POTENTIAL AND FIELD STRENGTH DISTRIBUTION IN A PLANE DIELECTRIC ASSUMING PARABOLIC TEMPERATURE DISTRIBUTION

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Introduction

The requirements for electrical insulations of high current, high voltage equipment are ever increasing. The load on the insulation materials has increased considerably, therefore an enhanced load capacity is sought for. The applicability of most insulation materials is limited by the maximum heating up specifications (according to heat resistance classes [24]). From economy aspects the insulation should operate at a temperature as near as possible to the maximum specified temperature. But for meeting this requirement the temperature distribution developing in the dielectric must be known, and so must be the potential and field strength distribution. These are accurately established by solving the KIRCHHOFF—FOURIER differential equation of temperature distribution (for given boundary conditions). There is no known general solution of this equation for the three-dimensional case, it can only be solved with simplifying assumptions [2]. In the plane one-dimensional case the temperature distribution can be determined by a computer according to [25]. In spite of accurate by delivering the temperature distribution in the dielectric, the disadvantage of this method is to require a computer, hence to be no fast and easy estimation method of the temperature distribution.

The method described and partly proved in the following permits easy estimation of the developing temperature distribution, while that of the potential and field strength distribution is facilitated by tables calculated in advance.

Notations

E — electric field strength [V/m]
$2U_0$ — dielectric energizing voltage [V]
p'_0 — dielectric losses per unit volume and per unit field strength at temperature ϑ_0 [W/mV²]
b — thermal coefficient of the temperature dependence of $p(\vartheta)$ [1/°C]
ϑ_0 — reference temperature [°C]
ϑ_a — ambient temperature [°C]
ϑ_1 — boundary temperature between the dielectric and the electrode [°C]
ϑ_2 — electrode outer surface temperature [°C]
ϑ_m — maximum temperature [°C]
F — heat transfer surface [m²]

h — dielectric thickness [m]
m — electrode thickness [m]
α — heat transfer coefficient [W/m² °C]
λ_1 — dielectric thermal conductivity [W/m °C]
λ_2 — electrode thermal conductivity [W/m °C]

1. The differential equation of temperature distribution

The temperature distribution arising in solids is obtained by solving the KIRCHHOFF—FOURIER differential equation [4]. The general form of this differential equation is:

$$\operatorname{div}(\lambda_1 \operatorname{grad} \vartheta) + q_b = c_f \cdot \gamma_f \cdot \frac{\partial \vartheta}{\partial t} \tag{1}$$

where q_b is the heat quantity forming in unit volume of the tested material. The analytical solution of the equation is very difficult even for $q_b = 0$. In practice, several simplifying assumptions are generally permissible, e.g. that of one-dimensional heat flow, a nearly infinite cylinder length, etc. Our investigations refers also to the one-dimensional case, assuming that the thermal conductivity of the dielectric is independent of the temperature. Although this condition is not perfectly true, this neglection is permissible for most insulations in the temperature range of interest (20 to —120 [°C]) varying by little. With the simplifying assumptions, our equation for the symmetrical plane model shown in Fig. 1 is as follows:

$$\frac{d^2\vartheta}{dx^2} + \frac{q_b}{\lambda_1} = 0 \tag{2}$$

The loss in dielectrics is generally an exponential function of temperature [6, 8]:*

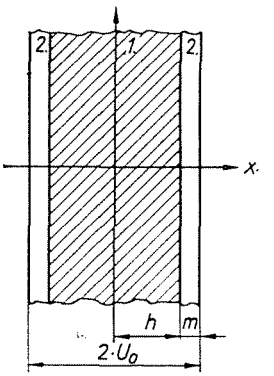

Fig. 1. Plane model: 1. dielectric; 2. electrode

* The more general temperature dependence is of the form $e^{T/\vartheta}$, but this expression is fairly approximated by $e^{b(\vartheta-\vartheta_0)}$ for a moderate range of temperature. For D.C. voltages the losses may be expressed in a form similar to that of the temperature dependence of the specific thermal conductivity [8, 11].

$$q_b(\vartheta) = p'(\vartheta) \cdot E^2 = p'_0 \cdot E^2 \cdot e^{b(\vartheta - \vartheta_0)} \tag{3}$$

where

$$p'_0 = \frac{f \cdot (\varepsilon \cdot tg\delta)\, \vartheta_0}{18.10^{11}} \tag{4}$$

Introducing relative variables, this differential equation reduces to:

$$\frac{d^2\Theta}{dz^2} + B \cdot e^{\Theta} = 0 \tag{5}$$

where

$$\Theta = b(\vartheta - \vartheta_a), \qquad z = \frac{x}{h}, \qquad B = \frac{p'_k \cdot E^2 \cdot h^2 \cdot b}{\lambda_1} \tag{6}$$

The stationary temperature distribution developing in the dielectric determined by solving differential equation (5). Requiring the following boundary conditions [4, 10, 20]:
from the symmetry of the model it follows that

$$\left(\frac{d\vartheta}{dx}\right)_{x=0} = 0 \quad \text{and} \quad \left(\frac{d\Theta}{dz}\right)_{z=0} = 0 \tag{7}$$

respectively.

From the outer surface of the electrodes heat is transferred to the constant temperature surroundings:

$$-\lambda_2 \cdot F \cdot \left(\frac{d\vartheta}{dz}\right)_{x=h+m-0} = \alpha \cdot F(\vartheta_2 - \vartheta_a) \tag{8}$$

and

$$-\frac{\lambda_2 \cdot F}{b \cdot h} \cdot \left(\frac{d\Theta}{dz}\right)_{z=1+\frac{m}{h}-0} = \frac{\alpha \cdot F}{b}\,\Theta_2 \tag{9}$$

respectively.

2. Assumption of parabolic temperature distribution

The assumption of parabolic temperature distribution is frequent in thermal engineering practice [17, 19]. Here — instead of exactly solving Eq. (5) — the temperature is assumed to vary parabolically as a function of the thickness:

$$\vartheta_p(x) = \vartheta_m - (\vartheta_m - \vartheta_1) \cdot \left(\frac{x}{h}\right)^2 ; \quad 0 \leqq x \leqq h \tag{10}$$

or expressed in relative units:

$$\Theta_p(z) = \Theta_m - (\Theta_m - \Theta_1) \cdot z^2 ; \quad 0 \leqq z \leqq 1 \tag{11}$$

The function $\Theta_p(z)$ satisfies automatically the boundary condition (7). Satisfying the boundary condition (8) as well, we have:

$$\Theta_p(z) = \Theta_m \cdot (1 - \nu \cdot z^2) \tag{12}$$

where

$$\nu = \frac{c}{2 + c} \quad \text{and} \quad c = \frac{\lambda_2}{\lambda_1} \cdot \frac{\alpha \cdot h}{\alpha \cdot m + \lambda_2} \tag{13}$$

The individual boundary temperatures may also be expressed as:

$$\Theta_1 = \Theta_m \cdot (1 - \nu) \tag{14}$$

$$\Theta_2 = \Theta_m \cdot (1 - \nu) \cdot \frac{\lambda_2}{\lambda_2 + \alpha \cdot m} \tag{15}$$

The expression (12) is easy to handle and to solve, and suits analytical calculations as well.

The question arises, however, how close — or at what an error — $\Theta(z)$ is approximated by $\Theta_p(z)$, exact solution of the differential equation.

3. The proof of the parabolic temperature distribution

The merit of the approximation $\Theta_p(z)$ can be proved in an indirect way. In their thermal breakdown studies involving derivation of the thickness dependence of the so-called thermal breakdown voltage GRINBERG, KONTOROVICH and LEBEDEV proved [11] a close agreement in a certain range of c between the so-called Fok-function ($\varphi(c)$, [27]) derived by applying the exact solution $\Theta(z)$, and the approximate function $\varphi_k(c)$ derived by applying $\Theta_p(z)$.

This is an indirect proof of the validity of assuming parabolic temperature distribution; the question is, however, in what range of c, and by what $\Theta(z)$ and $\Theta_p(z)$ differ.

The differential equation may be solved on a computer by the method given in [25] and the results have been compared with those obtained by assuming parabolic temperature distribution. The outputs are compiled in Table I, showing the relative temperature maxima and minima of the dielectric for various λ_1, and h values, the heat quantities transferred into the surroundings (Q_{ki}), the parameter c — expressing the geometrical and heat transfer conditions — and the maximum temperature deviation percentages:

$$Q_{ki} = \frac{\alpha \cdot F}{b} \cdot \Theta_{\min}[Ws] \tag{16}$$

$$\Delta\Theta[\%] = 100 \cdot \frac{\Theta(1) - \Theta_p(1)}{\Theta(1)} \tag{17}$$

It is of importance that the developing temperature distribution depends only on parameter c (with present boundary conditions), so that identical c values

Table I

Θ_{max}	Θ_{min}	Q_{out}	c	$\Delta\Theta(\%)$	λ_1 [W/m °C]
0.17042	0.07510	15.02	0.5	−0.86231	0.04
0.27071	0.16550	33.10	1.25	−0.65865	0.08
0.40107	0.28150	56.30	0.8333	−0.57194	0.12
0.59860	0.45353	90.71	0.625	−0.56005	0.16
0.77730	0.60463	120.93	0.5555	−0.60952	0.18
					α [W/m^2 °C]
0.93276	0.73199	124.44	0.5312	−0.68354	8.5
0.75027	0.58209	104.78	0.5625	−0.59960	9
0.59860	0.45353	90.71	0.625	−0.56005	10
0.35295	0.23889	71.67	0.9375	−0.59234	15
0.22790	0.12703	63.52	1.5625	−0.72025	25
0.15273	0.05905	59.06	3.125	−0.93015	50
0.13013	0.03852	57.77	4.6875	−1.0458	75
0.11922	0.02858	57.17	6.25	−1.1182	100
					h [m]
0.93276	0.73199	146.40	0.5313	−0.68354	0.0085
0.59860	0.45353	90.71	0.625	−0.56005	0.01
0.27071	0.16550	33.10	1.25	−0.65865	0.02
0.20147	0.10318	20.64	1.875	−0.77427	0.03
0.17042	0.07510	15.02	2.500	−0.86331	0.04
0.15273	0.05905	11.81	3.125	−0.93015	0.05

$Q_{b0} = 0$ $B = 0.165$

are associated with identical temperature distributions. Fig. 2 shows relative temperature maxima and minima versus c. As the computer outputs refer throughout to the constant value $B = 0.165$, of course, for lesser thicknesses (lower c) the temperature increases considerably, and vice versa. Fig. 3 presents the maximum temperature deviations in percentages versus c, which is of the highest interest for us. For B = const., the function is seen to have a minimum and in the range of c values met with in practice it hardly exceeds 1%, i.e. the approximation is rather good. Be e.g. the dielectric data: $\lambda_1 = 0.16$ [W/m °C], $\lambda_2 = 180$ [W/m °C], $\alpha = 10$ [W/m^2 °C], $h = 0.02$ [m], $m = 0.002$ [m], then $c = 1.25$ and this is associated with $\Delta\Theta = -0.66$ [%]. The negative sign

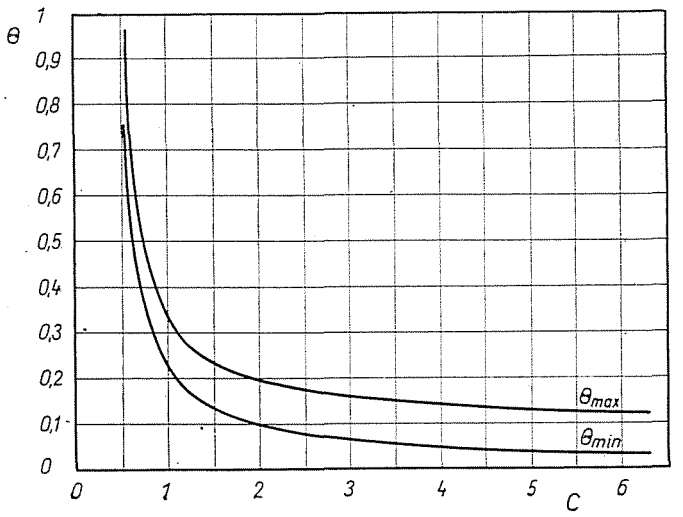

Fig. 2. Relative temperature maxima and minima of the plane dielectric versus c, $B = 0.165$

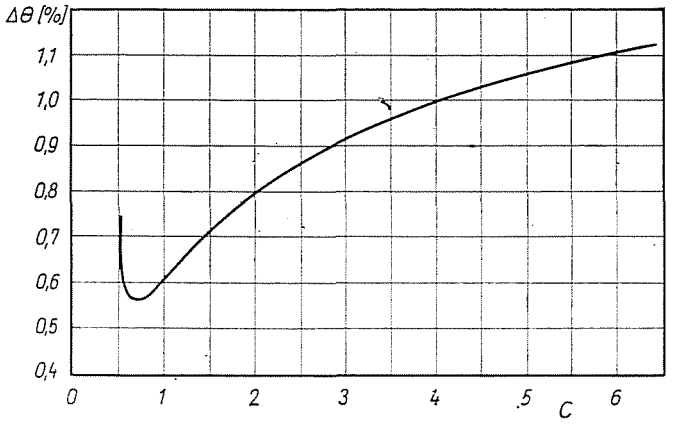

Fig. 3. Maximum percentage deviation between the temperature distribution developing in the plane dielectric and the parabolic temperature distribution versus c

shows that at $z = 1$ (where the temperature difference is at its maximum), the temperature $\Theta_p(1)$ computed from the parabolic temperature distribution is a little bit higher than the exact temperature $\Theta(1)$.

4. Calculation of the potential and field strength distribution

The potential and field strength distribution developing in the dielectric at D.C. voltages are governed first of all by the specific thermal conductivity, and at A.C. voltages by the permittivity. If a D.C. voltage is applied on the

dielectric, then, neglecting the temperature distribution in the dielectric, the developing potential distribution will be linear, and the electric field strength homogeneous. The specific thermal conductivity is highly temperature-dependent, therefore in reality the potential distribution is distorted by the temperature distribution developing in the dielectric. By taking an exponential temperature dependence of the specific thermal conductivity analogous to (3) into account, the relative potential distribution, exclusively due to $\gamma(\vartheta)$ and $\Theta(z) \neq$ const. can be derived to be of the form [27]:

$$\frac{U(z)}{U_0} = \frac{\int_0^z e^{-\Theta(z)} dz}{\int_0^1 e^{-\Theta(z)} dz} \tag{18}$$

and the relative field strength distribution:

$$\frac{E(z)}{E_0} = \frac{e^{-\Theta(z)}}{\int_0^1 e^{-\Theta(z)} dz} \tag{19}$$

These expressions are valid for the case of arbitrary general temperature distribution $\Theta(z)$. Assuming a parabolic $\Theta(z)$ distribution according to (12) and substituting it into (18) and (19) we have:

$$\frac{U(z)}{U_0} = \frac{\int_0^z e^{\nu\Theta_m z^2} dz}{\int_0^1 e^{\nu\Theta_m z^2} dz} \tag{20}$$

and

$$\frac{E(z)}{E_0} = \frac{-e^{\nu\Theta_m z^2}}{\int_0^1 e^{\nu\Theta_m z^2} dz} \tag{21}$$

respectively.

The value of the integral in the expressions can only be expressed by expanding into series [28]:

$$F(\nu, \Theta_m, z) = \int_0^z e^{\nu\Theta_m z^2} dz = \sum_{n=0}^{\infty} \frac{(\nu\Theta_m)^n \cdot z^{(2n+1)}}{(2n+1) \cdot n!} \tag{22}$$

The function $F(\nu, \Theta_m, z)$ is difficult to determine by the given series expansion, but it is easy to obtain in a computer. As the relative potential distribution developing in the dielectric depends only on ν in the case of a given Θ_m, so we

Table II

$\Theta_m = 1$	Relative potential distribution of plane dielectric									$F(\nu, \Theta_m, z)$
ν	$z = 0.1$	$z = 0.2$	$z = 0.3$	$z = 0.4$	$z = 0.5$	$z = 0.6$	$z = 0.7$	$z = 0.8$	$z = 0.9$	$z = 1.0$
0.025	0.09918	0.19840	0.29773	0.39720	0.49687	0.59679	0.69702	0.79759	0.89857	1.00000
0.050	0.09835	0.19680	0.29545	0.39440	0.49374	0.59358	0.69402	0.79516	0.89712	1.00000
0.075	0.09753	0.19521	0.29318	0.39159	0.49060	0.59035	0.69100	0.79271	0.89566	1.00000
0.100	0.09671	0.19361	0.29091	0.38879	0.48745	0.58711	0.68797	0.79025	0.89418	1.00000
0.125	0.09589	0.19202	0.28864	0.38598	0.48430	0.58586	0 68492	0.78776	0.89269	1.00000
0.150	0.09507	0.19043	0.28637	0.38317	0.48114	0.58059	0.68185	0 78526	0.89118	1.00000
0.175	0.09426	0.18885	0.28410	0.38036	0.47798	0.57732	0.67877	0.78274	0.88966	1.00000
0.200	0.09344	0.18726	0.28184	0.37755	0.47481	0.57404	0.67567	0.78020	0.88812	1.00000
0.225	0.09263	0.18568	0.27957	0.37474	0.47164	0.57074	0 67256	0.77764	0.88657	1.00000
0.250	0.09182	0.18410	0.27731	0.37193	0.46847	0.56744	0.66943	0.77506	0.88500	1.00000
0.275	0.09101	0.18253	0.27506	0.36912	0.46529	0.56413	0.66629	0 77246	0.88342	1.00000
0.300	0.09021	0.18096	0.27280	0.36631	0.46210	0.56081	0.66313	0.76985	0.88182	1.00000
0.325	0.08940	0.17939	0.27055	0.36351	0.45892	0.55748	0.65996	0.76722	0.88020	1.00000
0.350	0.08860	0.17782	0.26830	0.36070	0.45572	0.55414	0.65677	0.76457	0.87858	1.00000
0.375	0.08780	0.17626	0.26605	0.35789	0.45253	0.55079	0.65357	0.76190	0.87693	1.00000
0.400	0.08700	0.17470	0.26381	0.35509	0.44934	1.54743	0.65036	0.75922	0.87527	1.00000
0.425	0.08620	0.17314	0.26157	0.35228	0.44614	0.54407	0.64713	0.75651	0.87360	1.00000
0.450	0.08541	0.17159	0.25953	0.34948	0.44294	0.54070	0.64389	0.75379	0.87191	1.00000
0.475	0.08461	0.17004	0.25710	0.34668	0.43974	0.53732	0.64063	0.75106	0.87021	1.00000
0.500	0.08382	0.16849	0.25487	0.34388	0.43653	0.53394	0.63737	0.74830	0.86849	1.00000
0.525	0.08304	0.16695	0.25265	0.34109	0.43333	0.53054	0.63409	0.74553	0.86675	1.00000
0.550	0.08225	0.16541	0.25043	0.33830	0.43012	0.52714	0.63079	0.74274	0.86500	1.00000
0.575	0.08147	0.16388	0.24821	0.33551	0.42692	0.52374	0.62749	0.73994	0.86324	1.00000

0.600	0.08069	0.16235	0.24600	0.33272	0.42371	0.52033	0.62417	0.73712	0.86146	1.00000
0.625	0.07991	0.16083	0.24380	0.32994	0.42050	0.51691	0.62084	0.73428	0.85967	1.00000
0.650	0.07914	0.15931	0.24160	0.32716	0.41730	0.51349	0.61750	0.73143	0.85786	1.00000
0.675	0.07836	0.15780	0.23940	0.32439	0.41409	0.51007	0.61415	0.72856	0.85604	1.00000
0.700	0.07760	0.15629	0.23731	0.32162	0.41089	0.50664	0.61079	0.72568	0.85420	1.00000
0.725	0.07683	0.15478	0.23503	0.31885	0.40768	0.50320	0.60742	0.72278	0.85235	1.00000
0.750	0.07606	0.15328	0.23285	0.31609	0.40448	0.49977	0.60403	0.71986	0.85048	1.00000
0.775	0.07530	0.15179	0.23068	0.31333	0.40128	0.49632	0.60064	0.71693	0.84860	1.00000
0.800	0.07455	0.15030	0.22851	0.31058	0.39808	0.49288	0.59724	0.71399	0.84670	1.00000
0.825	0.07379	0.14881	0.22635	0.30784	0.39489	0.48943	0.59383	0.71103	0.84479	1.00000
0.850	0.07304	0.14733	0.22420	0.30510	0.39169	0.48598	0.59041	0.70806	0.84287	1.00000
0.875	0.07229	0.14586	0.22205	0.30236	0.38850	0.48253	0.58698	0.70507	0.84093	1.00000
0.900	0.07155	0.14439	0.21992	0.29963	0.38531	0.47907	0.58355	0.70207	0.83898	1.00000
0.925	0.07080	0.14293	0.21778	0.29691	0.38213	0.47562	0.58010	0.69905	0.83701	1.00000
0.950	0.07007	0.14148	0.21566	0.29420	0.37895	0.47216	0.57665	0.69602	0.83503	1.00000
0.975	0.06933	0.14003	0.21354	0.29149	0.37577	0.46870	0.57319	0.69298	0.83303	1.00000
1.000	0.06860	0.13858	0.21143	0.28879	0.37260	0.46525	0.56972	0.68993	0.83102	1.00000

Table II (cont.)

$\Theta_m = 2$	Relative potential distribution of plane dielectric									$F(v, \Theta_m, \nu)$
v	$z = 0.1$	$z = 0.2$	$z = 0.3$	$z = 0.4$	$z = 0.5$	$z = 0.6$	$z = 0.7$	$z = 0.8$	$z = 0.9$	$z = 1.0$
0.025	0.09835	0.19680	0.29545	0.39440	0.49374	0.59358	0.69402	0.79516	0.89712	1.00000
0.050	0.09671	0.19361	0.29091	0.38879	0.48745	0.58711	0.68797	0.79025	0.89418	1.00000
0.075	0.09507	0.19043	0.28637	0.38317	0.48114	0.58059	0.68185	0.78526	0.89118	1.00000
0.100	0.09344	0.18726	0.28184	0.37755	0.47481	0.57404	0.67567	0.78020	0.88812	1.00000
0.125	0.09182	0.18410	0.27731	0.37193	0.46847	0.56744	0.66943	0.77506	0.88500	1.00000
0.150	0.09021	0.18096	0.27280	0.36631	0.46210	0.56081	0.66313	0.76985	0.88182	1.00000
0.175	0.08860	0.17782	0.26830	0.36070	0.45572	0.55414	0.65677	0.76457	0.87858	1.00000
0.200	0.08700	0.17470	0.26381	0.35509	0.44934	0.54743	0.65036	0.75922	0.87527	1.00000
0.225	0.08541	0.17159	0.25933	0.34948	0.44294	0.54070	0.64389	0.75379	0.87191	1.00000
0.250	0.08382	0.16849	0.25487	0.34388	0.43653	0.53394	0.63737	0.74830	0.86849	1.00000
0.275	0.08225	0.16541	0.25043	0.33830	0.43012	0.52714	0.63079	0.74274	0.86500	1.00000
0.300	0.08069	0.16235	0.24600	0.33272	0.42371	0.52033	0.62417	0.73712	0.86146	1.00000
0.325	0.07914	0.15931	0.24160	0.32716	0.41730	0.51349	0.61750	0.73143	0.85786	1.00000
0.350	0.07760	0.15629	0.23721	0.32162	0.41089	0.50664	0.61079	0.72568	0.85420	1.00000
0.375	0.07606	0.15328	0.23285	0.31609	0.40448	0.49977	0.60403	0.71986	0.85048	1.00000
0.400	0.07455	0.15030	0.22851	0.31058	0.39808	0.49288	0.59724	0.71399	0.84670	1.00000
0.425	0.07304	0.14733	0.22420	0.30510	0.39169	0.48598	0.59041	0.70806	0.84287	1.00000
0.450	0.07155	0.14439	0.21992	0.29963	0.38531	0.47907	0.58355	0.70207	0.83898	1.00000
0.475	0.07007	0.14148	0.21566	0.29420	0.37895	0.47216	0.57665	0.69602	0.83503	1.00000
0.500	0.06860	0.13858	0.21143	0.28879	0.37260	0.46525	0.56972	0.68993	0.83102	1.00000
0.525	0.06714	0.13571	0.20723	0.28341	0.36627	0.45833	0.56277	0.68378	0.82697	1.00000
0.550	0.06570	0.13287	0.20307	0.27806	0.35997	0.45141	0.55579	0.67758	0.82285	1.00000
0.575	0.06428	0.13006	0.19894	0.27275	0.35368	0.44451	0.54879	0.67134	0.81869	1.00000

0.600	0.06287	0.12727	0.19484	0.26747	0.34743	0.43760	0.54178	0.66505	0.81447	1.00000
0.625	0.06147	0.12451	0.19078	0.26222	0.34120	0.43071	0.53474	0.65872	0.81020	1.00000
0.650	0.06009	0.12178	0.18676	0.25702	0.33500	0.42383	0.52770	0.65235	0.80589	1.00000
0.675	0.05873	0.11907	0.18277	0.25185	0.32884	0.41697	0.52064	0.64594	0.80152	1.00000
0.700	0.05738	0.11640	0.17882	0·24673	0.32271	0.41013	0.51358	0.63950	0.79710	1.00000
0.725	0.05605	0.11376	0.17492	0.24165	0.31662	0.40331	0.50652	0.63302	0.79264	1.00000
0.750	0.05474	0.11115	0.17106	0.23662	0.31056	0.39651	0.49945	0.62651	0.78814	1.00000
0.775	0.05344	0.10857	0.16724	0.23163	0.30456	0.38974	0.49238	0.61998	0.78358	1.00000
0.800	0.05216	0.10603	0.16346	0.22669	0.29859	0.38300	0.48532	0.61341	0.77899	1.00000
0.825	0.05090	0.10351	0.15973	0.22181	0.29267	0.37629	0.47827	0.60683	0.77436	1.00000
0.850	0.04966	0.10104	0.15604	0.21697	0.28680	0.36961	0.47122	0.60022	0.76968	1.00000
0.875	0.04843	0.09859	0.15240	0.21218	0.28098	0.36297	0.46419	0.59360	0.76497	1.00000
0.900	0.04722	0.09618	0.14881	0.20745	0.27521	0.35638	0.45718	0.58696	0.76022	1.00000
0.925	0.04603	0.09381	0.14526	0.20278	0.26950	0.34982	0.45018	0.58031	0.75543	1.00000
0.950	0.04486	0.09147	0.14177	0.19816	0.26384	0.34331	0.44320	0.57365	0.75061	1.00000
0.975	0.04371	0.08916	0.13832	0.19359	0.25823	0.33684	0.43625	0.56698	0.74576	1.00000
1.000	0.04258	0.08690	0.13492	0.18909	0.25269	0.33043	0.42933	0.56031	0.74088	1.00000

Table II (cont.)

$\Theta_m = 3$	Relative potential distribution of plane dielectric									$F(v, \Theta_m, v)$
v	$z = 0.1$	$z = 0.2$	$z = 0.3$	$z = 0.4$	$z = 0.5$	$z = 0.6$	$z = 0.7$	$z = 0.8$	$z = 0.9$	$z = 1.0$
0.025	0.09753	0.19521	0.29318	0.39159	0.49060	0.59035	0.69100	0.79271	0.89566	1.00000
0.050	0.09507	0.19043	0.28637	0.38317	0.48114	0.58059	0.68185	0.78526	0.89118	1.00000
0.075	0.09263	0.18568	0.27957	0.37474	0.47164	0.57074	0.67256	0.77764	0.88657	1.00000
0.100	0.09021	0.18096	0.27280	0.36631	0.46210	0.56081	0.66313	0.76985	0.88182	1.00000
0.125	0.08780	0.17626	0.26605	0.35789	0.45253	0.55079	0.65357	0.76190	0.87693	1.00000
0.150	0.08541	0.17159	0.25933	0.34948	0.44294	0.54070	0.64389	0.75379	0.87191	1.00000
0.175	0.08304	0.16695	0.25265	0.34109	0.43333	0.53054	0.63409	0.74553	0.86675	1.00000
0.200	0.08069	0.16235	0.24600	0.33272	0.42371	0.52033	0.62417	0.73712	0.86146	1.00000
0.225	0.07836	0.15780	0.23940	0.32439	0.41409	0.51007	0.61415	0.72856	0.85604	1.00000
0.250	0.07606	0.15328	0.23285	0.31609	0.40448	0.49977	0.60403	0.71986	0.85048	1.00000
0.275	0.07379	0.14881	0.22635	0.30784	0.39489	0.48943	0.59383	0.71103	0.84479	1.00000
0.300	0.07155	0.14439	0.21992	0.29963	0.38531	0.47907	0.58355	0.70207	0.83898	1.00000
0.325	0.06933	0.14003	0.21354	0.29149	0.37577	0.46870	0.57319	0.69298	0.83303	1.00000
0.350	0.06714	0.13571	0.20723	0.28341	0.36627	0.45833	0.56277	0.68378	0.82697	1.00000
0.375	0.06499	0.13146	0.20100	0.27540	0.35682	0.44796	0.55229	0.67446	0.82078	1.00000
0.400	0.06287	0.12727	0.19484	0.26747	0.34743	0.43760	0.54178	0.66505	0.81447	1.00000
0.425	0.06078	0.12314	0.18876	0.25962	0.33810	0.42727	0.53122	0.65554	0.80805	1.00000
0.450	0.05873	0.11907	0.18277	0.25185	0.32884	0.41697	0.52064	0.64594	0.80152	1.00000
0.475	0.05672	0.11508	0.17687	0.24419	0.31966	0.40671	0.51005	0.63626	0.79488	1.00000
0.500	0.05474	0.11115	0.17106	0.23662	0.31056	0.39651	0.49945	0.62651	0.78814	1.00000
0.525	0.05280	0.10729	0.16534	0.22916	0.30157	0.38636	0.48885	0.61670	0.78129	1.00000
0.550	0.05090	0.10351	0.15973	0.22181	0.29267	0.37629	0.47827	0.60683	0.77436	1.00000
0.575	0.04904	0.09981	0.15421	0.21457	0.28388	0.36629	0.46771	0.59692	0.76733	1.00000

0.600	0.04722	0.09618	0.14881	0.20745	0.27521	0.35638	0.45718	0.58696	0.76022	1.00000
0.625	0.04545	0.09263	0.14351	0.20046	0.26666	0.34656	0.44669	0.57698	0.75303	1.00000
0.650	0.04371	0.08916	0.13832	0.19359	0.25823	0.33684	0.43625	0.56698	0.74576	1.00000
0.675	0.04202	0.08578	0.13324	0.18686	0.24994	0.32724	0.42588	0.55697	0.73842	1.00000
0.700	0.04037	0.08247	0.12828	0.18026	0.24179	0.31775	0.41557	0.54695	0.73102	1.00000
0.725	0.03876	0.07925	0.12343	0.17380	0.23378	0.30839	0.40534	0.53694	0.72356	1.00000
0.750	0.03720	0.07611	0.11871	0.16748	0.22591	0.29915	0.39519	0.52695	0.71605	1.00000
0.775	0.03568	0.07306	0.11410	0.16130	0.21820	0.29006	0.38514	0.51697	0.70849	1.00000
0.800	0.03420	0.07009	0.10960	0.15527	0.21064	0.28111	0.37519	0.50703	0.70088	1.00000
0.825	0.03277	0.06720	0.10523	0.14938	0.20324	0.27230	0.36535	0.49713	0.69324	1.00000
0.850	0.03137	0.06440	0.10098	0.14365	0.19601	0.26365	0.35563	0.48727	0.68557	1.00000
0.875	0.03003	0.06168	0.09685	0.13806	0.18893	0.25516	0.34603	0.47746	0.67788	1.00000
0.900	0.02872	0.05905	0.09284	0.13262	0.18202	0.24682	0.33655	0.46772	0.67016	1.00000
0.925	0.02746	0.05649	0.08895	0.12733	0.17528	0.23866	0.32721	0.45805	0.66243	1.00000
0.950	0.02624	0.05403	0.08519	0.12218	0.16871	0.23066	0.31801	0.44845	0.65469	1.00000
0.975	0.02506	0.05164	0.08153	0.11719	0.16231	0.22283	0.30896	0.43893	0.64694	1.00000
1.000	0.02392	0.04933	0.07800	0.11235	0.15607	0.21518	0.30005	0.42950	0.63919	1.00000

Table II (cont.)

$\Theta_m = 4$	Relative potential distribution of plane dielectric									$F(v, \Theta_m, v)$
v	$z = 0.1$	$z = 0.2$	$z = 0.3$	$z = 0.4$	$z = 0.5$	$z = 0.6$	$z = 0.7$	$z = 0.8$	$z = 0.9$	$z = 1.0$
0.025	0.09671	0.19361	0.29091	0.38879	0.48745	0.58711	0.68797	0.79025	0.89418	1.00000
0.050	0.09344	0.18726	0.28184	0.37755	0.47481	0.57404	0.67567	0.78020	0.88812	1.00000
0.075	0.09021	0.18096	0.27280	0.36631	0.46210	0.56081	0.66313	0.76985	0.88182	1.00000
0.100	0.08700	0.17470	0.26381	0.35509	0.44934	0.54743	0.65036	0.75922	0.87527	1.00000
0.125	0.08382	0.16849	0.25487	0.34388	0.43653	0.53394	0.63737	0.74830	0.86849	1.00000
0.150	0.08069	0.16235	0.24600	0.33272	0.42371	0.52033	0.62417	0.73712	0.86146	1.00000
0.175	0.07760	0.15629	0.23721	0.32162	0.41089	0.50664	0.61079	0.72568	0.85420	1.00000
0.200	0.07455	0.15030	0.22851	0.31058	0.39808	0.49288	0.59724	0.71399	0.84670	1.00000
0.225	0.07155	0.14439	0.21992	0.29963	0.38531	0.47907	0.58355	0.70207	0.83898	1.00000
0.250	0.06860	0.13858	0.21143	0.28879	0.37260	0.46525	0.56972	0.68993	0.83102	1.00000
0.275	0.06570	0.13287	0.20307	0.27806	0.35997	0.45141	0.55579	0.67758	0.82285	1.00000
0.300	0.06287	0.12727	0.19484	0.26747	0.34743	0.43760	0.54178	0.66505	0.81447	1.00000
0.325	0.06009	0.12178	0.18676	0.25702	0.33500	0.42383	0.52770	0.65235	0.80589	1.00000
0.350	0.05738	0.11640	0.17882	0.24673	0.32271	0.41013	0.51358	0.63950	0.79710	1.00000
0.375	0.05474	0.11115	0.17106	0.23662	0.31056	0.39651	0.49945	0.62651	0.78814	1.00000
0.400	0.05215	0.10603	0.16346	0.22669	0.29859	0.38300	0.48532	0.61341	0.77899	1.00000
0.425	0.04966	0.10104	0.15604	0.21697	0.28680	0.36961	0.47122	0.60022	0.76968	1.00000
0.450	0.04722	0.09618	0.14881	0.20745	0.27521	0.35638	0.45718	0.58696	0.76022	1.00000
0.475	0.04486	0.09147	0.14177	0.19816	0.26384	0.34331	0.44320	0.57365	0.75061	1.00000
0.500	0.04258	0.08690	0.13492	0.18909	0.25269	0.33043	0.42933	0.56031	0.74088	1.00000
0.525	0.04037	0.08247	0.12828	0.18026	0.24179	0.31775	0.41557	0.54695	0.73102	1.00000
0.550	0.03823	0.07819	0.12184	0.17168	0.23114	0.30529	0.40195	0.53361	0.72106	1.00000
0.575	0.03618	0.07407	0.11562	0.16335	0.22076	0.29308	0.38848	0.52030	0.71101	1.00000

0.600	0.03420	0.07009	0.10960	0.15527	0.21064	0.28111	0.37519	0.50703	0.70088	1.00000
0.625	0.03220	0.06626	0.10380	0.14745	0.20081	0.26940	0.36210	0.49384	0.69069	1.00000
0.650	0.03047	0.06258	0.09822	0.13990	0.19127	0.25797	0.34921	0.48073	0.68044	1.00000
0.675	0.02872	0.05905	0.09284	0.13262	0.18202	0.24682	0.33655	0.46772	0.67016	1.00000
0.700	0.02705	0.05566	0.08769	0.12559	0.17307	0.23597	0.32413	0.45484	0.65985	1.00000
0.725	0.02545	0.05243	0.08274	0.11884	0.16442	0.22542	0.31196	0.44209	0.64952	1.00000
0.750	0.02392	0.04933	0.07800	0.11235	0.15607	0.21518	0.30005	0.42950	0.63919	1.00000
0.775	0.02247	0.04638	0.07347	0.10613	0.14803	0.20525	0.28842	0.41707	0.62888	1.00000
0.800	0.02108	0.04357	0.06915	0.10016	0.14029	0.19564	0.27707	0.40482	0.61858	1.00000
0.825	0.01977	0.04090	0.06502	0.09446	0.13285	0.18634	0.26600	0.39276	0.60832	1.00000
0.850	0.01852	0.03835	0.06109	0.08901	0.12571	0.17737	0.25523	0.38091	0.59809	1.00000
0.875	0.01734	0.03594	0.05735	0.08381	0.11886	0.16871	0.24476	0.36926	0.58793	1.00000
0.900	0.01622	0.03365	0.05380	0.07885	0.11231	0.16037	0.23459	0.35783	0.57782	1.00000
0.925	0.01516	0.03148	0.05043	0.07413	0.10604	0.15235	0.22473	0.34663	0.56778	1.00000
0.950	0.01416	0.02944	0.04724	0.06964	0.10005	0.14463	0.21518	0.33567	0.55783	1.00000
0.975	0.01321	0.02750	0.04422	0.06538	0.09434	0.13723	0.20593	0.32494	0.54795	1.00000
1.000	0.01232	0.02567	0.04136	0.06133	0.08890	0.13013	0.19698	0.31445	0.53818	1.00000

Table II (cont.)

$\Theta_m = 5$	Relative potential distribution of plane dielectric									$F(v, \Theta_m, v)$
v	$z = 0.1$	$z = 0.2$	$z = 0.3$	$z = 0.4$	$z = 0.5$	$z = 0.6$	$z = 0.7$	$z = 0.8$	$z = 0.9$	$z = 1.0$
0.025	0.09589	0.19202	0.28864	0.38598	0.48430	0.58386	0.68492	0.78776	0.89269	1.00000
0.050	0.09182	0.18410	0.27731	0.37193	0.46847	0.56744	0.66943	0.77506	0.88500	1.00000
0.075	0.08780	0.17626	0.26605	0.35789	0.45253	0.55079	0.65357	0.76190	0.87693	1.00000
0.100	0.08382	0.16849	0.25487	0.34388	0.43653	0.53394	0.63737	0.74830	0.86849	1.00000
0.125	0.07991	0.16083	0.24380	0.32994	0.42050	0.51691	0.62084	0.73428	0.85967	1.00000
0.150	0.07606	0.15328	0.23285	0.31609	0.40448	0.49977	0.60403	0.71986	0.85048	1.00000
0.175	0.07229	0.14586	0.22205	0.30236	0.38850	0.48253	0.58698	0.70507	0.84093	1.00000
0.200	0.06860	0.13858	0.21143	0.28879	0.37260	0.46525	0.56972	0.68993	0.83102	1.00000
0.225	0.06499	0.13146	0.20100	0.27540	0.35682	0.44796	0.55229	0.67446	0.82078	1.00000
0.250	0.06147	0.12451	0.19078	0.26222	0.34120	0.43071	0.53474	0.65872	0.81020	1.00000
0.275	0.05806	0.11773	0.18079	0.24929	0.32577	0.41355	0.51711	0.64272	0.79932	1.00000
0.300	0.05474	0.11115	0.17106	0.23662	0.31056	0.39651	0.49945	0.62651	0.78814	1.00000
0.325	0.05153	0.10477	0.16159	0.22424	0.29562	0.37964	0.48179	0.61012	0.77668	1.00000
0.350	0.04843	0.09859	0.15240	0.21218	0.28098	0.36297	0.46419	0.59360	0.76497	1.00000
0.375	0.04545	0.09263	0.14351	0.20046	0.26666	0.34656	0.44669	0.57698	0.75303	1.00000
0.400	0.04258	0.08690	0.13492	0.18909	0.25269	0.33043	0.42933	0.56031	0.74088	1.00000
0.425	0.03983	0.08139	0.12665	0.17809	0.23910	0.31462	0.41215	0.54362	0.72854	1.00000
0.450	0.03720	0.07611	0.11871	0.16748	0.22591	0.29915	0.39519	0.52695	0.71605	1.00000
0.475	0.03469	0.07107	0.11109	0.15726	0.21315	0.28407	0.37850	0.51034	0.70342	1.00000
0.500	0.03230	0.06626	0.10380	0.14745	0.20081	0.26940	0.36210	0.49384	0.69069	1.00000
0.525	0.03003	0.06168	0.09685	0.13806	0.18893	0.25516	0.34603	0.47746	0.67788	1.00000
0.550	0.02788	0.05734	0.09024	0.12907	0.17751	0.24136	0.33031	0.46126	0.66501	1.00000
0.575	0.02584	0.05322	0.08395	0.12050	0.16656	0.22803	0.31498	0.44527	0.65210	1.00000

0.600	0.02392	0.04933	0.07800	0.11235	0.15607	0.21518	0.30005	0.42950	0.63919	1.00000
0.625	0.02212	0.04567	0.07237	0.10461	0.14607	0.20282	0.28556	0.41399	0.62630	1.00000
0.650	0.02042	0.04222	0.06706	0.09728	0.13653	0.19095	0.27150	0.39877	0.61344	1.00000
0.675	0.01883	0.03898	0.06206	0.09035	0.12746	0.17958	0.25790	0.38385	0.60065	1.00000
0.700	0.01734	0.03594	0.05735	0.08381	0.11886	0.16871	0.24476	0.36926	0.58793	1.00000
0.725	0.01595	0.03310	0.05294	0.07764	0.11071	0.15833	0.23210	0.35501	0.57530	1.00000
0.750	0.01465	0.03045	0.04882	0.07185	0.10301	0.14845	0.21992	0.34112	0.56279	1.00000
0.775	0.01344	0.02797	0.04496	0.06642	0.09575	0.13905	0.20821	0.32760	0.55041	1.00000
0.800	0.01232	0.02567	0.04136	0.06133	0.08890	0.13013	0.19698	0.31445	0.53818	1.00000
0.825	0.01128	0.02354	0.03801	0.05658	0.08246	0.12168	0.18623	0.30169	0.52609	1.00000
0.850	0.01032	0.02156	0.03489	0.05214	0.07642	0.11368	0.17595	0.28932	0.51418	1.00000
0.875	0.00943	0.01972	0.03200	0.04800	0.07076	0.10612	0.16613	0.27734	0.50244	1.00000
0.900	0.00861	0.01803	0.02932	0.04415	0.06546	0.09899	0.15677	0.26576	0.49088	1.00000
0.925	0.00785	0.01646	0.02684	0.04057	0.06051	0.09227	0.14785	0.25457	0.47952	1.00000
0.950	0.00715	0.01502	0.02455	0.03725	0.05588	0.08595	0.13937	0.24377	0.46836	1.00000
0.975	0.00651	0.01369	0.02244	0.03428	0.05157	0.08001	0.13130	0.23336	0.45740	1.00000
1.000	0.00592	0.01247	0.02049	0.03133	0.04756	0.07444	0.12365	0.22333	0.44664	1.00000

Table II (cont.)

$\Theta_m = 6$	Relative potential distribution of plane dielectric									$F(r, \Theta_m, r)$
r	$z = 0.1$	$z = 0.2$	$z = 0.3$	$z = 0.4$	$z = 0.5$	$z = 0.6$	$z = 0.7$	$z = 0.8$	$z = 0.9$	$z = 1.0$
0.025	0.09507	0.19043	0.28637	0.38317	0.48114	0.58059	0.68185	0.78526	0.89118	1.00000
0.050	0.09021	0.18096	0.27280	0.36631	0.46210	0.56081	0.66313	0.76985	0.88182	1.00000
0.075	0.08541	0.17159	0.25933	0.34948	0.44294	0.54070	0.64389	0.75379	0.87191	1.00000
0.100	0.08069	0.16235	0.24600	0.33272	0.42371	0.52033	0.62417	0.73712	0.86146	1.00000
0.125	0.07606	0.15328	0.23285	0.31609	0.40448	0.49977	0.60403	0.71986	0.85048	1.00000
0.150	0.07155	0.14439	0.21992	0.29963	0.38531	0.47907	0.58355	0.70207	0.83898	1.00000
0.175	0.06714	0.13571	0.20723	0.28341	0.36627	0.45833	0.56277	0.68378	0.82697	1.00000
0.200	0.06287	0.12727	0.19484	0.26747	0.34743	0.43760	0.54178	0.66505	0.81447	1.00000
0.225	0.05873	0.11907	0.18277	0.25185	0.32884	0.41697	0.52064	0.64594	0.80152	1.00000
0.250	0.05474	0.11115	0.17106	0.23662	0.31056	0.39651	0.49945	0.62651	0.78814	1.00000
0.275	0.05090	0.10351	0.15973	0.22181	0.29267	0.37629	0.47827	0.60683	0.77436	1.00000
0.300	0.04722	0.09618	0.14881	0.20745	0.27521	0.35638	0.45718	0.58696	0.76022	1.00000
0.325	0.04371	0.08916	0.13832	0.19359	0.25823	0.33684	0.43625	0.56698	0.74576	1.00000
0.350	0.04037	0.08247	0.12828	0.18026	0.24179	0.31775	0.41557	0.54695	0.73102	1.00000
0.375	0.03720	0.07611	0.11871	0.16748	0.22591	0.29915	0.39519	0.52695	0.71605	1.00000
0.400	0.03420	0.07009	0.10960	0.15527	0.21064	0.28111	0.37519	0.50703	0.70088	1.00000
0.425	0.03137	0.06440	0.10098	0.14365	0.19601	0.26365	0.35563	0.48727	0.68557	1.00000
0.450	0.02872	0.05905	0.09284	0.13262	0.18202	0.24682	0.33655	0.46772	0.67016	1.00000
0.475	0.02624	0.05403	0.08519	0.12218	0.16871	0.23066	0.31801	0.44845	0.65469	1.00000
0.500	0.02392	0.04933	0.07800	0.11235	0.15607	0.21518	0.30005	0.42950	0.63919	1.00000
0.525	0.02177	0.04496	0.07128	0.10311	0.14412	0.20041	0.28271	0.41092	0.62372	1.00000
0.550	0.01977	0.04090	0.06502	0.09446	0.13285	0.18634	0.26600	0.39276	0.60832	1.00000
0.575	0.01792	0.03713	0.05920	0.08638	0.12225	0.17300	0.24996	0.37506	0.59300	1.00000

0.600	0.01622	0.03365	0.05380	0.07885	0.11231	0.16037	0.23459	0.35783	0.57782	1.00000
0.625	0.01465	0.03045	0.04882	0.07185	0.10301	0.14845	0.21992	0.34112	0.56279	1.00000
0.650	0.01321	0.02750	0.04422	0.06538	0.09434	0.13723	0.20593	0.32494	0.54795	1.00000
0.675	0.01189	0.02480	0.03999	0.05939	0.08628	0.12670	0.19263	0.30930	0.53332	1.00000
0.700	0.01069	0.02233	0.03611	0.05388	0.07879	0.11683	0.18001	0.29422	0.51892	1.00000
0.725	0.00960	0.02008	0.03256	0.04880	0.07186	0.10760	0.16806	0.27971	0.50477	1.00000
0.750	0.00861	0.01803	0.02932	0.04415	0.06546	0.09899	0.15677	0.26576	0.49088	1.00000
0.775	0.00770	0.01616	0.02637	0.03989	0.05956	0.09098	0.14612	0.25238	0.47727	1.00000
0.800	0.00689	0.01448	0.02368	0.03599	0.05412	0.08353	0.13609	0.23956	0.46395	1.00000
0.825	0.00615	0.01295	0.02125	0.03244	0.04913	0.07662	0.12666	0.22730	0.45092	1.00000
0.850	0.00549	0.01157	0.01904	0.02921	0.04456	0.07023	0.11781	0.21558	0.43819	1.00000
0.875	0.00489	0.01032	0.01704	0.02627	0.04037	0.06431	0.10952	0.20439	0.42577	1.00000
0.900	0.00435	0.00920	0.01524	0.02360	0.03654	0.05885	0.10175	0.19373	0.41365	1.00000
0.925	0.00387	0.00820	0.01361	0.02119	0.03305	0.05381	0.09448	0.18357	0.40184	1.00000
0.950	0.00344	0.00729	0.01215	0.01900	0.02986	0.04918	0.08770	0.17390	0.39034	1.00000
0.975	0.00305	0.00648	0.01083	0.01703	0.02697	0.04491	0.08136	0.16471	0.37914	1.00000
1.000	0.00270	0.00576	0.00965	0.01524	0.02433	0.04099	0.07546	0.15597	0.36824	1.00000

Table II (cont.)

$\Theta_m = 7$	Relative potential distribution of plane dielectric									$F(r, \Theta_m, r)$
r	$z = 0.1$	$z = 0.2$	$z = 0.3$	$z = 0.4$	$z = 0.5$	$z = 0.6$	$z = 0.7$	$z = 0.8$	$z = 0.9$	$z = 1.0$
0.025	0.09426	0.18885	0.28410	0.38036	0.47798	0.57732	0.67877	0.78274	0.88966	1.00000
0.050	0.08860	0.17782	0.26830	0.36070	0.45572	0.55414	0.65677	0.76457	0.87858	1.00000
0.075	0.08304	0.16695	0.25265	0.34109	0.43333	0.53054	0.63409	0.74553	0.86675	1.00000
0.100	0.07760	0.15629	0.23721	0.32162	0.41089	0.50664	0.61079	0.72568	0.85420	1.00000
0.125	0.07229	0.14586	0.22205	0.30236	0.38850	0.48253	0.58698	0.70507	0.84093	1.00000
0.150	0.06714	0.13571	0.20723	0.28341	0.36627	0.45833	0.56277	0.68378	0.82697	1.00000
0.175	0.06217	0.12588	0.19280	0.26484	0.34431	0.43416	0.53826	0.66189	0.81234	1.00000
0.200	0.05738	0.11640	0.17882	0.24673	0.32271	0.41013	0.51358	0.63950	0.79710	1.00000
0.225	0.05280	0.10729	0.16534	0.22916	0.30157	0.38636	0.48885	0.61670	0.78129	1.00000
0.250	0.04843	0.09859	0.15240	0.21218	0.28098	0.36297	0.46419	0.59360	0.76497	1.00000
0.275	0.04428	0.09031	0.14004	0.19587	0.26103	0.34007	0.43973	0.57032	0.74819	1.00000
0.300	0.04037	0.08247	0.12828	0.18026	0.24179	0.31775	0.41557	0.54695	0.73102	1.00000
0.325	0.03668	0.07508	0.11716	0.16541	0.22333	0.29611	0.39183	0.52362	0.71353	1.00000
0.350	0.03324	0.06815	0.10668	0.15133	0.20569	0.27522	0.36862	0.50042	0.69580	1.00000
0.375	0.03003	0.06168	0.09685	0.13806	0.18893	0.25516	0.34603	0.47746	0.67788	1.00000
0.400	0.02705	0.05566	0.08769	0.12559	0.17307	0.23597	0.32413	0.45484	0.65985	1.00000
0.425	0.02430	0.05009	0.07917	0.11395	0.15813	0.21771	0.30301	0.43263	0.64178	1.00000
0.450	0.02177	0.04496	0.07128	0.10311	0.14412	0.20041	0.28271	0.41092	0.62372	1.00000
0.475	0.01945	0.04025	0.06402	0.09307	0.13103	0.18407	0.26328	0.38978	0.60576	1.00000
0.500	0.01734	0.03594	0.05735	0.08381	0.11886	0.16871	0.24476	0.36926	0.58793	1.00000
0.525	0.01542	0.03201	0.05126	0.07529	0.10758	0.15432	0.22717	0.34941	0.57028	1.00000
0.550	0.01368	0.02845	0.04571	0.06748	0.09716	0.14090	0.21051	0.33027	0.55288	1.00000
0.575	0.01211	0.02523	0.04067	0.06036	0.08758	0.12841	0.19480	0.31187	0.53575	1.00000

0.600	0.01069	0.02233	0.03611	0.05388	0.07879	0.11683	0.18001	0.29422	0.51892	1.00000
0.625	0.00943	0.01972	0.03200	0.04800	0.07076	0.10612	0.16613	0.27734	0.50244	1.00000
0.650	0.00830	0.01739	0.02831	0.04269	0.06344	0.09626	0.15315	0.26124	0.48632	1.00000
0.675	0.00729	0.01530	0.02500	0.03790	0.05678	0.08719	0.14103	0.24590	0.47058	1.00000
0.700	0.00639	0.01344	0.02203	0.03359	0.05075	0.07887	0.12974	0.23132	0.45523	1.00000
0.725	0.00559	0.01179	0.01939	0.02973	0.04529	0.07126	0.11925	0.21749	0.44029	1.00000
0.750	0.00489	0.01032	0.01704	0.02627	0.04037	0.06431	0.10952	0.20439	0.42577	1.00000
0.775	0.00427	0.00903	0.01495	0.02318	0.03594	0.05798	0.10050	0.19200	0.41166	1.00000
0.800	0.00372	0.00788	0.01310	0.02043	0.03195	0.05222	0.09217	0.18029	0.39797	1.00000
0.825	0.00324	0.00688	0.01147	0.01799	0.02838	0.04700	0.08447	0.16924	0.38470	1.00000
0.850	0.00282	0.00599	0.01003	0.01582	0.02518	0.04226	0.07738	0.15883	0.37184	1.00000
0.875	0.00245	0.00521	0.00876	0.01389	0.02232	0.03797	0.07085	0.14902	0.35939	1.00000
0.900	0.00212	0.00453	0.00764	0.01219	0.01977	0.03409	0.06484	0.13979	0.34734	1.00000
0.925	0.00184	0.00393	0.00666	0.01069	0.01750	0.03059	0.05931	0.13111	0.33568	1.00000
0.950	0.00159	0.00341	0.00580	0.00936	0.01547	0.02743	0.05424	0.12296	0.32441	1.00000
0.975	0.00138	0.00296	0.00504	0.00819	0.01367	0.02459	0.04959	0.11529	0.31352	1.00000
1.000	0.00119	0.00256	0.00438	0.00717	0.01207	0.02203	0.04533	0.10810	0.30299	1.00000

Table III

$z=1$	Values of the integral $\int_0^1 e^{\nu\Theta_m} dz^2$						$F(\nu, \Theta_m, 1)$
ν	$\Theta_m=1$	$\Theta_m=2$	$\Theta_m=3$	$\Theta_m=4$	$\Theta_m=5$	$\Theta_m=6$	$\Theta_m=7$
0.025	1.00840	1.01692	1.02557	1.03436	1.04328	1.05233	1.06153
0.050	1.01692	1.03436	1.05233	1.07086	1.08997	1.10968	1.13001
0.075	1.02557	1.05233	1.08035	1.10968	1.14042	1.17262	1.20639
0.100	1.03436	1.07086	1.10968	1.15098	1.19496	1.24181	1.29175
0.125	1.04328	1.08997	1.14042	1.19496	1.25399	1.31796	1.38734
0.150	1.05233	1.10968	1.17262	1.24181	1.31796	1.40191	1.49457
0.175	1.06153	1.13001	1.20639	1.29175	1.38734	1.49457	1.61510
0.200	1.07086	1.15098	1.24181	1.34503	1.46265	1.59700	1.75083
0.225	1.08035	1.17262	1.27896	1.40191	1.54450	1.71038	1.90393
0.250	1.08997	1.19496	1.31796	1.46265	1.63351	1.83603	2.07693
0.275	1.09975	1.21801	1.35890	1.52757	1.73043	1.97548	2.27274
0.300	1.10968	1.24181	1.40191	1.59700	1.83603	2.13041	2.49472
0.325	1.11977	1.26638	1.44709	1.67129	1.95121	2.30276	2.74675
0.350	1.13001	1.29175	1.49457	1.75083	2.07693	2.49472	3.03334
0.375	1.14042	1.31796	1.54450	1.83603	2.21428	2.70874	3.35969
0.400	1.15098	1.34503	1.59700	1.92736	2.36445	2.94764	3.73183
0.425	1.16172	1.37300	1.65224	2.02531	2.52878	3.21459	4.15674
0.450	1.17262	1.40191	1.71038	2.13041	2.70874	3.51318	4.64255
0.475	1.18370	1.43178	1.77158	2.24325	2.90597	3.84751	5.19865
0.500	1.19496	1.46265	1.83603	2.36445	3.12228	4.22221	5.83596
0.525	1.20639	1.49457	1.90393	2.49472	3.35969	4.64255	6.56717
0.550	1.21801	1.52757	1.97548	2.63478	3.62044	5.11450	7.40703
0.575	1.22981	1.56170	2.05089	2.78546	3.90703	5.64488	8.37269
0.600	1.24181	1.59700	2.13041	2.94764	4.22221	6.24140	9.48409
0.625	1.25399	1.63351	2.21428	3.12228	4.56908	6.91287	10.76445
0.650	1.26638	1.67129	2.30276	3.31041	4.95105	7.66928	12.24082
0.675	1.27896	1.71038	2.39614	3.51318	5.37193	8.52203	13.94467
0.700	1.29175	1.75083	2.49472	3.73183	5.83596	9.48409	15.91274
0.725	1.30475	1.79270	2.59880	3.96768	6.34785	10.57022	18.18780
0.750	1.31796	1.83603	2.70874	4.22221	6.91287	11.79725	20.81980
0.775	1.33139	1.88090	2.82489	4.49701	7.53685	13.18437	23.86696
0.800	1.34503	1.92736	2.94764	4.79381	8.22631	14.75343	27.39728
0.825	1.35890	1.97548	3.07740	5.11450	8.98852	16.52939	31.49017
0.850	1.37300	2.02531	3.21459	5.46114	9.83156	18.54059	36.23834
0.875	1.38734	2.07693	3.35969	5.83596	10.76445	20.81980	41.75017
0.900	1.40191	2.13041	3.51318	6.24140	11.79725	23.40377	48.15233
0.925	1.41672	2.18582	3.67561	6.68013	12.94117	26.33492	55.59290
0.950	1.43178	2.24325	3.84751	7.15503	14.20872	29.66157	64.24510
0.975	1.44709	2.30276	4.02950	7.66928	15.61386	33.43893	74.31160
1.000	1.46265	2.36445	4.22221	8.22631	17.17216	37.73006	86.02957

have determined the relative potential distribution for various Θ_m values and tabulated in the Appendix, permitting to establish the potential distribution in the given temperature range for dielectrics, of arbitrary thickness. A separate table contains the function values $F(v, \Theta_m, 1)$ for the determination of the field strength distribution.

As an illustration, the developing potential and field strength distributions have been plotted in Figs 4 and 5 for $c = 0.625$, and in Figs 6 and 7 for

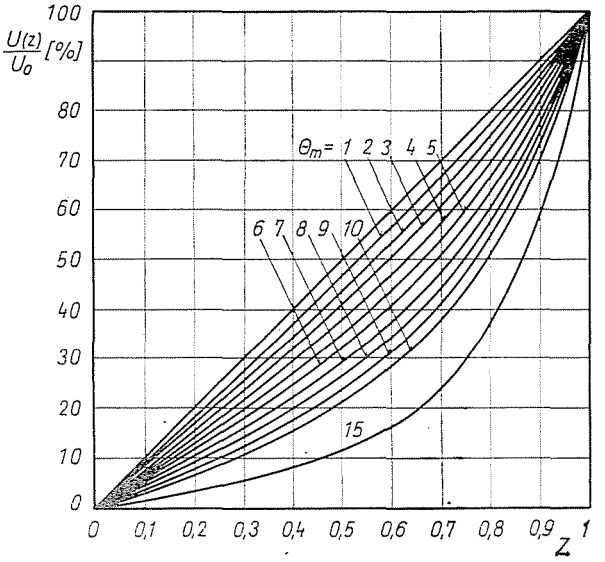

Fig. 4. Relative potential distribution developing in the plane dielectric, the maximum relative temperature (Θ_m), as parameter $c = 0.625$

$c = 1.25$ respectively. It is very interesting to observe that — because of the potential distortion, — the load in the dielectric is shifted towards the colder regions. The maximum local field strength may appear at the outer edge of the dielectric, that may be as high as the 3 to 5-fold. This is a very remarkable fact with regard to possibility of local discharge, with resulting partial breakdown. Figs 8 and 9 present the potential and field strength distributions, respectively, versus, the thickness $c \sim h$, of the dielectric. Of course there is no dielectric of infinite thickness, but the rate of the potential distortion with increasing c is very remarkable.

Acknowledgement

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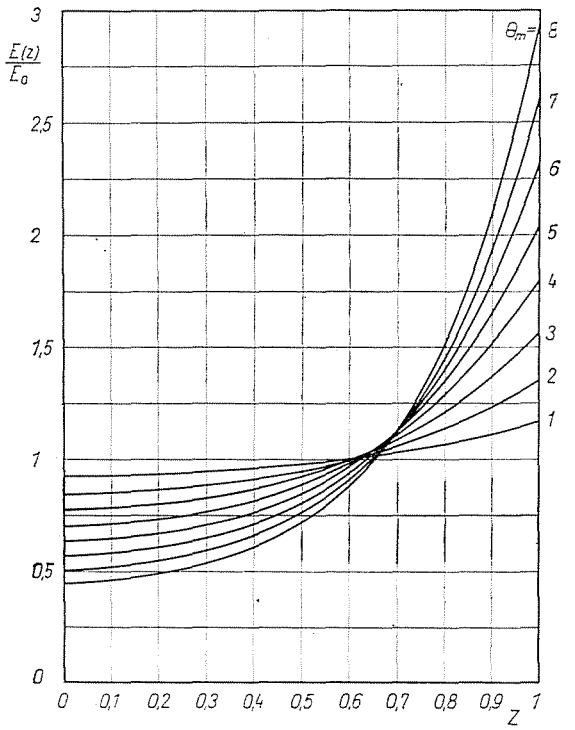

Fig. 5. Relative field strength distribution developing in the plane dielectric, parameter the maximum relative temperature (Θ_m). $c = 0.625$

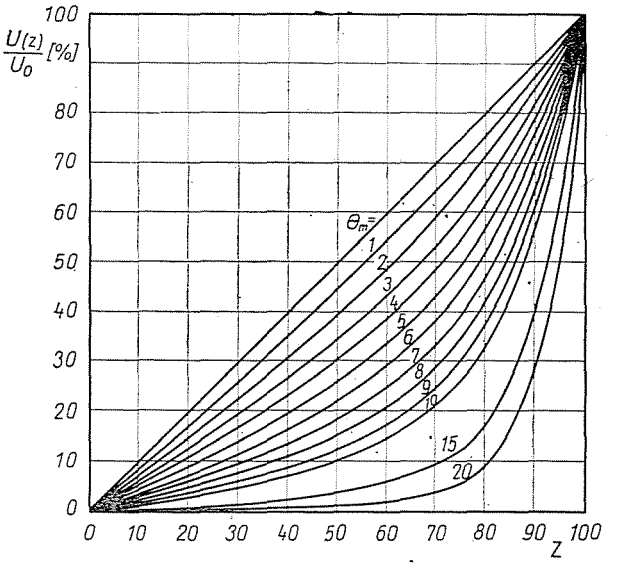

Fig. 6. Relative potential distribution developing in the plane dielectric, parameter the maximum relative temperature (Θ_m), $c = 1.25$

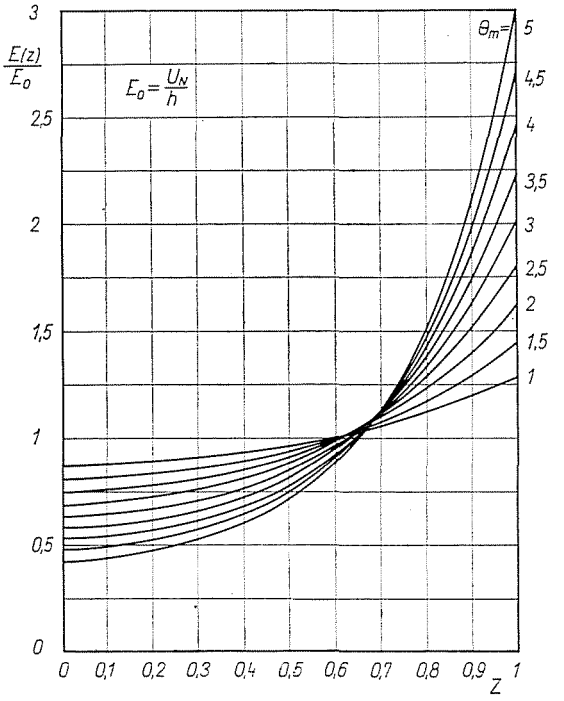

Fig. 7. Relative field strength distribution developing in the plane dielectric, parameter the maximum relative temperature (Θ_m), $c = 1.25$

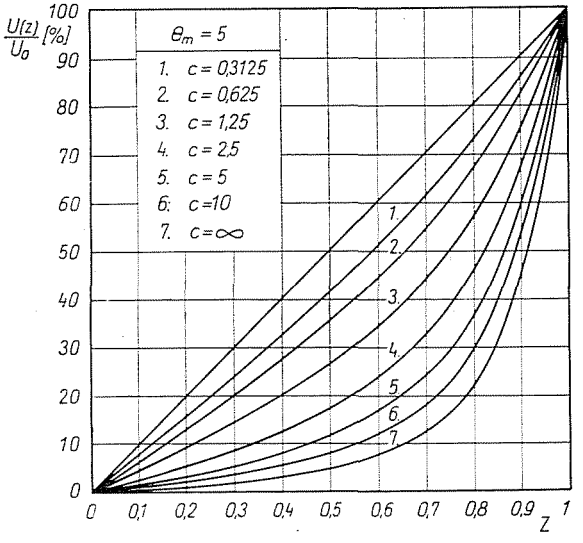

Fig. 8. Relative potential distribution developing in the plane dielectric, parameter the constant maximum relative temperature ($\Theta_m = 5$)

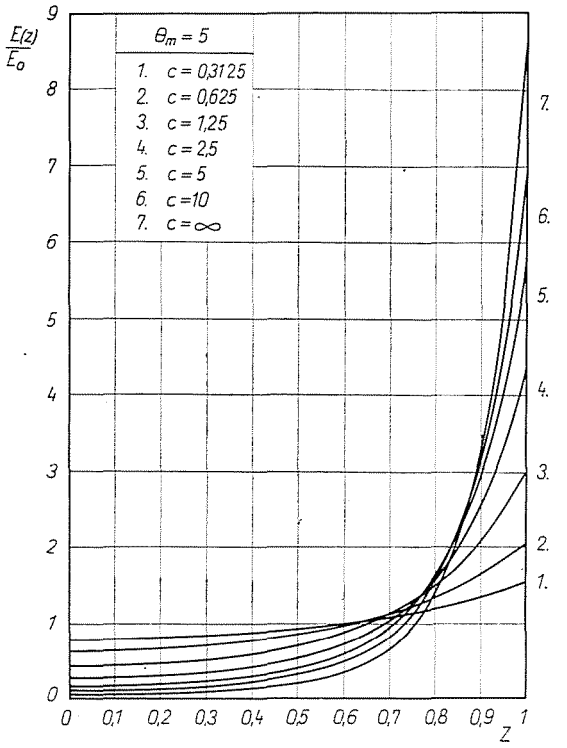

Fig. 9. Relative field strength distribution developing in the plane dielectric, parameter the constant maximum relative temperature ($\Theta_m = 5$)

Summary

Differential equation for the thermal conduction of solid dielectrics has been written. Instead of its exact solution, the parabolic temperature distribution has been expressed. The parabolic temperature distribution has been proved indirectly, partly by the Fok-function known from the thermal breakdown theories and partly by comparing it with the computerized numerical solution of the differential equation. The outputs permit to express the potential and field strength distributions in the dielectric due to the temperature dependence of specific conductivity. As the relative potential distribution depends only on the dielectric data (c) and the maximum relative temperature (Θ_m), the results could be tabulated permitting to establish the potential and field strength distortions for any thickness of the dielectric. Remind however, that our results obtained by applying parabolic temperature distribution are the most accurate in the range $0.5 \leqq c \leq 10$, as we have proved.

Appendix

For the expression (20) given in item 4 a program has been prepared in ALGOL-60 language, run on a computer RAZDAN-3 of the University Computing Centre. The outputs were compiled in tables.

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