CALCULATION OF COMPENSATING CURRENTS FLOWING IN MULTIPOLAR LAP WINDINGS, BY THE HELP OF THE SYMMETRICAL COMPONENTS TRANSFORMATION

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Introduction

In multipolar direct current machines pole flux values are not equal because of differences in pole excitation, of production and assembly inaccuracies resulting in different air gaps, and of other reasons. Consequently, voltages induced in the parallel branches of lap type armature windings are also different, resulting in compensating circulating currents to flow in the windings, causing additional losses, increasing the current load on brushes, impairing commutation. Use of equalizing connections, that is connecting the theoretically equipotential points of the winding by a conductor of possibly low resistance permits compensating currents to close inside the winding, without loading the brushes. It follows from Lenz's law that circulating currents closed through the equalizing connections produce a magnetic field tending to reduce differences in pole fluxes, the asymmetry of the flux [1]. Of course, compensating currents closed through the brushes in lack of equalizing connections behave similarly. In the following, a calculation method for compensating currents flowing in the multipolar simple-lap windings without equalizing connections will be described. The calculation method is based on the symmetrical components transformation.

1. Symmetrical components transformation

Any quadratic matrix $\mathbf{A}$ of n order can be diagonalized by means of the similitude transformation, if it has n linearly independent eigenvectors, i.e. it is always possible to find a non-singular transformation matrix $\mathbf{T}$ where the relationship

$$\mathbf{T}^{-1}\mathbf{A}\mathbf{T} = \operatorname{diag}(\lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_{n-1}) \tag{1}$$

is valid. The elements $\lambda_0, \lambda_1, \ldots, \lambda_{n-1}$ of the diagonal matrix are the eigenvalues of matrix $\mathbf{A}$ and the transformation matrix $\mathbf{T}$ can be formed of the eigenvectors $\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_{n-1}$ pertaining to these eigenvalues [3].

$$\mathbf{T} = [\mathbf{s}_0, \mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_{n-1}] \tag{2}$$

The diagonalizing process, for which the eigenvalues and eigenvectors are sought for is considerably simplified, if the matrix to be diagonalized is cyclically symmetrical. Let $\mathbf{C}$ be a cyclically symmetrical matrix of n order,

$$\mathbf{C} = \begin{bmatrix} C_0 & C_1 & C_2 \dots & C_{n-1} \\ C_{n-1} & C_0 & C_1 \dots & C_{n-2} \\ C_{n-2} & C_{n-1} & C_0 \dots & C_{n-3} \\ \vdots & \vdots & \vdots & \vdots \\ C_1 & C_2 & C_3 \dots & C_0 \end{bmatrix}$$

The eigenvectors are independent of the elements of matrix $\mathbf{C}$ and they contain the various powers of the n-th unit root [4] [5]. To obtain the individual components in the forms usual in electrical engineering, the conjugate of the n-th root will be used, a unit root itself. The i-th eigenvector is found to be

$$\mathbf{s}_{ii} = [1 \hat{g}_i \hat{g}_i^2 \dots \hat{g}_i^{n-1}] \qquad i = 0, 1, 2, \dots, n-1 \tag{3}$$

where

$$\hat{g}_i = e^{-j\frac{2\pi}{n}i} \tag{4}$$

The relationship $\mathbf{C}\mathbf{s}_i = \lambda_i \mathbf{s}_i$ valid for the eigenvalue leads to:

$$\lambda_i = C_0 + C_1 \hat{g}_i + C_2 \hat{g}_i^2 + \dots + C_{n-1} \hat{g}_i^{n-1} \qquad i = 0, 1, 2, \dots, n-1 \tag{5}$$

By inversion, we obtain:

$$\mathbf{T}^{-1} = \frac{1}{n} \hat{\mathbf{T}}_i . \tag{6}$$

The connection between the original quantities $x_1, x_2, \dots x_n$ and the new quantities $x'_0, x'_1, x'_2, \dots, x'_{n-1}$ introduced by the transformation is given by matrix $\mathbf{T}$:

$$\mathbf{x} = \mathbf{T}\mathbf{x}' \tag{7}$$

$$\mathbf{x}' = \mathbf{T}^{-1}\mathbf{x} . \tag{8}$$

The new quantities introduced by the transformation are named the symmetrical components of order $0, 1, 2, \dots, n-1$. The component of the order number 1 is usually named a component of positive order, while that of order $n-1$ a component of negative order. Relationship (7) yields the original quantities from the symmetrical components, while relationship (8) gives the rule of decomposition to symmetrical components. By using (2), (3), (6), on the basis of (7) and (8):

$$x_{i+1} = x'_0 + \hat{g}_1^i x'_1 + \hat{g}_2^i x'_2 + \dots + x'_{n-1} \hat{g}_{n-1}^i \tag{9}$$

$$x'_i = 1/n(x_1 + g_i x_2 + g_i^2 x_3 + \dots + g_i^{n-1} x_n) \qquad i = 0, 1, 2, \dots, n-1. \tag{10}$$

Applying the transformation, equation e.g. $\mathbf{y} = \mathbf{C}\mathbf{x}$ is transformed to

$$\mathbf{y}' = \mathbf{T}^{-1}\mathbf{C}\mathbf{T}\mathbf{x}' = \operatorname{diag}(\lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_{n-1})\mathbf{x}' .$$

This corresponds to n equations relating the symmetrical components of various orders:

$$y'_i = \lambda_i x'_i \qquad i = 0, 1, 2, \ldots, n-1.$$

In the following let us suppose that quantities $x_1, x_2, \ldots, x_n$ are all real and $n = 2p$, i.e. n is even number. Thus on the basis of (4):

$$\hat{g}_i = e^{-j\frac{\pi}{p}i} . \tag{11}$$

By using relationship (10) we find that the 0-th and p-th components are real, while the others are complex:

$$x'_0 = \frac{1}{2p}(x_1 + x_2 + \ldots + x_{2p}) \tag{12}$$

$$x'_p = \frac{1}{2p}(x_1 - x_2 + x_3 - x_4 + \ldots - x_{2p}). \tag{13}$$

On the basis of (11) the relationship

$$g^k_{2p-1} = \hat{g}^k_i \;\; i = 1, 2, \ldots, p-1 . \tag{14}$$

is seen to be valid for any k value. By using this, it follows from (10) that

$$x'_{2p-i} = \hat{x}'_i \;\; i = 1, 2, \ldots, p-1 . \tag{15}$$

According to this relationship, the couples of symmetrical components 1 and $2p - 1$, 2 and $2p - 2$, etc. are conjugates of each other.

2. The calculation method

a) *Asymmetrical pole excitation*

Fig. 1 shows a part of the scheme of a direct current machine of $2p$ poles, as developed into the plane. The ideal pole arch is b, the pole pitch τ, the ideal length l. The air gaps δ below the poles are identical. A number of z conductors are arranged along the circumference of the armature.

Fig. 2 is the schematic drawing of the winding. There are $2p$ parallel branches formed, accordingly the resistance of one branch of the winding is $R = R_a \cdot 2p$, where R_a denotes the armature resistance. Contact resistance of the brushes is neglected. The magnetic circuit is considered as linear. Both figures are indicating the directions of excitations, fluxes, currents and voltages

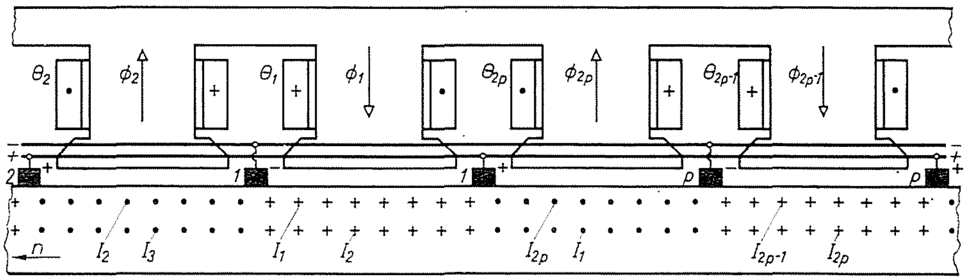

Fig. 1

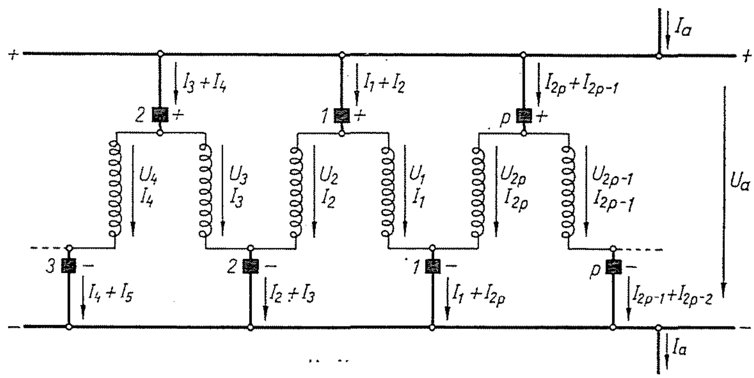

Fig. 2

assumed as positive. Let us determine the distribution of armature current I_a for different pole excitations $\Theta_1, \Theta_2, \ldots, \Theta_{2p}$. Only the steady-state is examined. To solve the problem, the excitation law will be written for the magnetic circuit, and the voltage equation for the winding.

Let us take e.g. the magnetic circuit closed through the poles 1 and 2. Neglecting the magnetic resistance of the iron, the excitation law for the closed loop at a distance x from the axis of the poles is:

$$\Theta_1 + \Theta_2 + \frac{z}{8p}(I_1 - I_3)\left(1 - \frac{2x}{\tau}\right) = \delta(H_1 + H_2)$$

Integrating for the complete pole arch b:

$$\Theta_1 + \Theta_2 + \frac{z}{8p}(I_1 - I_3) = \Lambda_m^{-1}(\Phi_1 + \Phi_2) \tag{16}$$

where Λ_m denotes the magnetic conductivity

$$\Lambda_m = \frac{\mu_0 b l}{\delta} \tag{17}$$

Writing the excitation law also for the other magnetic circuits relationships similar to (16) are obtained. Or, in matrix form:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \dots 0 \\ 0 & 1 & 1 & 0 \dots 0 \\ 0 & 0 & 1 & 1 \dots 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 \dots 1 \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \\ \vdots \\ \Theta_{2p} \end{bmatrix} + \frac{z}{8p} \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \dots 0 \\ 0 & 1 & 0 & -1 & 0 \dots 0 \\ 0 & 0 & 1 & 0 & -1 \dots 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -1 & 0 & 0 & 0 \dots 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_{2p} \end{bmatrix} =$$

$$= \Lambda_m^{1} \begin{bmatrix} 1 & 1 & 0 & 0 \dots 0 \\ 0 & 1 & 1 & 0 \dots 0 \\ 0 & 0 & 1 & 1 \dots 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 \dots 1 \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \vdots \\ \Phi_{2p} \end{bmatrix}. \tag{18}$$

By summarizing the voltages induced in the conductors belonging to e.g. winding branch 1, and adding to this the ohmic voltage drop, we obtain:

$$U_1 = \frac{zn}{2} (\Phi_1 + \Phi_{2p}) + I_1 R$$

By summing up the voltage equations in matrix form:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ \vdots \\ U_{2p} \end{bmatrix} = \frac{zn}{2} \begin{bmatrix} 1 & 0 & 0 \dots 0 & 1 \\ 1 & 1 & 0 \dots 0 & 0 \\ 0 & 1 & 1 \dots 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 \dots 1 & 1 \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \vdots \\ \Phi_{2p} \end{bmatrix} + R \begin{bmatrix} 1 & 0 & 0 \dots 0 \\ 0 & 1 & 0 \dots 0 \\ 0 & 0 & 1 \dots 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 \dots 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_{2p} \end{bmatrix}. \tag{19}$$

Quadratic matrices in (18) and (19) are cyclic matrices of $2p$ order. Introducing cyclic matrices $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{C}_3$ the two matrix equations are:

$$\mathbf{C}_1 \mathbf{\Theta} + \frac{z}{8p} \mathbf{C}_2 \mathbf{I} = \Lambda_m^{-1} \mathbf{C}_1 \mathbf{\Phi} \tag{20}$$

$$\mathbf{U} = \frac{zn}{2} \mathbf{C}_3 \mathbf{\Phi} + R\mathbf{E}\mathbf{I} \tag{21}$$

By symmetrical component transformation, that is, introducing the symmetrical component vectors $\mathbf{\Theta}'$, $\mathbf{U}'$, $\mathbf{I}'$, $\mathbf{\Phi}'$ according to (7), by means of transformation matrx $\mathbf{T}$:

$$\mathbf{C}_1 \mathbf{T} \mathbf{\Theta}' + \frac{z}{8p} \mathbf{C}_2 \mathbf{T} I' = \Lambda_m^{-1} \mathbf{C}_1 \mathbf{T} \mathbf{\Phi}'$$

$$\mathbf{ETU}' = \frac{zn}{2} \mathbf{C}_3 \mathbf{T} \mathbf{\Phi}' + R\mathbf{ETI}'$$

Multiplying both equations by $\mathbf{T}^{-1}$ in front, and using relationship (1), we obtain:

$$\lambda_{(1)i}\Theta_i' + \frac{z}{8p}\lambda_{(2)i}I_i' = \Lambda_m^{-1}\lambda_{(1)i}\Phi_i' \tag{22a}$$

$$U_i' = \frac{zn}{2}\lambda_{(3)i}\Phi_i + RI_i' \tag{22b}$$

$$i = 0, 1, 2, \ldots, 2p-1 \tag{22b}$$

The eigenvalues of matrices $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{C}_3$, on the basis of (5) are:

$$\begin{aligned} &\lambda_{(1)i} = 1 + \hat{g}_i \\ &\lambda_{(2)i} = 1 - \hat{g}_i^2 \qquad i = 0, 1, 2, \ldots, 2p-1 \\ &\lambda_{(3)i} = 1 + \hat{g}_i^{2p-1} \end{aligned} \tag{23}$$

The problem can be solved by equations (22a) and (22b) instead of the original matrix equations (18) and (19). The resulting symmetrical components lead to the solution by using Eq. (7).

Let us examine each equation obtained for the individual components. Let $i = 0$. From (11) and (23):

$$\lambda_{(1)0} = \lambda_{(3)0} = 2, \quad \lambda_{(2)0} = 0\,.$$

Equations for the quantities of order 0 are

$$\Theta_0' = \Lambda_m^{-1}\Phi_0'$$

$$U_0' = zn\Phi_0' + I_0'R$$

The obtained equations correspond to those valid in normal state of operation. Current I_0' does not react on flux Φ_0', this latter is determined by excitation Θ_0' alone. There is no armature reaction. On the basis of Fig. 2, if all the brushes are conducting current,

$$I_a = I_1 + I_2 + \ldots + I_{2p} \tag{24a}$$

$$U_a = U_1 = U_2 = \ldots = U_{2p} \tag{24b}$$

From relationship (12), $U_0' = U_a$, $I_0' = I_a/2p$.

Let $i = p$. The eigenvalues are $\lambda_{(1)p} = \lambda_{(2)p} = \lambda_{(3)p} = 0$. From (24b) and (13), $U_p' = 0$.

Accordingly, relationship (22a) does not determine the magnitude of Φ_p', while from (22b) it follows that $I_p' = 0$. If the fluxes have only p order components, then on the basis of (9) $\Phi_{i+1} = \hat{g}_p^i\Phi_p' = (-1)^i\Phi_p'$, where Φ_p' is seen to be real. Accordingly, the pole fluxes are $\Phi_1 = -\Phi_2 = \Phi_3 = -\Phi_4 = \ldots = -\Phi_{2p} = \Phi_p'$, i.e. the flux is passing below each pole in the same direction. The axial homopolar fluxes can be closed only through the bearings and end-shields, their intensity being determined by the magnetic resistance of this circuit. The homopolar fluxes do not produce compensating currents.

Let $i = 1, 2, 3, \ldots, p-1$. The i-th and $(2p-i)$-th components were seen to be conjugates and the same is true for their equations (22a) and (22b), consequently it is sufficient to discuss the equations for the components of order $1, 2, 3, \ldots, p-1$. Dividing Eq. (22a) by $\lambda_{(1)i}$, and multiplying Eq. (22b) by $\lambda_{(2)i}/\lambda_{(1)i}$, we obtain:

$$\Theta_i' + \frac{z}{8p}(1-\hat{g}_i)I_i' = \Lambda_m^{-1}\Phi_i' \tag{25a}$$

$$U_i'(1-\hat{g}_i) = \mathrm{jzn}\ \sin\left(\frac{\pi}{p}\,i\right)\Phi_i' + RI_i'(1-\hat{g}_i) \tag{25b}$$

$$i = 1, 2, \ldots, p-1.$$

The equations are further simplified by introducing the reduced currents and voltages I_{ir}' and U_{ir}', respectively:

$$I_{ir}' = (1-\hat{g}_i)I_i' \tag{26a}$$

$$U_{ir}' = (1-\hat{g}_i)U_i' \tag{26b}$$

$$\Theta_i' + \frac{z}{8p}I_{ir}' = \Lambda_m^{-1}\Phi_i' \tag{27a}$$

$$U_{ir}' = \mathrm{jzn}\ \sin\left(\frac{\pi}{p}i\right)\Phi_i' + RI_{ir}' \tag{27b}$$

$$i = 1, 2, \ldots, p-1. \qquad (27b)$$

Eliminating Φ_i' from Eq. (27a), and making use of (27b):

$$U_{ir}' = j\frac{8p}{z}\Theta_i'X_i + (R + jX_i)I_{ir}' \tag{27c}$$

where X_i denotes rotation reactance,

$$X_i = n\frac{z^2}{8p}\Lambda_m \sin\left(\frac{\pi}{p}i\right). \tag{28}$$

If relationship (24b) is valid, then on the basis of (10):

$$U_i' = \frac{1}{n}U_a(1 + g_i + g_i^2 \ldots + g_i^{n-1}) = 0 \quad i \neq 0.$$

Using relationship $U_i' = U_{ir}' = 0$, Eqs. (27a) and (27c) simplify to:

$$\Phi_i' = \Theta_i'\Lambda_m\frac{1}{1 + j\xi_i} \tag{29a}$$

$$I'_{ir} = -j\frac{8p}{z}\Theta'_i\frac{\xi_i}{1+j\xi_i} \tag{29b}$$

where $\xi_i = X_i/R$. Current and flux loci plotted from (29a) and (29b) respectively are shown in Fig. 3. The loci descriptively show the course of Φ'_i and I'_{ir}. The parameter is ξ_i. Eq. (25a) demonstrates the examined current components to react on the formation of the flux. From relationship (27a) those symmetrical component systems with identical I'_{ir} appear to exert an equal armature reaction. On the basis of relationship (27c), in turn, it can be stated

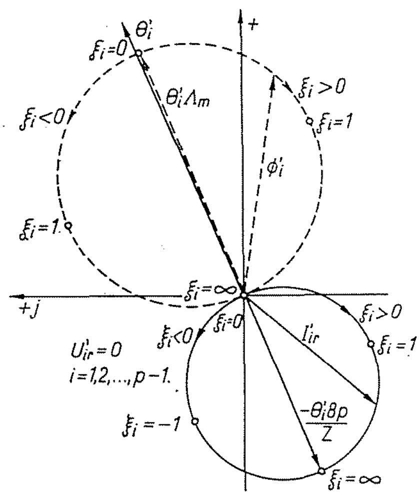

Fig. 3

that the reduced current I'_{ir} is limited not only by the ohmic resistance but also by rotation reactance X_i. Accordingly, systems with identical X_i values behave identically with respect to armature reaction. In the table below, the $\sin(\pi/p)i$ values occurring in the expression for X_i are compiled up to $p = 10$. In the case of an e.g. 10 pole machine not only the systems 1 and 9, but also the systems 1, 9, 4, 6 and 2, 8, 3, 7 are seen to behave identically.

Accordingly, for a given value of Θ'_i, flux Φ'_i is diminishing with the increase of ξ_i, i.e. of the speed of rotation. If the armature is standing, no compensating current is flowing, while the compensating current tends to a constant value with increasing speed of rotation. For $n \to \infty$, Θ'_i and the excitation of currents I'_{ir} are in equilibrium and $\Phi'_i = 0$. The ξ_{in} value pertaining to the rated speed of rotation n_n can be estimated. The rated pole excitation and the armature excitation for the rated armature current are

$$\Theta_{gn} = \Phi_n\Lambda_m^{-1}; \Theta_{an} = \frac{I_{an}z}{8p^2},$$

$\sin\left(\frac{\pi}{p}i\right) \quad i = 1, 2, 3, \ldots p-1.$

p \ i	1	2	3	4	5	6	7	8	9
2	1								
3	0.866	0.866							
4	0.707	1	0.707						
5	0.588	0.951	0.951	0.588					
6	0.5	0.866	1	0.866	0.5				
7	0.434	0.782	0.975	0.975	0.782	0.434			
8	0.383	0.707	0.924	1	0.924	0.707	0.383		
9	0.342	0.643	0.866	0.985	0.985	0.866	0.643	0.342	
10	0.309	0.588	0.809	0.951	1	0.951	0.809	0.588	0.309

respectively. Supposing $R_a \cong 0.05\ U_{an}/I_{an}$, and using (28) we find:

$$\xi_{in} = \frac{n_n z^2 \Lambda_m}{8pR} \sin\left(\frac{\pi}{p}i\right) = \frac{z}{8p} \frac{z n_n \Phi_n}{\Lambda_m^{-1}\Phi_n 2pR_a} \sin\left(\frac{\pi}{p}i\right) \cong$$

$$\cong 9{,}5 \frac{\Theta_{an}}{\Theta_{gn}} \sin\left(\frac{\pi}{p}i\right).$$

Since for direct current machines $\Theta_{an} \approx \Theta_{gn}$, at the rated speed of rotation $\xi_{in} \cong 9.5 \sin(\pi/p)i$. Considering the values of $\sin(\pi/p)i$ at the rated speed of rotation reactance X_i rather than the ohmic resistance of the armature is seen to be the factor determining the current magnitude.

These results permit to establish a physical model simulating the physical phenomena. Let us decompose each quantity x_i' to a real and an imaginary part,

$$x_i' = x_{id}' + jx_{iq}'. \tag{30}$$

In Eq. (27c), replace rotation reactance X_i by rotation inductivity $L_i = X_i/\omega = X_i/2\pi pn$ and decompose quantities U_{ir}', Θ_{ir}', I_{ir}' using relationship (30).

From the equality of the real and imaginary parts we obtain:

$$U_{ird}' = -\frac{8p}{z}\Theta_{iq}'\omega L_i - \omega L_i I_{irq}' + RI_{ird}' \tag{31a}$$

$$U_{irq}' = \frac{8p}{z}\Theta_{id}'\omega L_i + \omega L_i I_{ird}' + RI_{irq}'. \tag{31b}$$

From the equality of the real and imaginary parts we obtain on the basis of Eq. (27a):

$$\Phi'_{id} = \Lambda_m \left[\Theta'_{id} + \frac{z}{8p} I'_{ird} \right] \tag{32a}$$

$$\Phi'_{iq} = \Lambda_m \left[\Theta'_{iq} + \frac{z}{8p} I'_{irq} \right] \tag{32b}$$

The voltage equation of the windings dq of the four-winding $(DQdq)$, $2p$-pole, commutator primitive machine (the currents being direct currents, thus transformer voltages are zero) is found to be [2]:

$$U_d = -\omega L_m I_Q - \omega L_r I_q + R I_d \tag{33a}$$

$$U_q = \omega L_m I_D + \omega L_r I_d + R I_q \tag{33b}$$

Flux relationships:

$$\psi_d = N_r \Phi_d = L_m I_D + L_r I_d \tag{34a}$$

$$\psi_q = N_r \Phi_q = L_m I_Q + L_r I_q \tag{34b}$$

Relating Eqs (31), (32), (33), (34) permits to draw the primitive machine shown in Fig. 4. The armature resistance of the machine is R, and the rotation inductivity

$$L_r = L_i = \frac{z^2}{16\pi p^2} \Lambda_m \sin \left(\frac{\pi}{p} i \right) = N_r^2 \Lambda_m$$

Accordingly, the component couples of various order numbers can be simulated by means of four-winding commutator models having different rotation inductivities L_r. For $U_{ir} = 0$, brushes are short-circuited in Fig. 4

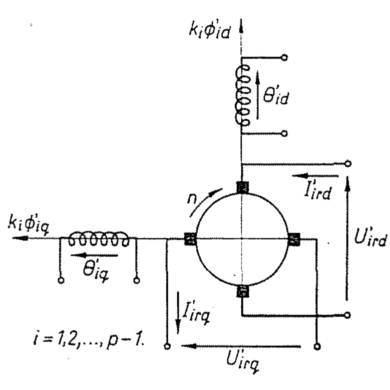

Fig. 4

b) *Examination of small excitation and air gap asymmetries*

Suppose excitation Θ_p and flux Φ_p of each pole to be identical, then armature current I_a is uniformly distributed among the parallel branches, hence only quantities of 0 order exist. If the excitation of the poles differs from the value Θ_p by small $\Delta\Theta_1, \Delta\Theta_2, \ldots, \Delta\Theta_{2p}$ values, and the air gap between the individual poles differs from δ by the small values $\Delta\delta_1, \Delta\delta_2, \ldots, \Delta\delta_{2p}$, then this causes pole fluxes to differ from their operating point value by the small $\Delta\Phi_1, \Delta\Phi_2, \ldots, \Delta\Phi_{2p}$ values. Fluxes $\Delta\Phi$ cause compensating currents $\Delta I_1, \Delta I_2, \ldots, \Delta I_{2p}$ to flow in the armature. According to the excitation law, the magnetic circuit closing through e.g. poles 1 and 2 is ruled by the following relationship neglecting the small quantities of second order:

$$(\Delta\Theta_1 + \Delta\Theta_2) + \frac{z}{8p}(\Delta I_1 - \Delta I_3) = \Lambda_{mp}^{-1}(\Delta\Phi_1 + \Delta\Phi_2) + \frac{B_\delta}{\mu_0}(\Delta\delta_1 + \Delta\delta_2)$$

where

$$\Lambda_{mp} = \left[\frac{\Delta\Phi}{\Delta\Theta}\right]_{\Theta_p}$$

is the operating point slope of the characteristic curve Φ_p (Θ_p) and B_δ the magnitude of air gap induction at the operating point. In case of small changes the characteristic curve is well approximated by its tangent to the operating point. Using notations in (20), equations valid also for the other magnetic circuits lead to the matrix equation:

$$\mathbf{C}_1\boldsymbol{\Delta\Theta} + \frac{z}{8p}\mathbf{C}_2\boldsymbol{\Delta}\mathbf{I} = \Lambda_{\mathrm{mp}}^{-1}\mathbf{C}_1\boldsymbol{\Delta\Phi} + \frac{B_\delta}{\mu_0}\mathbf{C}_1\boldsymbol{\Delta\delta}.$$

Rearranging:

$$\mathbf{C}_1\left(\boldsymbol{\Delta\Theta} - \frac{B_\delta}{\mu_0}\boldsymbol{\Delta\delta}\right) + \frac{z}{8p}\mathbf{C}_2\boldsymbol{\Delta}\mathbf{I} = \Lambda_{\mathrm{mp}}^{-1}\mathbf{C}_1\boldsymbol{\Delta\Phi}. \tag{35}$$

The voltage equation:

$$\boldsymbol{\Delta}\mathbf{U} = \frac{zn}{2}\mathbf{C}_3\boldsymbol{\Delta\Phi} + R\mathbf{E}\boldsymbol{\Delta}\mathbf{I}. \tag{36}$$

From Eq. (35) air gap asymmetries $\Delta\delta_1, \Delta\delta_2, \Delta\delta_3, \ldots, \Delta\delta_{2p}$ are seen to be reducible to fictitious excitation asymmetries. The vector of the fictitious excitation is $-(B_\delta/\mu_0)\boldsymbol{\Delta\delta}$. Thereby, Eqs (35) and (36) valid for small changes, are formally quite identical with Eqs (20) and (21), thus the results obtained in the previous item can be used directly. In the case of higher asymmetries the obtained results can only be regarded as first approximations.

3. Application of this method for a four-pole machine

For the sake of simplicity, the application of the method is illustrated on a four-pole machine.

The four-pole machine will be examined by the four phase symmetrical components. For $2p = 4$, on the basis of (11) $\hat{g}_i = (-j)^i$. Using relationships (2), (3) and (6), the transformation matrix $\mathbf{T}$ and its inverse can be written. Hence according to (7) and (8), the transformation rules for arbitrary quantities x_1, x_2, x_3, x_4, either excitations, fluxes, voltages, or currents, are:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} x'_0 \\ x'_1 \\ x'_2 \\ x'_3 \end{bmatrix} \tag{37}$$

$$\begin{bmatrix} x'_0 \\ x'_1 \\ x'_2 \\ x'_3 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{38}$$

Let us examine now the case where there are only components of 1 and 3 order, that is, components of 0 and 2 order are missing from each quantity. From Eq. (38), since $x'_0 = x'_2 = 0$,

$$x'_1 = \frac{1}{2}(x_1 + jx_2) = \hat{x}'_3 \tag{39}$$

$$x_1 = -x_3 \tag{40a}$$

$$x_2 = -x_4 \tag{40b}$$

From Eq. (37), using also Eq. (39) we obtain:

$$x_1 = x'_1 + x'_3 = 2\,\mathrm{Re}\,(x'_1) \tag{41a}$$

$$x_2 = j(x'_3 - x'_1) = 2Im(x'_1)\,. \tag{41b}$$

If all the brushes are conducting current, then, as it was seen, $U'_1 = 0$. On the basis of (29a) and (29b) the loci for the component of $i = 1$ order are shown in Fig. 5. From (28) the parameter is found to be:

$$\xi_1 = \frac{X_1}{R} = \frac{nz^2}{16R} \Lambda_m$$

Component I'_1 of current I'_{1r} is furnished by relationship (26a):

$$I'_1 = \frac{I'_{1r}}{1+j} = \frac{1}{\sqrt{2}} I'_{1r} e^{-j45^\circ} \tag{42}$$

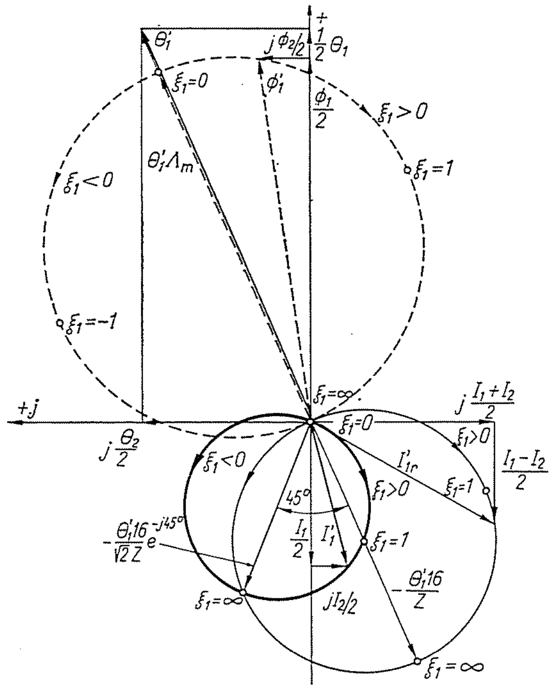

Fig. 5

The current locus for current component I_1' is again a circle, obtained from the locus of I_{1r}' by rotating by 45° and reducting by $\sqrt{2}$. It follows from (42) and (39) that

$$I_{1r}' = (1+j)I_1' = \frac{I_1 - I_2}{2} + j\,\frac{I_1 + I_2}{2}\,. \tag{43}$$

According to relationship (41), the real parts in the locus of Φ_1' and I_1' show the course of Φ_1 and I_1, while the imaginary parts the course of Φ_2, and I_2, respectively. On the basis of Eq. (43) the course of the currents $(I_1 + I_2)$ and $(I_1 - I_2)$ can be read off the locus of I_{1r}', just equal to the currents flowing on the positive and negative brush couples, respectively. In Fig. 6 these quantities are plotted as function of ξ_1, for $\Theta_2 = 0$. $\xi_{1n} \approx 10$ is assigned to the nominal speed of rotation.

The physical model which can be drawn in the present case is shown in Fig. 7. Assuming $\Theta_2 = 0$, the course of the individual quantities shown in Fig. 6 can also be followed on the basis of the model.

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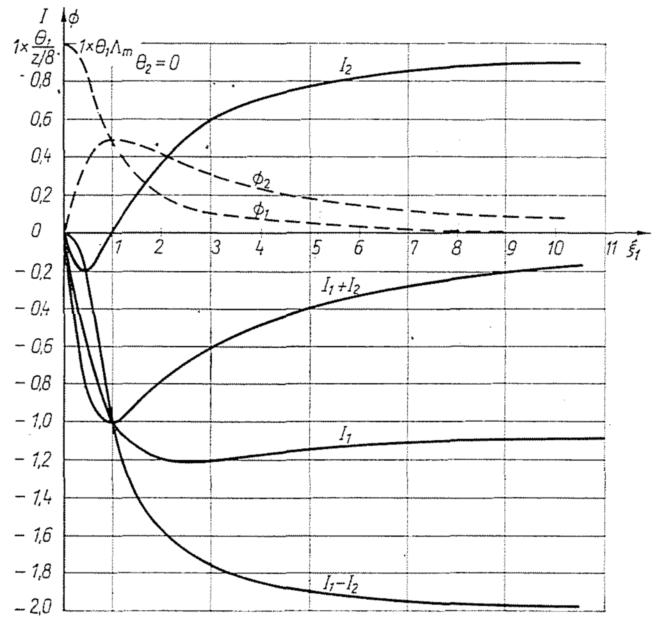

Fig. 6

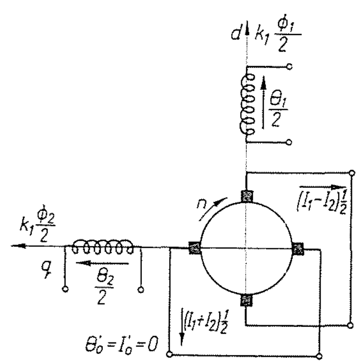

Fig. 7

Summary

In multipolar lap windings compensating currents are flowing on account of the asymmetry of pole excitations and air gap dimensions. Air gap asymmetry can be reduced to a fictitious excitation asymmetry. Compensating currents flowing upon the effect of asymmetrical excitations can be calculated by the method of decomposing to symmetrical components. Compensating currents generated by the individual symmetrical component excitation systems can be examined separately. Flux asymmetry existing in the steady state is reduced during rotation by the reaction of compensating currents in the armature, to be taken into consideration when calculating compensating currents. The degree of reaction is different in the case of systems of various orders.

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