ANALYSIS OF LINEAR CONTROL SYSTEMS WITH DEAD TIME IN THE TIME DOMAIN

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(Received September 13, 1973)

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Two possibilities offer themselves for the analysis of the control systems in the time domain: either in an indirect way, conclusions concerning their behaviour in the time domain may be drawn on the basis of the frequency domain studies with the aid of empirical relationships [1, 5, 6, 7, 8, 9, 10, 11, 12], or the static and the dinamic states of the system may be determined directly by establishing the time function of the control system [1, 2, 3, 4, 12]. The calculation by the indirect method is more convenient, but the conclusions drawn concerning the transient behaviour of the system are not always sufficiently accurate in this case.

The study of the dynamic state of linear, concentrated-parameter, invariant systems, even in the single-loop case is made difficult by the presence of dead time, as is met with in the frequently applied modelling of the control system time behaviour on the analogue computer, due to the difficulty of transforming the dead time. But at present the optimum adjustment according to specified aspects of a control system with dead time by the method of digital simulation and optimization does not cause any more difficulties. For applying this method the use of the digital computer is indispensable.

The results obtained by the analysis may be well utilized for the rapid planning of industrial control processes. But as plants of the most various character are found in practice, no rules of general validity for the type of the compensating element and the adjustment of its parameters can be given. But the grapho-analytical identification processes in most cases lead to systems of first order lag with dead time, or even more frequently of a second order lag with dead time. Therefore, we will describe the behaviour of the plant in the simpler cases by a first order lag with dead time. In this case we generally assume that if one time constant of the process is considered, the other time constants may be neglected. The analysis of the first order lag, single-loop, concentrated-parameter control system with dead time has already been met with in the literature [2, 4].

The process to be controlled is approximated in many cases by choosing its transfer function as a second order lag element with dead time [1]. Therefore, the determination of the dynamic characteristics (maximum overshot, control time) seems to be desirable when the plant is a second order lag with dead time.

Determination of the Dynamic Characteristics of a Linear Control System with Second Order Lag and Dead Time

In the following the course of the transient process of the control system with dead time shown in Fig. 1 versus the dead time and the time constants of

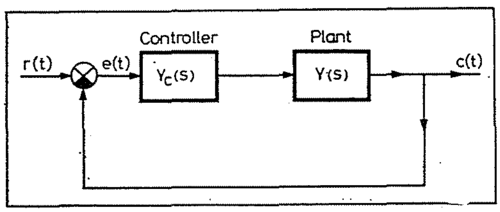

Fig. 1

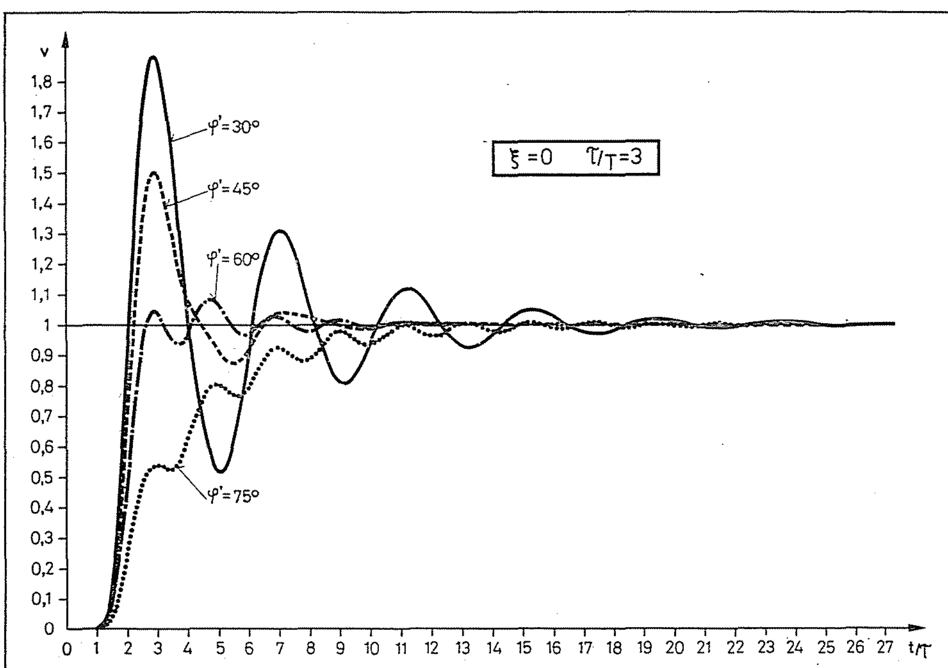

Fig. 2

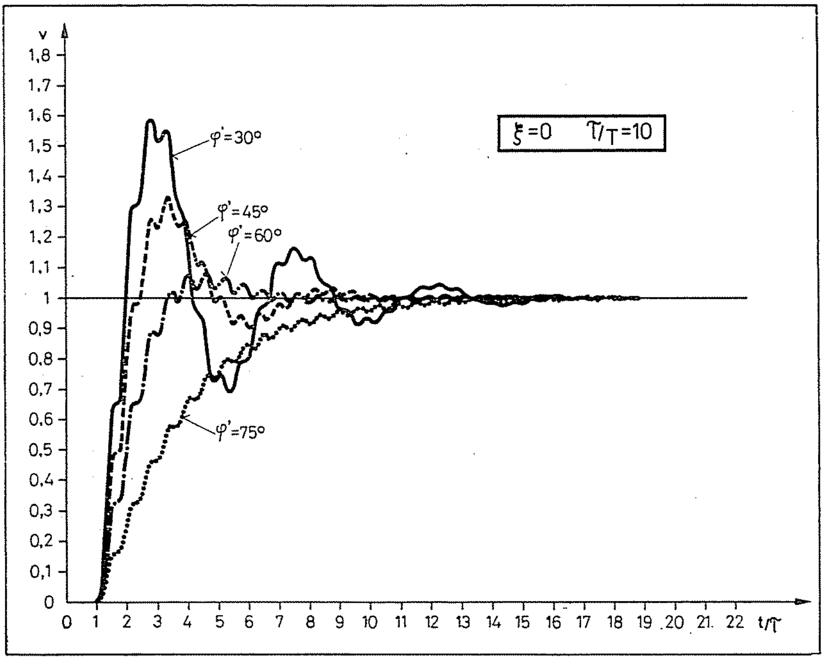
ξ=0 τ/T=10
φ'=30°
φ'=45°
φ'=60°
φ'=75°
v
1,8
1,7
1,6
1,5
1,4
1,3
1,2
1,1
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 t/τ

Fig. 3

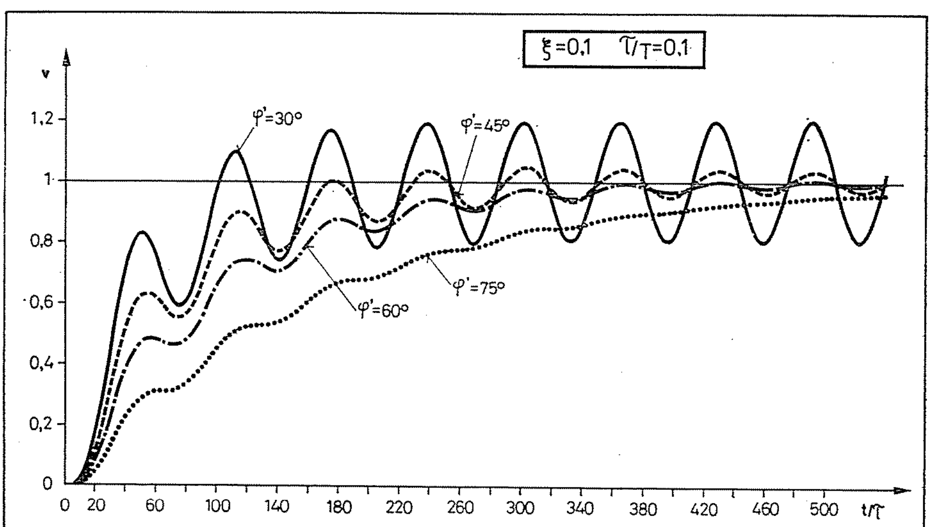
ξ=0,1 τ/T=0,1
φ'=30°
φ'=45°
φ'=60°
φ'=75°
v
1,2
1
0,8
0,6
0,4
0,2
0
0 20 60 100 140 180 220 260 300 340 380 420 460 500 t/τ

Fig. 4

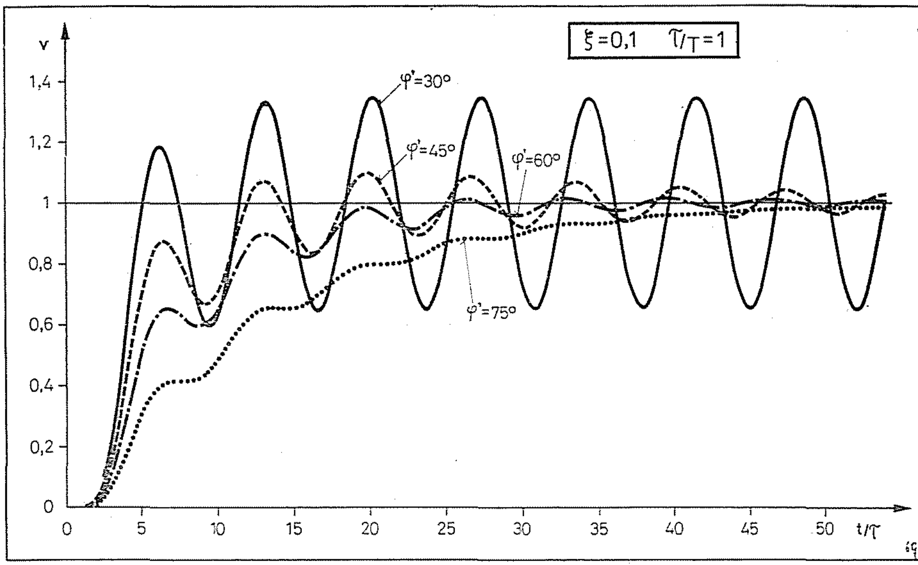
ξ=0,1 τ/T=1
v
1,4
1,2
1
0,8
0,6
0,4
0,2
0
φ'=30°
φ'=45°
φ'=60°
φ'=75°
0 5 10 15 20 25 30 35 40 45 50 t/τ

Fig. 5

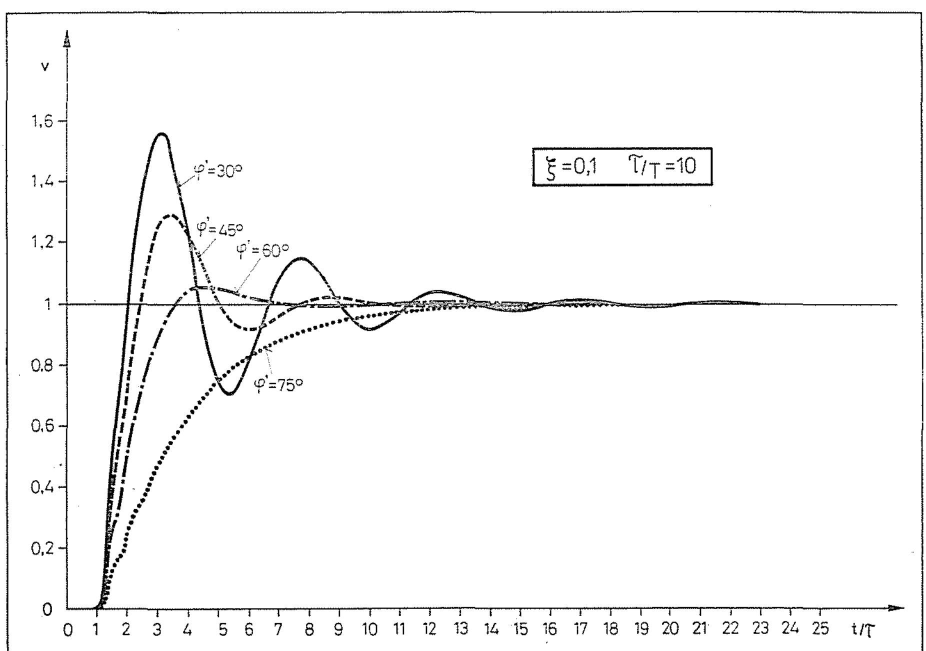
ξ=0,1 τ/T=10
v
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
φ'=30°
φ'=45°
φ'=60°
φ'=75°
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 t/τ

Fig. 6

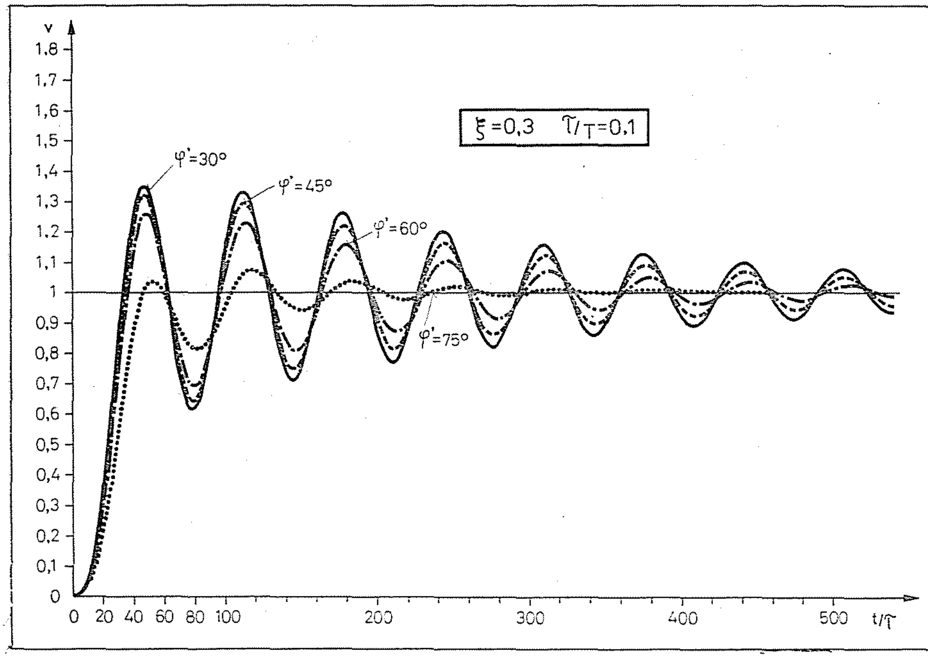
v
1,8
1,7
1,6
1,5
1,4
1,3
1,2
1,1
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0 20 40 60 80 100 200 300 400 500 t/τ
ξ=0,3 τ/T=0,1
φ'=30°
φ'=45°
φ'=60°
φ'=75°

Fig. 7

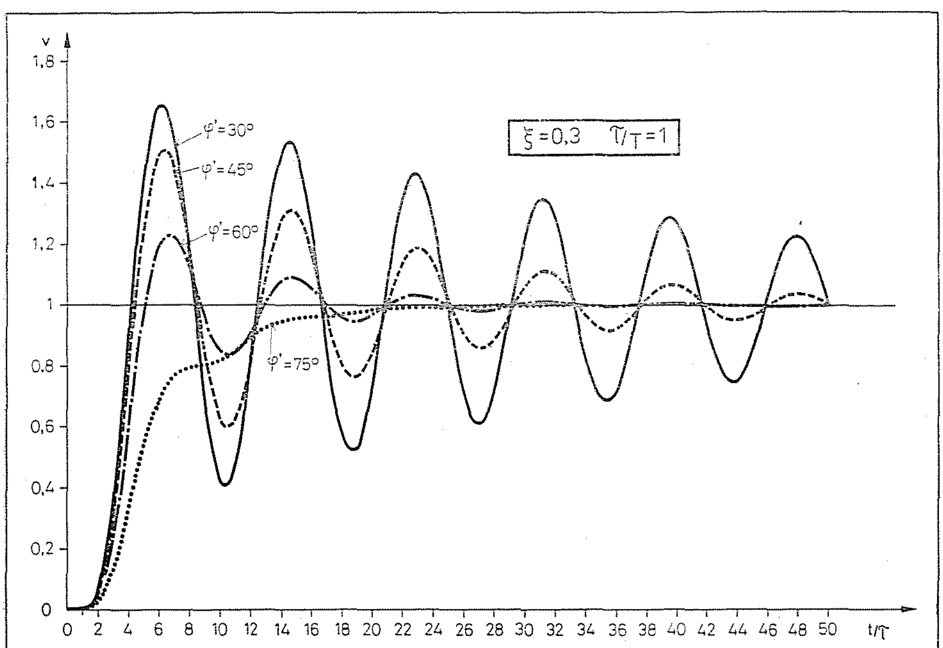
v
1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 t/τ
ξ=0,3 τ/T=1
φ'=30°
φ'=45°
φ'=60°
φ'=75°

Fig. 8

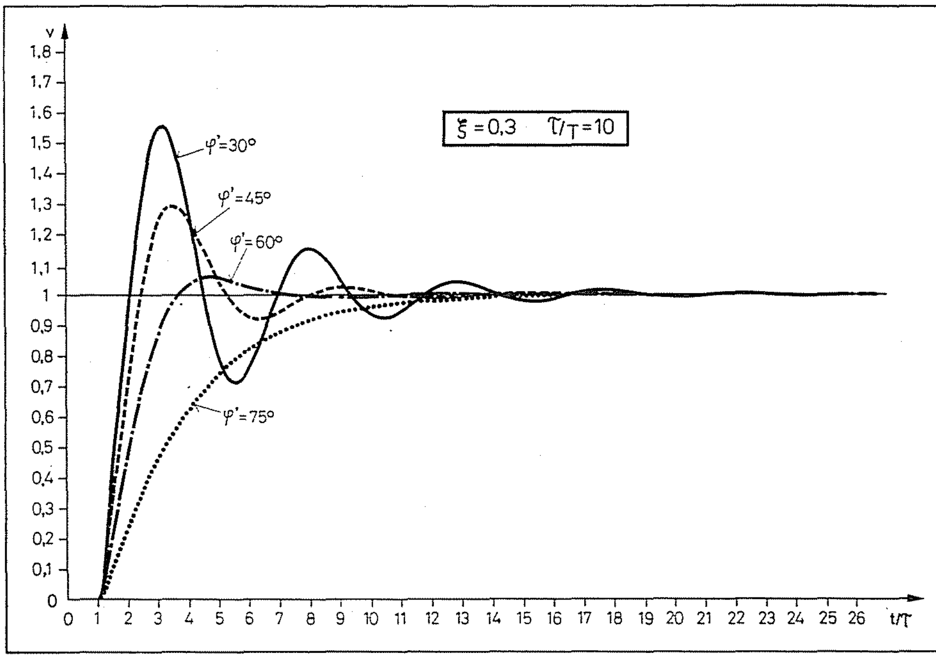
v
1,8
1,7
1,6
1,5
1,4
1,3
1,2
1,1
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
ξ =0,3 τ/T=10
φ'=30°
φ'=45°
φ'=60°
φ'=75°
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 t/T

Fig. 9

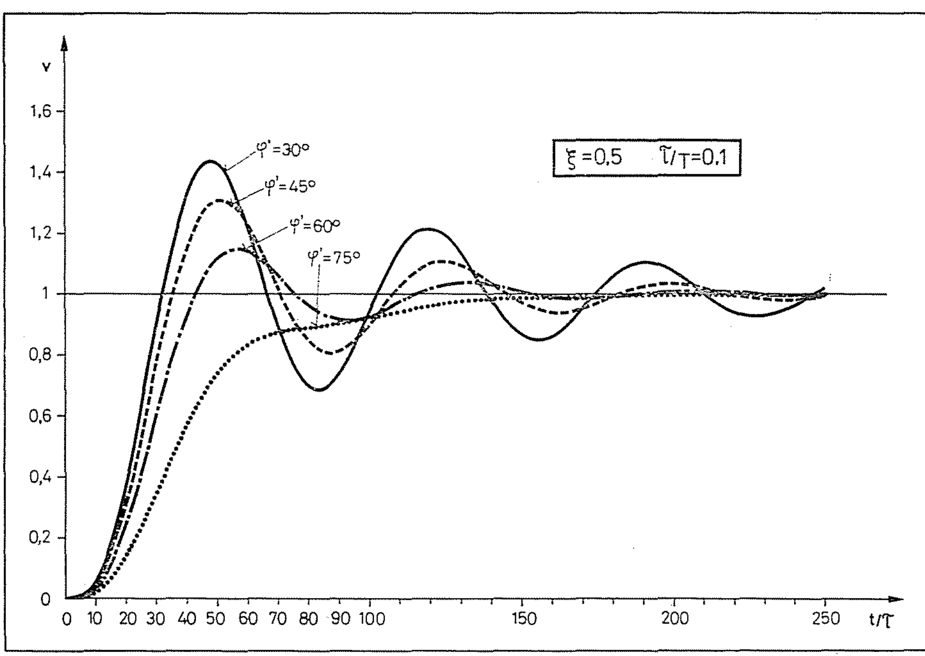
v
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
ξ =0,5 τ/T=0,1
φ'=30°
φ'=45°
φ'=60°
φ'=75°
0 10 20 30 40 50 60 70 80 90 100 150 200 250 t/T

Fig. 10

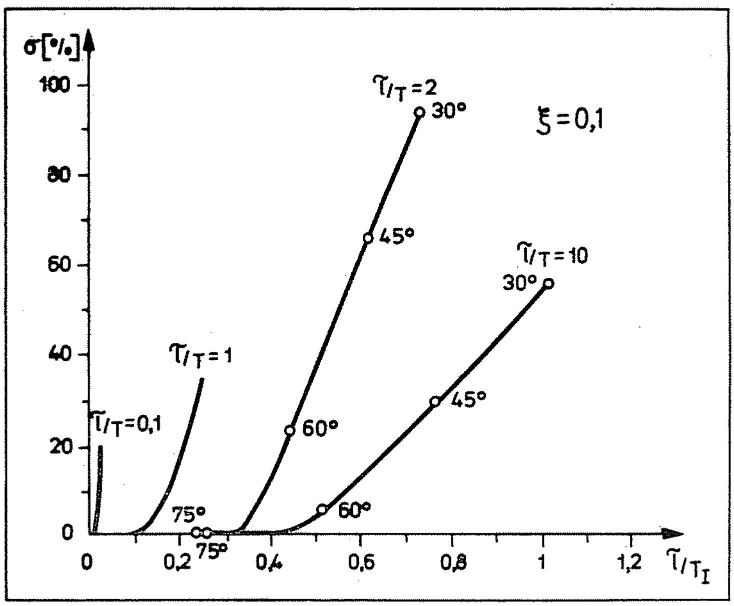
σ [%]
100
80
60
40
20
0
τ/T = 2
30°
ξ = 0,1
45°
τ/T = 10
30°
τ/T = 1
45°
τ/T = 0,1
60°
75°
60°
75°
0
0,2
0,4
0,6
0,8
1
1,2
τ/T_I

Fig. 11

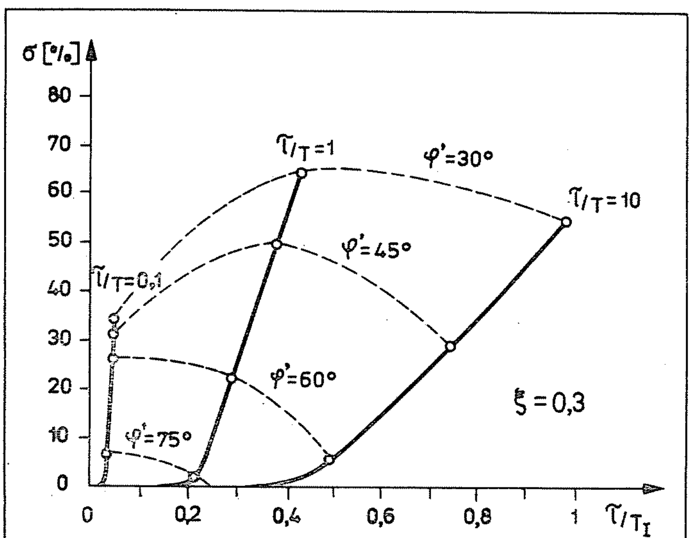
σ [%]
80
70
60
50
40
30
20
10
0
τ/T = 1
φ' = 30°
τ/T = 10
φ' = 45°
τ/T = 0,1
φ' = 60°
ξ = 0,3
φ' = 75°
0
0,2
0,4
0,6
0,8
1
τ/T_I

Fig. 12

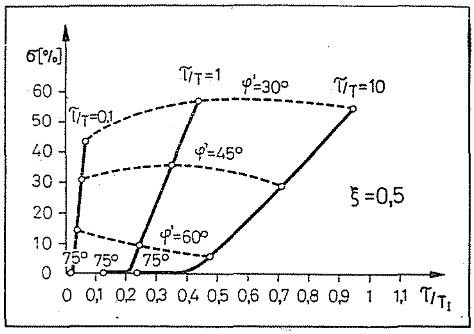

Fig. 13

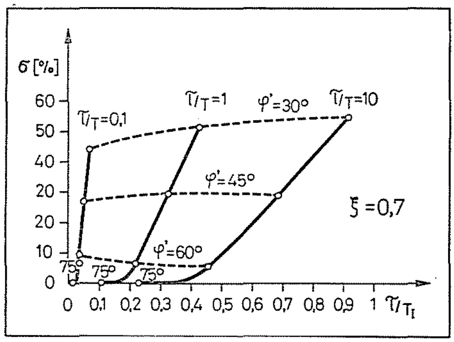

Fig. 14

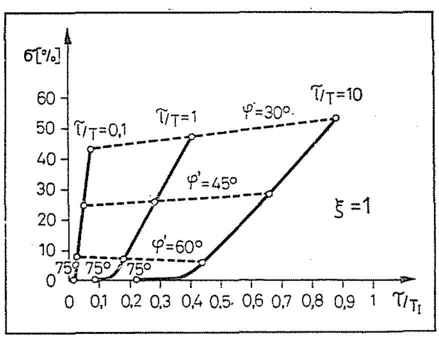

Fig. 15

the system, when the input signal is the unit step function, is investigated. The transfer function of the plant is:

$$Y(s) = \frac{e^{-s\tau}}{1 + 2\zeta Ts + T^2 s^2}$$

where τ is the dead time, T the time constant of the second order lag and ζ the damping factor.

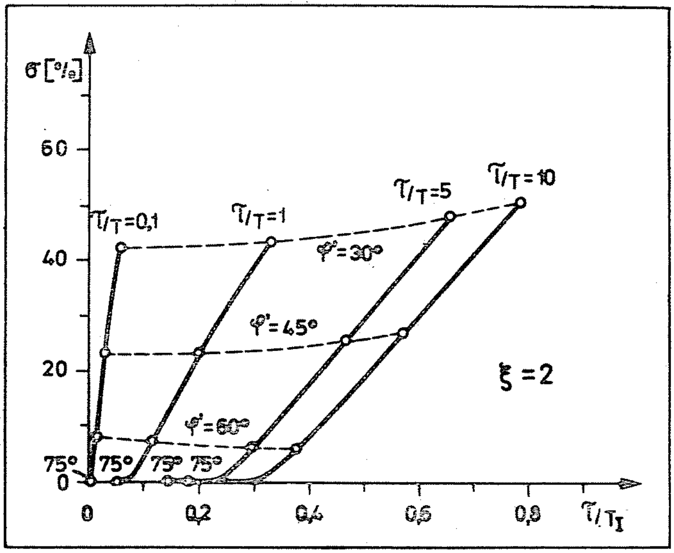

Fig. 16

The transfer function of the control integrating element is:

$$Y_c(s) = \frac{1}{sT_1}$$

T_I is the integral time constant.

The step response has been determined by the digital simulation method. The numerical integration of the differential equations was carried out by the fourth order RUNGE—KUTTA method.

The transfer function was taken up with $\zeta = 0$, 0.1, 0.3, 0.5, 0.7, 1, 2, $\tau/T = 0.1$, 1, 10 and $\varphi' = 30°$, 45°, 60° and 75°. From among the great number of the time functions only the characteristic types are given (Figs 2—10). In the case of $\zeta > 0.5$ damping factors the character of the transfer function for the values $\tau/T = 0.1$, 1, 10 agrees with the course of the transfer function shown in Fig. 10, when $\varphi' = 30°$, 45°, 60° and 75°, respectively.

The high frequency vibrations in Figs 2 and 3 ($\zeta = 0$) are caused by the fact that the own vibrations of the second order lag is superimposed on the time function of the closed control circuit. If e.g. $\zeta = 0$ and $\tau/T = 3$, then $\omega = 1/T = 3 = 2\pi/T_p$, from which $T_p = 2.0733$, as can be read off the diagram in Fig. 2.

The maximum overshoot (σ) and the controlling time (t_c) versus the limit of the stability region ensuring the arbitrary phase margin were plotted in diagrams (Figs 11—16 and 17—21, respectively). The points belonging to identical φ' values are connected. For plotting the functions $\sigma = \sigma(\tau/T_I)$ and

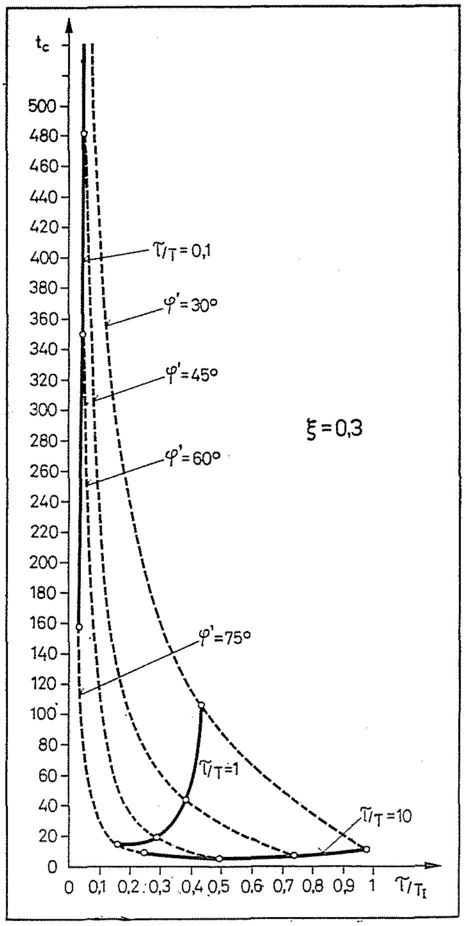

Fig. 17

$t_c = t_c(\tau/T_I)$ only four points were available, therefore, the intermediate points of the curves may slightly deviate from the real course.

Conclusions drawn from the maximum overshot diagrams:

1. σ — as expected — increases with the decrease of ζ.

2. With the decrease of ζ the σ values belonging to an identical φ' value show ever higher deviations from a specified value.

3. We have compared the results with the values obtained by applying the M—α curves and the empirical relationship [1] given by the literature [12] and have found that the best agreement with the σ values obtained by the digital simulation method is supplied by the estimated values in the proximity of the $\varphi' = 45°$ phase margin.

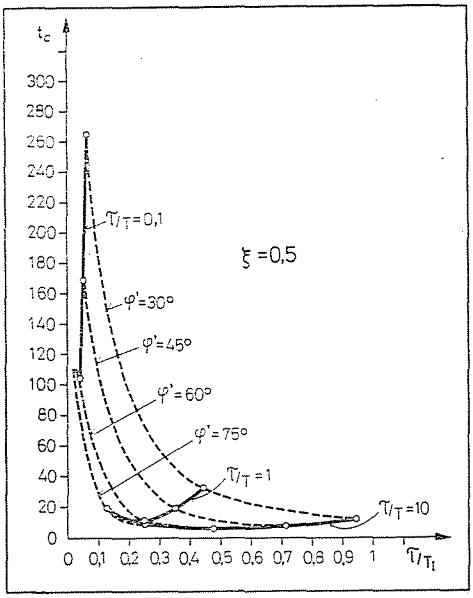

Fig. 18

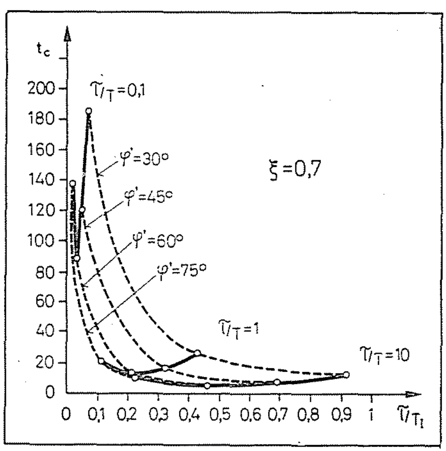

Fig. 19

In plotting the diagrams $t_c/\tau = f(\tau/T_I)$ the time passing until the deviation of the transfer function from the final value becomes less than $\pm 5\%$, was accepted as control time.

The curves of the control time versus τ/T_I with the parameter τ/T show a minimum in the proximity of $\varphi' = 60°$. The exact location of the minimum cannot be determined by the diagrams, as only four points were available for plotting the curves. But in planning the approximate knowledge of the location of the minimum can be well utilized, too.

We should note that the curve t_c/τ and its differential quotient, respectively, versus the time constants of the system is not continuous [12].

We have compared the obtained values of the control time with the t_c values estimated by the empirical relationship [1]

$$\frac{\pi}{\omega_c} \leqq t_c \leqq \frac{3\pi}{\omega_c}$$

known from the literature [12]. Here ω_c denotes the intersection angular frequency of the open loop frequency characteristic curve. It can be established that in the region $45^\circ \leqq \varphi' \leqq 60^\circ$ the control time estimated by the empirical relationship can be regarded as a relatively good approximation. In the case of $\varphi' < 45^\circ$ the t_c value derived from the empirical relationship is lower (i.e. considerably lower) and in the case of $\varphi' > 60^\circ$ it is higher (i.e. considerably higher) than the effective control time. The deviations are caused by the fact that the empirical relationship assumes a unidirectional variation of the control time which in reality does not exist.

Conclusions

By comparing the σ and t_c values — determined by the digital computer and estimated, respectively, with the aid of the empirical relationship — of the linear, unit-feedback control with dead time, containing a second order lag as shown in Fig. 1, it could be established that the σ values calculated by the approximative relationship show a good agreement with the results obtained

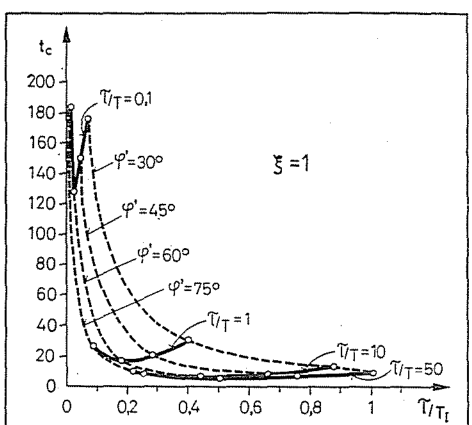

Fig. 20

by the computer in the interval $45° \leqq \varphi' \leqq 60°$. Regarding that in most cases the quality requirements are satisfied by the course of the transfer function for the values of the interval $45° \leqq \varphi' \leqq 60°$, so for less demanding controls the empirical relationship may well be utilized for the estimation of σ and t_c. But for more demanding controls the values of the parameters of the compensating element can be adjusted only with the required accuracy by using the results obtained with the aid of the digital computer.

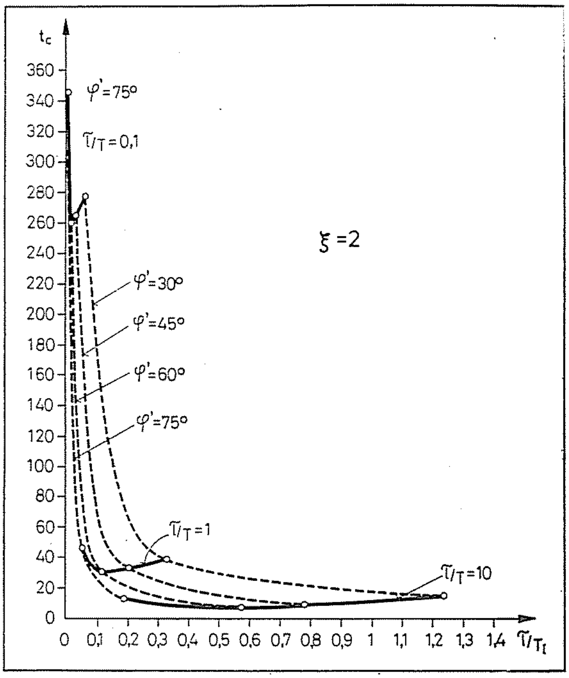

Fig. 21

We should mention that in the knowledge of the modern methods of synthesis (optimization, digital simulation, hybrid computation, etc.) the planning of systems with dead time means no difficulty for the theoreticans. At the same time for the practical experts and those of other professional fields a great assistance is given by the precalculated curves in planning control systems with dead time, moreover the precalculated curves may very well be utilized, even by the theoreticans, for choosing the initial points for the modern synthesis methods.

Summary

In the present paper the variation of the performance characteristics (maximum overshot, control time) versus the time constants and the phase margin of a linear, single-loop, concentrated-parameter control system with dead time containing a second order lag plant

and compensated by a serial integrating element has been determined; the results calculated by a digital computer were plotted in diagrams; the obtained maximum overshot and control time values were compared with the σ and t_c values calculated by the empirical relationships given by the literature.

References

1. Dr. Csáki, F.—Bars, R.: Automatics. (In Hungarian.) Tankönyvkiadó, Budapest, 1972.
2. Goscinski, A.: Przeregulowania w ukladach z opoznieniem. Automatyki i Telemechaniki. TOM-XV ZESZYT 1—1970.
3. Schmidt, G.: Vergleich verschiedener Totzeitregelsysteme. Messen, Steuern, Regeln. **10**, 1967, H. 2.
4. СЕРЕБРЯНСКИЙ, А. Я.: Метод определения величины максимального отклонения в системах автоматического регулирования, включающих объект с запаздыванием. Приборы и системы управления. 1968, II.
5. Csáki, F.—Habermayer, M.: Stability test of linear control systems with dead time. Periodica Polytechnica, El. **12**, 1968, 3.
6. Habermayer, M.: Stability test of linear control systems with dead time by digital computer. Periodica Polytechnica, El. **12**, 1968, 4.
7. Habermayer, M.: Stability test of linear control systems with dead time compensated by PI controller. Periodica Polytechnica, El. **14**, 1970, 1.
8. Habermayer, M.: Stability test of linear control system with dead time compensated by PID action controller. Periodica Polytechnica, El. **14**, 1970, 2.
9. Habermayer, M.: Stability degree analysis of linear control systems with dead time. Periodica Polytechnica, El. **15**, 1971, 3.
10. Habermayer, M.: Stability degree analysis of linear control systems with dead time compensated by a proportional-plus-integral controller. Periodica Polytechnica, El. **15**, 1971, 3.
11. Habermayer, M.: Stability degree analysis of linear control systems with dead time by a digital computer. Periodica Polytechnica, El. **16**, 1972, 4.
12. Habermayer, M.: Analysis of linear, single-loop control systems with dead time in the frequency and time domain. Doctoral dissertation. Technical University, Budapest, 1972.

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