rotor bar with admittance Y_d differs from the others, with admittances Y_r, then if the first phase of the rotor coincides with the phase "a" of the stator, and the real axis of the rotor co-ordinate system is symmetrical to the rotor bar with differing impedance, then

$$Y_i' = \frac{Y_r}{m}\left[\sum_{t=0}^{(m-1)/2} e^{-j\frac{2\Pi}{m}it} + \sum_{t=1}^{(m-1)/2} e^{j\frac{2\Pi}{m}it} - \frac{Yd}{Y_r}\right] = Y_D' + Y_{im}'. \tag{5}$$

From (5)

$$Y'_{m-1}\frac{Y_r}{m}\left[\sum_{t=0}^{(m-1)/2} e^{-j\frac{2\Pi}{m}(m-i)t}+\sum_{t=1}^{(m-1)/2} e^{-j\frac{2\Pi}{m}(m-i)t}-\frac{Y_d}{Y_r}\right]=Y'_{im}+Y'_n \qquad (6)$$

so in (3) **Y** will be a matrix with cyclic symmetry.

In a case where more $(x-1)$ of equal rotor bars differ from the symmetrical bars, then if x is uneven:

$$Y'_i=\frac{Y_r}{m}\left[\sum_{t=0}^{\frac{m-x-1}{2}} e^{-j\frac{2\Pi}{m}it}+\sum_{t=1}^{\frac{m-x-1}{2}} e^{j\frac{2\Pi}{m}it}+\right.$$
$$\left.+\frac{Y_d}{Y_r}\sum_{t=\frac{m-x+1}{2}}^{\frac{m}{2}-1} e^{-j\frac{2\Pi}{m}it}+\frac{Y_d}{Y_r}\sum_{t=\frac{m-x+1}{2}}^{\frac{m}{2}-1} e^{j\frac{2\Pi}{2}it}-\frac{Y_d}{Y_r}\right] \qquad (7)$$

that is:

$$Y'_i=\frac{Y_r}{m}\left[1+\sum_{1}^{\frac{m-x+1}{2}} 2\cos\frac{2\Pi}{m}it+\right.$$
$$\left.+\frac{Y_d}{Y_r}\left(\sum_{\frac{m-x+1}{2}}^{\frac{m}{2}-1} 2\cos\frac{2\Pi}{m}it\right)-1\right]=Y'_{im(\text{uneven})}+Y'_{D(\text{uneven})}. \qquad (8)$$

From (8) it follows that

$$Y'_{m-i}=Y'_{im(\text{uneven})}+Y'_{D(\text{uneven})}. \qquad (9)$$

If x is even, then

$$Y'_i=\frac{Y_r}{m}\left[\sum_{t=0}^{\frac{m-x}{2}-1} 2\cos\Pi(2t+1)i/m+\frac{Y_d}{Y_r}\sum_{\frac{m-x}{2}}^{\frac{m}{2}} 2\cos\Pi(2t+1)i/m\right]=$$
$$=Y'_{im(\text{even})}+Y'_{D(\text{even})} \qquad (10)$$

so again

$$Y'_{m-i}=Y'_{im(\text{even})}+Y'_{D(\text{even})}. \qquad (11)$$

In a general case if x is either uneven or even, let

$$Y'_i=Y''_{im}+Y''_{iD} \qquad (11a)$$

therefore:

$$U'=\mathbf{Z}'I'\,;\ \mathbf{Z}'=\begin{bmatrix} Z_0 & Z_{m-1}\\ Z_{m-1} & Z_0\end{bmatrix} \qquad (12)$$

where

$$U'=\begin{bmatrix}U'_1\\ U'_{m-1}\end{bmatrix};\ I'=\begin{bmatrix}I'_1\\ I'_{m-1}\end{bmatrix};\ Z'=\frac{1}{Y}\begin{bmatrix}Z_{11}\,Z_{12}\\ Z_{21}\,Z_{22}\end{bmatrix};$$

$$\begin{aligned} Z_{11} &= (Y''_{0m}+Y''_{0D}+Y''_{1m}+Y''_{1D})(Y''_{0m}+Y'_{0D}-Y'_{1m}-Y''_{1D})\\ Z_{12} &= (Y''_{1m}+Y''_{1D})^2-(Y''_{0m}+Y''_{0D})(Y''_{2m}+Y''_{2D})\\ Z_{21} &= (Y''_{1m}+Y''_{1D})^2-(Y''_{0m}+Y''_{0D})(Y''_{2m}+Y''_{2D})\\ Z_{22} &= (Y''_{0m}+Y''_{0D}-Y''_{1m}-Y''_{1D})(Y''_{0m}+Y''_{0D}+Y''_{1m}+Y''_{1D}) \end{aligned} \tag{13}$$

and

$$\begin{aligned} Y = (Y''_{0m}+Y''_{0D})\,[(Y''_{0m}+Y''_{0D})^2-(Y''_{2m}+Y''_{2D})^2-\\ -2\,(Y''_{1m}+Y''_{1D})^2]+2\,(Y''_{2m}+Y''_{2D})(Y''_{1m}+Y''_{1D})\,. \end{aligned} \tag{13a}$$

According to (13) the equivalent circuit can be realized (Fig. 1).

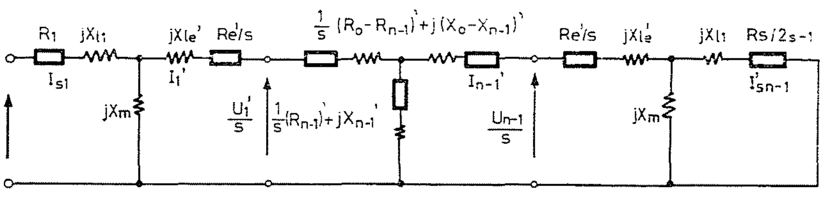

Fig. 1. An equivalent circuit of the squirrel-cage induction motor with asymmetrical rotor bars

In the equivalent circuit, R_1 and X_{l1} are stator resistance and leakage reactance of one stator phase resp., R'_e and X'_e are the same end ring quantities of the symmetrical machine but referred to the stator side, X_n is the magnetizing reactance. The elements of the T-network connecting the machine's modified positive and negative sequence network can be calculated from (12). The effect of skew can be considered in the well-known way.

4. Conclusions

The equivalent circuit can also be applied if the impedances of the rotor bars which differ from the symmetrical machines bars, are not equal, but are symmetrically spaced around the real axis of the rotor co-ordinate system or around the two rotor axes, i.e., if the asymmetry is of the concentrated type. Of course, in this case (8) and (10) should be modified.

For double-cage machine with the two cages connected with one end ring each, then the bars can be considered to be parallel connected, but the equations include an additive admittance for the leakage flux lines coupling the upper and lower rotor bars. If there exist two end rings, then the derived equation will be formally the same as (12), but the 1 and $m-1$ sequence rotor voltages refer separately to the inner and outer cage, resp.

From the equivalent circuit the rotor current distribution is

$$I_{Bk} = Y_{rK}\,(U'_0 + U'_{m-1}\,e^{-j2\Pi(k-1)(m-1)/m} + U'_1\,e^{-j2\Pi\,(k-1)/m}) \tag{14}$$

where

$$U'_0 = [(Y'_1\,Y'_{m-2} - Y'_0\,Y'_{m-1})\,I'_1 + (Y'_2\,Y'_{m-1} - Y'_0\,Y'_1)\,I'_{m-1}]\,\frac{1}{Y}\,.$$

In the stator of the machine a balanced current of line frequency (I_{s1}) and of $(2s-1)$ times line frequency (I'_{s1}) appear, so

$$I_s = \sqrt{|I_{s1}|^2 + |I'_{sm-1}|^2} \qquad \text{for } s \neq 1\,.$$

A current of frequency $(2s-1)$ flows through the impedance of the stator winding and the power lines supplying the stator. Generally the impedance of the supply network is lower than the stator impedance, so the $(2s-1)$ currents can be assumed as short circuited through the stator impedance.

Both the rotor voltages and currents are of slip frequency. The stator field reacting with the positive sequence rotor current develops the positive sequence torque, which helps the motor in accelerating. The negative sequence rotor field induces the negative sequence stator currents, and produces a resulting torque containing a dip around the half-speed region. The value of the dip is a function of the degree of asymmetry. The asymmetry results in pulsating torques of frequencies $2sf_1$ (s is the slip, and f_1 is the line frequency), which superposes on the average torque. This torque results from the interaction of the negative sequence rotor fields with the positive sequence rotor *mmf*, and vice versa. The main value of this pulsating torque is zero, it does not contribute to the motor output, but causes undesirable noises and vibrations. Thus, in need of a squirrel-cage induction motor with low noise level and vibration, great care should be taken of the die-casting process. Equations of the rotor currents demonstrate that damage in one rotor bar can damage surrounding ones, as asymmetry causes unsymmetrical current distribution.

If all rotor bars of the induction motor would be different, then of course symmetrical component method could not be applied. In this case the only possibility seems to be to solve differential equations of the machine by means of digital computer. Relevant research work is under way, although this method yields a poorer physical insight into the operation of the machine than does symmetrical component theory.

If some of the rotor slots are not filled with conducting material, then a cage system similar to that of the synchronous machine results, with special assumptions the derived equivalent circuit is valid for this case too.

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Summary

An equivalent circuit is derived for the rotor asymmetries of squirrel-cage induction machines based on the symmetrical component theory. The model can be used in case of rotor bars with different impendances, even with some of the rotor bars broken. The model is applicable for both single and double cage induction motors. The effects of unbalance on the machine's performance are studied. The method can also be used for studying the effects of different damper windings in synchronous motors. The equations are easy to solve on a digital computer.

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