SOME PROBLEMS OF ADAPTIVE OPTIMAL PROCESS CONTROL

By

L. KEVICZKY, J. KOCSIS and Cs. BÁNYÁSZ*

Department of Automation, Technical University, Budapest

(Received July 10, 1973)

Presented by Prof. F. CSÁKI

Introduction

To state the problem in general, let us consider the idealized process model in Fig. 1. There $\mathbf{u}$, $\mathbf{z}_1$, $\mathbf{z}_2$ are an $(M \times 1)$ vector of control action, an $(M \times 1)$ vector of disturbances in control action and $(K \times 1)$ vector of observable but uncontrollable input variables, respectively; $\mathbf{x}_0$ — is an $(N \times 1)$ vector of ideal input signals, $(N = M+K)$; $\boldsymbol{\xi}$ — is an $(N \times 1)$ vector of input noise; $\mathbf{x}$ — is an $(N \times 1)$ vector of the measured input signals; $\boldsymbol{\eta}$ — is an $(P \times 1)$ vector of non-observable disturbances; $\mathbf{v}$ — is an $(L \times 1)$ vector of "inner" state-variables; y_0 — is the ideal output signal; ε — is the output noise; y — is the measured output signal. Let us assume that

$$E\{\mathbf{z}_1\} = \mathbf{0}\ ; \quad E\{\mathbf{0}\} = \mathbf{0} \quad \text{and} \quad E\{\varepsilon\} = 0 \tag{1}$$

where $E\{\ldots\}$ is the expected value.

According to a recently generalized conception, the inner structure of process can be divided to a linear dynamic and a non-linear static part by means of state variables $\mathbf{v}$ which can be chosen in several ways. Here $\mathbf{G}(s)$ is an $(L \times N)$ transfer function matrix of dynamics, $y_0(\mathbf{v})$ is a scalar-vector function assumed to be unimodal, describing the nonlinear characteristics.

First, let us investigate the adaptive optimal control of a quasistationary process. Then $\mathbf{G}(s) = \mathbf{I}$ (identity matrix), i.e. $\mathbf{v}$ equals $\mathbf{x}$. The purpose of the control is to determine an effect $\mathbf{u}$ observing $\mathbf{x}$ and y which guarantees minimum expected value of y (case of the cost performance index). The strategy of the adaptive optimum control is realized according to the idea of dual control where the decision on the control is made by means of an adaptive model computed on the basis of observed input and output values. The usual complete quadratic form is used for the quasistationary process with large-scale signal

* Computer and Automation Research Institute, Hungarian Academy of Sciences, Budapest

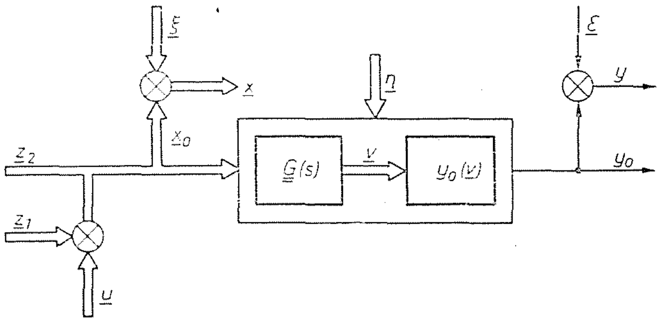

Fig. 1. Model of process

as an adaptive model. For sake of simplicity let us introduce notations $\mathbf{z}_2 = \mathbf{z}$ and

$$\mathbf{x} = [\mathbf{u}^T, \mathbf{z}^T]^T \quad (N \times 1) \quad \text{vector} \tag{2}$$

where T refers to the transposition and $N = M + K$. According to the above, our static adaptive model is

$$\hat{y} = c_0 + \mathbf{c}^T\mathbf{x} + \mathbf{x}^T\mathbf{C}\mathbf{x} = \mathbf{a}^T\boldsymbol{\varphi}(\mathbf{x}) \tag{3}$$

where

$$\mathbf{c} = [\mathbf{c}_u^T, \mathbf{c}_z^T]^T \tag{4}$$

and

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{uu} & \mathbf{C}_{uz} \\ \mathbf{C}_{zu} & \mathbf{C}_{zz} \end{bmatrix}. \tag{5}$$

According to Eq. (2) $\mathbf{c}_u$ and $\mathbf{c}_z$ are $(M \times 1)$, $(M \times K)$ vectors, $\mathbf{C}_{vu}$, $\mathbf{C}_{zz}$ and $\mathbf{C}_{uz} = \mathbf{C}_{zu}^T$ are $(M \times M)$, $(K \times K)$ and $(M \times K)$ matrices, respectively. The right side of Eq. (3) means the linearized form of the quadratic expression. The elements in the diagonal line and above the diagonal of matrix $\mathbf{x}'\mathbf{x}'^T$ used to be assigned to the vector of function components $\boldsymbol{\varphi}(\mathbf{x})$ row by row, where $\mathbf{x}' = [1, \mathbf{x}^T]^T$. Accordingly, there are $q = (N+1)(N+2)/2$ elements in $\boldsymbol{\varphi}(\mathbf{x})$ and in vector $\mathbf{a}$. Unambiguous relationships can be established between the left and right side of (3) that are simple to convert.

The algorithm of optimal control

Considering the adaptive optimal control as a discrete-time process — according to the process control by computer — $\mathbf{x}[n\Delta t]$ is replaced by the simpler $\mathbf{x}[n]$ for the discrete-time signals, i.e. sampling time Δt is taken unit. As it was mentioned the purpose of control is to provide for the condition

$$E\{y(\mathbf{u}, \mathbf{z}, \ldots)\} = \min_{\mathbf{u}} \tag{6}$$

in spite of disturbances. The general form of relevant algorithms [1]:

$$\mathbf{u}[n+1] = \mathbf{u}[n] - \mathbf{R}_1[n]\nabla_u y(\mathbf{x}[n], \mathbf{a}[n]) . \tag{7}$$

(Here ∇ means the gradient vector.) This expression can be considered as a general form of stochastic approximation which may include other methods [5], too — considered as classical — by suitably choosing $\mathbf{R}_1[n]$.

Let us determine

$$\nabla_u y(\mathbf{x}[n], \mathbf{a}[n]) \cong \nabla_u \hat{y}(\mathbf{x}[n], \mathbf{a}[n]) \tag{8}$$

from the adaptive model according to the idea of dual control. Differentiating (3) the optimal control is:

$$\mathbf{u}[n+1] = \mathbf{u}[n] - \mathbf{R}_1[n]\{\mathbf{c}_u[n] + 2\mathbf{C}_{uu}[n]\,\mathbf{u}\,[n] + 2\mathbf{C}_{uz}[n]\,\mathbf{z}\,[n]\} \tag{9}$$

where $\mathbf{R}_1[n]$ is an $(M \times M)$, so called weighting or convergence matrix and more or less general convergence conditions are formulated for it in [1], [3], [5].

The optimal convergence matrix in quadratic sense is:

$$\mathbf{R}_1[n] = \left[\sum_{m=1}^{n} \mathbf{H}_u\{\hat{y}(\mathbf{x}[n], \mathbf{a}[m])\}\right]^{-1} \tag{10}$$

where

$$\mathbf{H}_u\{\hat{y}(\mathbf{x}[n], \mathbf{a}[m])\} = \left[\frac{\delta^2 \hat{y}}{\delta u_i\, \delta u_j}\right] = 2\mathbf{C}_{uu}[m] \tag{11}$$

is the Hessian-matrix of second order derivatives by $\mathbf{u}$. Unfortunately $\mathbf{R}_1[n]$ in (10) cannot be computed by iteration. The algorithm becomes simpler if the control action intervenes only after coefficients $\mathbf{a}[n]$ reached a certain accuracy. In this case

$$\mathbf{R}_1[n] = \frac{1}{2}\,\mathbf{C}_{uu}^{-1}[n] \tag{12}$$

is also suitable. This latter $\mathbf{R}_1[n]$ has an expressive meaning, namely in this case

$$\mathbf{u}[n+1] = -\mathbf{C}_{uu}^{-1}[n]\{\frac{1}{2}\,\mathbf{c}_u[n] + \mathbf{C}_{uz}[n]\mathbf{z}[n]\} \tag{13}$$

which gives an estimation for the extremum of quadratic surface under restriction $\mathbf{z}[n]$.

Algorithm of adaptive identification

The least-squares model is applied to identify the static characteristic of process, i.e. to determine the adaptive model. (This method gives optimal, unbiased estimation for the model parameters in the case of independent output noise of normal distribution and identical variance if the input variables are measured without error [4].) In case of LS method, the expected value of performance index

$$Q(\mathbf{x}[n], \mathbf{a}[n-1]) = \frac{1}{2}(y[n] - \hat{y}[n])^2 = \frac{1}{2}(y[n] - \mathbf{a}^T[n-1]\,\boldsymbol{\varphi}(\mathbf{x}[n]))^2 \quad (14)$$

is minimized. The general algorithm of stochastic approximation is used for the minimization (see in [1]) similarly to (7), according to which

$$\mathbf{a}[n] = \mathbf{a}[n-1] - \mathbf{R}_2[n]\,\nabla_a Q(\mathbf{x}[n], \mathbf{a}[n-1]) \,. \quad (15)$$

Convergence conditions relating to $(q \times q)$ convergence matrix $\mathbf{R}_2[n]$ can be found in [1], [3], [5]. Applying the rules of vector differentiation [6] the term $\nabla_a Q(\mathbf{x}[n], \mathbf{a}[n-1])$ in (15) is:

$$\nabla_a Q(\mathbf{x}[n], \mathbf{a}[n-1]) = -\left(y[n] - \mathbf{a}^T[n-1]\,\varphi(\mathbf{x}[n])\right) \cdot \cdot \left\{ \varphi(\mathbf{x}[n]) + \mathbf{S}[n] \frac{d\,\hat{y}(\mathbf{x}[n], \mathbf{a}[n-1])}{d\,\mathbf{u}[n]} \right\} \quad (16)$$

where the $(q \times M)$ sensitivity matrix [1]

$$\mathbf{S}[n] = \mathbf{J}^T(\mathbf{u}[n], \mathbf{a}[n-1]) = \frac{d\mathbf{u}^T[n]}{d\mathbf{a}[n-1]} \quad (17)$$

is introduced to cope with the fact that during the identification also the control is changing, in general case. Here $\mathbf{J}^T$ means the transpose of Jacobian-matrix [6]. In control stragies where the intervals of two control actions permit the model to become sufficiently accurate during which the control is constant, the $\mathbf{S}[n]$ is zero. From (15), (16) and (3):

$$\mathbf{a}[n] = \mathbf{a}[n-1] + \mathbf{R}_2[n]\left(y[n] - \mathbf{a}^T[n-1]\,\varphi(\mathbf{x}[n])\right)$$

$$\left\{\varphi(\mathbf{x}[n]) + \mathbf{S}[n]\left[\mathbf{c}_u[n-1] + 2\mathbf{C}_{uu}[n-1]\mathbf{u}[n] + 2\,\mathbf{C}_{uz}[n-1]\mathbf{z}[n]\right]\right\}. \quad (18)$$

The convergence matrix of identification algorithm can be chosen in several ways. If $\mathbf{R}_2[n]$ is optimized in quadratic sense, then the result of stochastic approximation corresponds to the generally known recursive solution of LS method [3], [5]. In this case the optimal weighting matrix can be computed step by step, recursively:

$$\mathbf{R}_2[n] = \mathbf{R}_2[n-1] - \frac{[\mathbf{R}_2[n-1]\,\varphi(\mathbf{x}[n])]\,[\mathbf{R}_2[n-1]\,\varphi(\mathbf{x}[n])]^T}{1+\varphi^T(\mathbf{x}[n])\,\mathbf{R}_2[n-1]\,\varphi(\mathbf{x}[n])}\,. \quad (19)$$

In the recursive algorithm (19) $\mathbf{R}_2[0]$ can be chosen as a result of an off-line LS estimation

$$\mathbf{R}_2[0] = \left[\sum_{m=-j}^{0} \varphi(\mathbf{x}[m])\varphi^T(\mathbf{x}[m])\right]^{-1} \quad (20)$$

or as a diagonal matrix of a sufficiently large constant value.

The suboptimal scalar convergence coefficient $r[n]$ needs less operations than (19), and it can also be used but it provides a lower convergence speed. Determining the suboptimal value $r[n]$ by the steepest-descent method we get

$$r[n] = \frac{1}{y[n]-\mathbf{a}^T[n-1]\,\varphi(\mathbf{x}[n])} \cdot \frac{\varphi^T(\mathbf{x}[n])[\mathbf{k}[n]-\mathbf{G}[n]\,\mathbf{a}[n-1]]}{\varphi^T(\mathbf{x}[n])\,\mathbf{G}[n]\,\varphi(\mathbf{x}[n])} \quad (21)$$

where

$$\mathbf{k}[n] = \mathbf{k}[n-1] + y[n]\,\varphi(\mathbf{x}[n])\,; \quad \mathbf{k}[0] = \mathbf{0} \quad (22)$$

$$\mathbf{G}[n] = \mathbf{G}[n-1] + \varphi(\mathbf{x}[n])\,\varphi^T(\mathbf{x}[n])\,; \quad \mathbf{G}[0] = \mathbf{0}\,. \quad (23)$$

The adaptive model can be investigated for adequacy by a dynamic quadratic sum of residuals

$$d_k^2[n] = \frac{\sum_{i=n+1-k}^{n} (y[i]-\hat{y}[i])^2}{k-q}\,, \quad k > q \quad (24)$$

where q is the number of model parameters. For $k = n$, i.e. every measurement is taken into account, $d_n^2[n] = d^2[n]$ can be computed by iteration:

$$d^2[n] = d^2[n-1] + \frac{1}{n-q}\left[-d^2[n-1] + \frac{(y[n]-\mathbf{a}^T[n-1]\,\varphi(x[n]))^2}{1+\varphi^T(\mathbf{x}[n])\,\mathbf{R}_2[n-1]\varphi(\mathbf{x}[n])}\right]. \quad (25)$$

In adequacy testing either we consider whether $d^2[n]$ is become constant enough or one of the statistical tests is applied to compare $d^2[n]$ with the variance of y.

Determination of sensitivity model

In the general case of dual control the control action is changing during identification, therefore — as it was shown in (18) — the sensitivity matrix $\mathbf{S}[n]$ appears in the algorithm of identification. The sensitivity model — which produces the recursive computation of sensitivity matrix $\mathbf{S}[n]$ — is obtained by differentiating $\mathbf{u}^T[n]$ by $\mathbf{a}[n-1]$:

$$\mathbf{S}[n] = \mathbf{S}[n-1] - \{\mathbf{J}(\varphi(\mathbf{x}[n-1]), \mathbf{u}[n-1]) + 2\mathbf{S}[n-1]\mathbf{C}_{uu}[n-1]\}\mathbf{R}_1[n-1]. \tag{26}$$

Here $\mathbf{J}(\varphi(\mathbf{x}[n-1]), \mathbf{u}[n-1])$ is a $(q\times M)$ Jacobian matrix of $\varphi(\mathbf{x}[n-1])$ with respect to $\mathbf{u}[n-1]$ easy to compute on the basis of (3) [1], [8].

Remind that if $\mathbf{R}_1[n]$ is chosen according to (12) and there is no observable, uncontrollable input then the sensitivity matrix $\mathbf{S}[n]$ is obviously irrelevant for (18).

Utilization of input signal synthesis

In case of control strategies where the control is delayed until a certain accuracy of identification, the convergence rate of identification is very important. The convergence speed and information about the process can be maximized by the input signal synthesis. In case of $\mathbf{R}_2[n]$ in (19) the optimization of identification can only be ensured by $\mathbf{x}[n]$. The global maximum of quadratic form $\varphi^T(\mathbf{x}[n])\mathbf{R}_2[n-1]\varphi(\mathbf{x}[n])$ is ensured by $\mathbf{z}_1[n]$ in a given confined environment of the actual $\mathbf{u}[n]$ as working point. In this way the determinant of $\mathbf{R}_2[n]$ ($\mathbf{R}_2[n]$ is proportional to covariance matrix of estimation $\mathbf{a}[n]$) can be ensured to decrease at a maximum rate in every step [9]. This method can also be applied when the control is changing in every step but then the situation of so-called triple control is brought about. Thus, after all, the optimization of perturbing test signals, entering the system from outside, is due to the triple control method.

Consideration of restrictions

Let us suppose that the control problem (6) has to be solved under restrictions

$$E\{\mathbf{g}(\mathbf{x})\} = \mathbf{0} \tag{27}$$

where $\mathbf{g}(\mathbf{x})$ is an $(F\times 1)$ vector. Assuming the $\hat{y}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ to be continuously differentiable, furthermore $\mathbf{g}(\mathbf{x})$ to satisfy the Slaterian regularity condition, the optimal control can be obtained by local Kuhn-Tucker's theory solved by stochastic approximation as [1]:

$$\mathbf{u}[n+1] = \mathbf{u}[n] - \mathbf{R}_1[n]\{\mathbf{c}_u[n] + 2\mathbf{C}_{uu}[n]\mathbf{u}[n] + \\ + 2\mathbf{C}_{uz}[n]\mathbf{z}[n] + \mathbf{J}^T(\mathbf{g}(\mathbf{x}[n]),\ \mathbf{u}[n])\boldsymbol{\lambda}[n]\} \tag{28}$$

where

$$\boldsymbol{\lambda}[n+1] = \max\{\mathbf{0}; \boldsymbol{\lambda}[n] + \mathbf{R}_3[n]\mathbf{g}(\mathbf{x}[n])\}\ ;\ \boldsymbol{\lambda}[0] \geq \mathbf{0}\,. \tag{29}$$

Here the note "max" relates to each component, $\mathbf{J}$ means — as mentioned above — the Jacobian-matrix (derivatives of $\mathbf{g}^T$ with respect to $\mathbf{u}$). (10) and (12) can be chosen for $\mathbf{R}_1[n]$ but for $\mathbf{R}_3[n]$ the scalar convergence coefficient $r[n] = 1/(n^z+\beta)$ proved to be the best one.

The influence of process dynamics

As far as the transfer functions (or impulse responses) of "channels" referred to the input variables of process are a priori known, the algorithms in the previous section can be simply modified for the dynamic case. Then the transfer function matrix $\mathbf{G}(s) = \mathrm{diag}[W_1(s), \ldots W_N(s)]$ for vector $\mathbf{v}$ is assumed to be a diagonal matrix. In this case the discrete convolution model for components of state vector is:

$$v_i[n] = \sum_{m=0}^{n} w_i[m]x_i[n-m]\,, \qquad i = 1, \ldots, N \tag{30}$$

where w_i is the impulse response of the ith "channel". Accordingly

$$\frac{d\mathbf{v}_u^T[n]}{d\mathbf{u}[n-1]} = \mathbf{D}(w[1]) = \mathrm{diag}\,[w_1[1], \ldots, w_n[1]] \tag{31}$$

where $\mathbf{v}_u$ contains only the part relating to $\mathbf{u}$ of vector $\mathbf{v}$, since similarly to $\mathbf{x}$

$$\mathbf{v} = [\mathbf{v}_u^T, \mathbf{v}_z^T]^T\,. \tag{32}$$

Taking into account expressions (30), (31) and (32), Eqs. (9), (18) and (26) are modified as:

$$\mathbf{u}[n+1] = \mathbf{u}[n] - \mathbf{R}_1[n]\mathbf{D}(w[1])\{c_u[n] + 2\mathbf{C}_{uu}[n]\mathbf{v}_u[n] + 2\mathbf{C}_{uz}[n]\mathbf{v}_z[n]\} \tag{33}$$

$$\mathbf{a}[n] = \mathbf{a}[n-1] + \mathbf{R}_2[n](y[n] - \mathbf{a}^T[n-1]\boldsymbol{\varphi}(\mathbf{v}[n])) \\ \{\boldsymbol{\varphi}(\mathbf{v}[n]) + \mathbf{S}[n]\mathbf{D}(w[1])\,[\mathbf{c}_u[n-1] + 2\mathbf{C}_{uu}[n-1]\mathbf{v}_u[n] + 2\mathbf{C}_{uz}[n-1]\mathbf{v}_z[n]]\} \tag{34}$$

$$\mathbf{S}[n] = \mathbf{S}[n-1] - \{\mathbf{J}(\boldsymbol{\varphi}(\mathbf{v}[n-1]), \mathbf{u}[n-1]) + \\ + 2\mathbf{S}[n-1]\mathbf{D}(w[1])\mathbf{C}_{uu}[n-1]\}\mathbf{D}(w[1])\mathbf{R}_1[n-1] \tag{35}$$

since now $\mathbf{v} \neq \mathbf{x}$ and

$$\hat{y} = c_0 + \mathbf{c}^T\mathbf{v} + \mathbf{v}^T\mathbf{C}\mathbf{v} = \mathbf{a}^T\boldsymbol{\varphi}(\mathbf{v}). \tag{36}$$

It is obvious also in the dynamic case, that use of $\mathbf{R}_1[n]$ in (12) eliminates the sensitivity model, makes it irrelevant for the identification.

Simulation results

A simulator PCSP (Process Control Simulator Program) has been constructed for investigating ideas and algorithms worked out for adaptive optimal control. Our results given by this program will be briefly reviewed.

The static characteristic is represented by a positive quadratic form; for sake of simplicity, the dynamics of process in every "channel" is chosen as first-order lag, hence the state-variables $\mathbf{v}$ are produced by the following equation [7]:

$$v_i[n] = e^m v_i[n-1] + (1 - e^{-m})x_i[n] \tag{37}$$

where

$$m = \Delta t/T. \tag{38}$$

Here T means the time constant of first order lag (for sake of simplicity, it is identical for every "channel").

Let us first investigate the case where the identification at certain working points determined by the control occurs in time t_i, i.e. $0 < n \leq t_i/\Delta t$ in (18). The necessary condition of this strategy is the existence of vector $\mathbf{z}_1$ in Fig. 1, the perturbation part of control signal. Now let us consider the case where $m \to \infty$, i.e. that of quasistationary control ($\mathbf{S}[n] \equiv \mathbf{0}$; $\mathbf{D}(w[1]) \equiv \mathbf{I}$). The influence of variance of the output noise on control is shown in Fig. 2 for cases of optimal convergence matrix (19) and suboptimal convergence coefficient (21) controlled by (9), $\mathbf{R}_1[n]$ corresponds to (12). The optimal identification algorithm is presented in Fig. 2/a and the suboptimal one in Fig. 2/b. For sake of comparability, the identification time is fixed at $t_i = 30\Delta t$. The variance of output error has been referred to constant term a_0 of static characteristics (meanwhile dispersion of $\mathbf{z}_1$ is constant). Possibilities of choosing initial values $\mathbf{R}_2[0]$, $\mathbf{G}[0]$ and $\mathbf{k}[0]$ differently have no great influence on the convergence rate of identification. It can be established from the figure that the variance of output noise does not influence significantly the convergence rate of control. Accordingly further on the optimal convergence matrix will be used also for identification.

The case where the algorithms suppose a quasistationary process, while in fact it has dynamics, has been investigated. The results are surprising: the control did make the system to tend to optimum, if not in the suboptimal (21) case by using the optimal convergence matrix (19) even in the case of $m = 0.5$

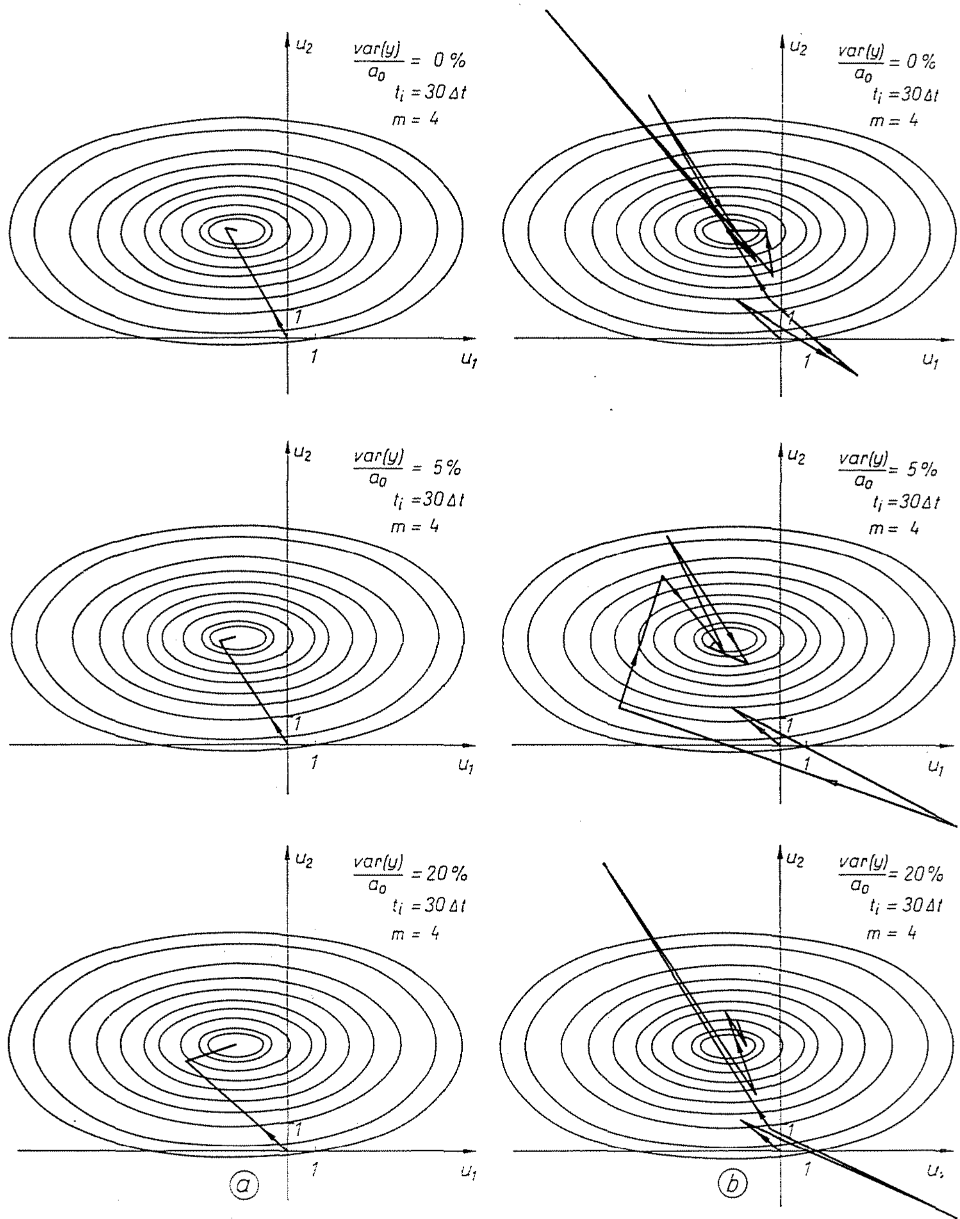

Fig. 2. Comparison of optimal and suboptimal strategies for identification

(see Fig. 3). Obviously, the result can be improved by increasing the identification time even for $m = 0.5$, Fig. 4. Figs 3 and 4 lead to the conclusion that the convergence rate of control depends slightly on the variance of output noise. Omitting the process dynamics in the algorithms acts as if a "dynamic noise" would appear inside the process, equivalent to input variables measured with error.

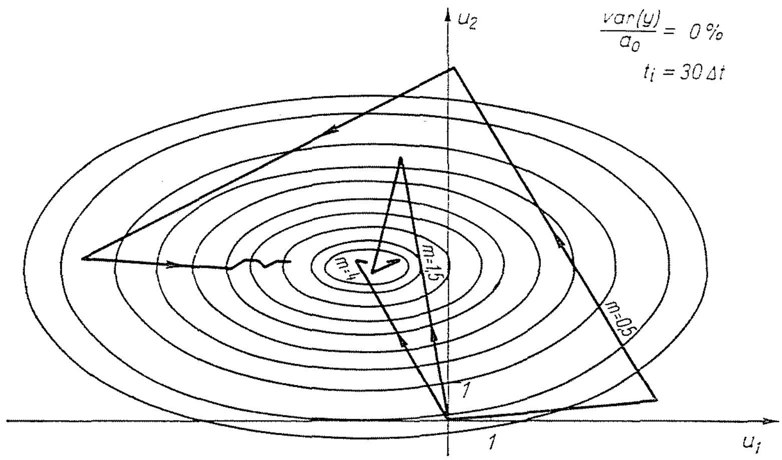

Fig. 3. Influence of process dynamics on the control

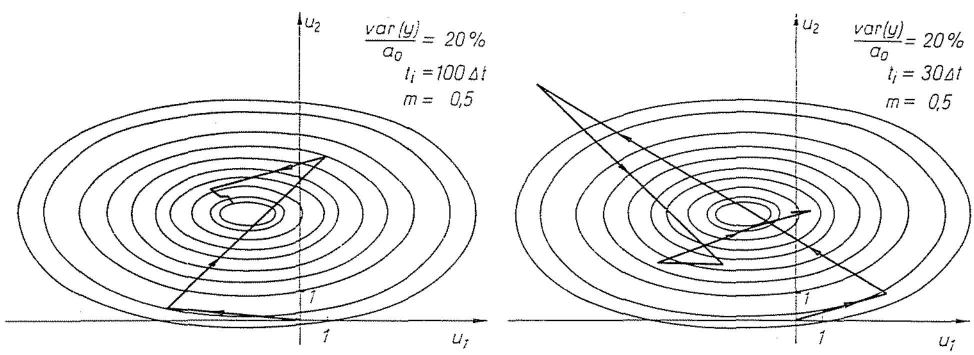

Fig. 4. Influence of identification time on the control

In case of small time constants the quasistationary control is still admissible but for higher values the identification becomes so unreliable that the control is completely bad. In case of large time constants of a priori known values the algorithms (33), (34), (35) can be used. The influence of a priori knowledge of time constants (m values) on the control is presented in Fig. 5. On the left side of Fig. 5, $m \neq \hat{m} \to \infty$, on the right one $m = \hat{m}$ (here $\hat{m}$ is an a priori known value). Control by the algorithms relating to the dynamic case is seen to work well.

In the classical case of dual control, i.e. for $t_i = \Delta t$, the experiences are equivalent to the previous ones but the control is more sensitive to the process dynamics.

The control under restriction has also been investigated by PCSP on the basis of algorithm (28). Using $r_3[n] = 1/n$, the control is shown in Fig. 6

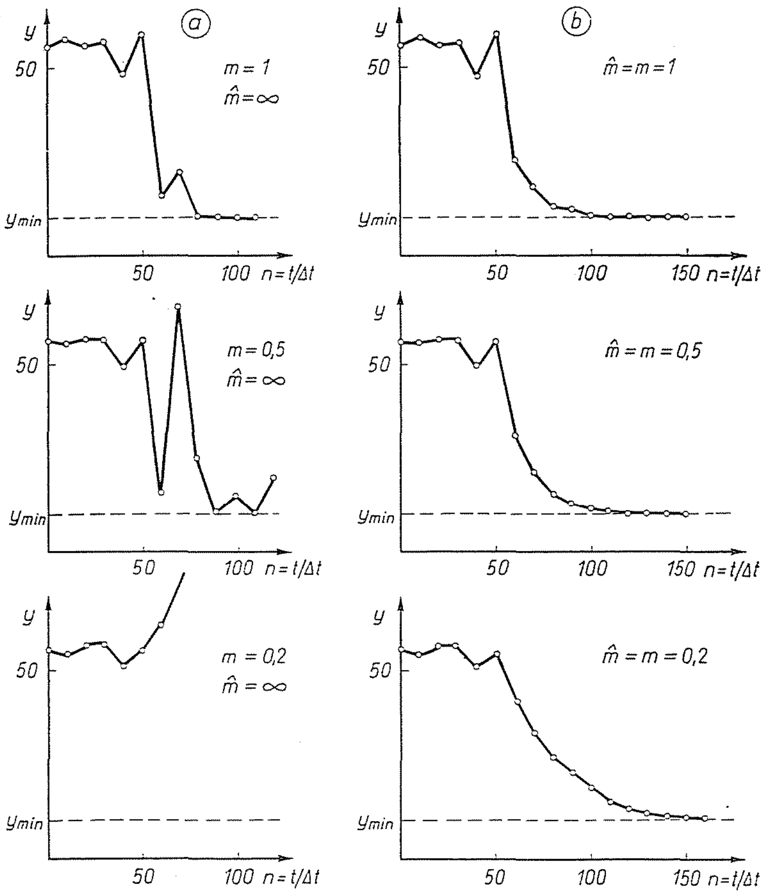

Fig. 5. Comparison of quasistationary and dynamic control algorithms

for convergence matrices $\mathbf{R}_1[n] = 1/n\ \mathbf{I}$, $\mathbf{R}_1[n] = 5/(4+n)\ \mathbf{I}$ and $\mathbf{R}_1[n]$ chosen according to (12). The control is rather efficient (though the convergence rate depends considerably on $\mathbf{R}_1$) but the restrictions are contravened during the intermediate steps, as expected from theoretical consideration. Thus this algorithm can be applied only in knowledge of $\mathbf{g}(\mathbf{x})$, by introducing another iteration cycle where first the conditional extremum must be computed on the model to determine the actual control.

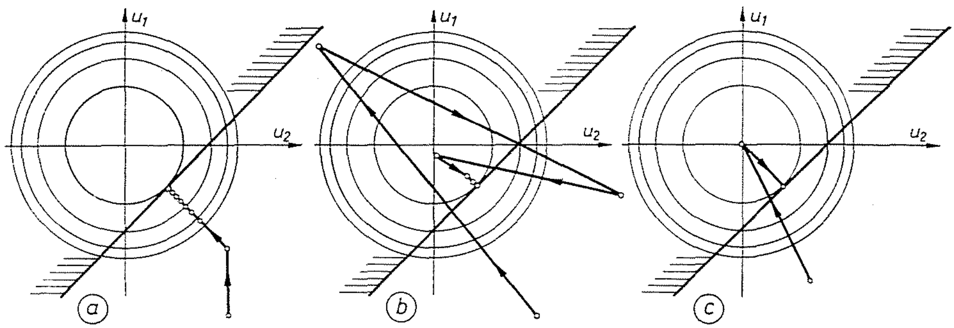

Fig. 6. Control under restrictions by different strategies

Conclusions

In this paper some adaptive optimal control algorithms elaborated for quadratic cost function are presented for quasistationary and dynamic cases where the dynamics in every "channel" is a priori known. The importance of optimally choosing convergence matrices has been established and illustrated by simulation examples. In case of optimal convergence matrix, the sensitivity model becomes unnecessary. We have shown the influence of process dynamics on the control that can be eliminated by the developed algorithms. The adaptive control is applied in case of explicit restrictions, too, and we have shown that it can be used only by the extremum-seeking performed on the model under restriction.

In most problems of adaptive optimal process control the quadratic form used in this paper is sufficient to define a system in its operation range. It is advantageous by requiring the least of necessary parameters — of importance in case of great many variables — and by lending the algorithms of adaptive optimal control and they have a simple easy to handle form in case of this simple, fixed structure.

The simulator PCSP has been constructed on the basis of these algorithms providing a multitude of useful experiences helping to form practical process control software.

Summary

Tsypkin has shown an algorithm based on dual control for the adaptive control of a dynamic non-linear single input — single output system [1] — with known dynamic lag series, connected with the input and quadratic performance index (loss function). This method has been generalized for adaptive control of multiple input — single output system [2]. Since then this idea has been developed and investigated in detail. In this paper the problems of choosing convergence matrices of dual control with two perceptrons (identification and control) and sensitivity model, furthermore the influence of a priori knowledge of dynamics on adaptive control have been considered. By means of the developed PCSP (Process Control Simulator Program), the issue of adaptive control has been simulated in several cases of different strategies. These experiences can be used in the development of process control software.

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Dr. Csilla BÁNYÁSZ
Dr. László KEVICZKY } H-1521 Budapest,
Dr. János KOCSIS