SOME REMARKS ON DIGITAL COMPUTER NUMERICAL METHOD FOR THE SOLUTION OF A SECOND ORDER NONLINEAR DIFFERENTIAL EQUATION

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Introduction

In recent years the problem of thermal breakdown in the design and operation of high current, high voltage and high power equipments and cables, respectively, is increasingly gaining in importance. This type of breakdown has been known since the turn of the century and has an extensive theory [2, 3, 5, 10, 11, 12, 13, 18—24, 28, 29, 32, 46—49]. But this theory can by no means be regarded as closed and several calculation varieties of one of the characteristic parameters of breakdown, the so-called thermal breakdown voltage is known, depending on the choice of the model. The Russian physicist V. A. FOK developed the function, named after him, for symmetrical plane and cylindrical arrangements, respectively, permitting to express the thermal breakdown voltage by the electrical and thermal parameters of insulation [8, 16, 28, 29, 35]. The accurate table of the Fok-function may be found in ref. [30, 41, 43].

For the case of asymmetrical cooling and for allowing, respectively, for the frequently appearing additional heat flow analitically, only rather vague approximations, are known [2]. The numerical methods given in refs. [36] and [42, 43] may be applied more generally and are more accurate than the formerly known methods and require the use of a digital computer.

We contribute some complementary remarks to the calculation process [42] as described in the following.

Notations

E — electric field strength [V/m]
$2U_0$ — dielectric energizing voltage [V]
U_{th} — thermal breakdown voltage [V]
p_0' — dielectric losses per unit volume and per unit field strength at temperature ϑ_0 [W/mV^2]
b — thermal coefficient of the temperature dependence of $p'(\vartheta)$ [1/°C]
ϑ_0 — reference temperature [°C]
ϑ_a — ambient temperature [°C]
ϑ_m — maximum temperature [°C]
$2h$ — dielectric thickness [m]

m — electrode thickness [m]
F — heat transfer surface [m^2]
α — heat transfer coefficient [$W/m^2 {}^\circ C$]
λ_1 — dielectric thermal coefficient [$W/m^\circ C$]
λ_2 — electrode thermal coefficient [$W/m^\circ C$]
Q_{in} — additional heat inflow to the dielectric [Ws]
$\text{tg}\,\delta$ — power factor
v — relative thermal breakdown voltage
$\dot{\Theta}$ — notations of derivation
Θ, z, R — relative variable.

1. Differential equation, the applied model

As is known, the thermal breakdown is a consequence of the thermal instability due to the nonlinear (generally exponential) thermal dependence of the losses occurring in the dielectric. The study of this phenomenon requires the knowledge of the stationary temperature distribution arising in the dielectric with given boundary conditions and of the parameter value where this solution disappears [3, 5, 8, 15, 40, 45]. Without violating generalities we restrict ourselves to the study of one-dimensional conditions. Fig. 1 shows our plane and cylindrical models with the note that additional heat flows

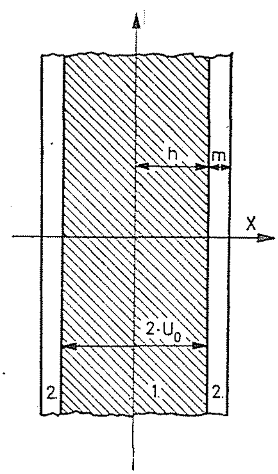

Fig. 1. Symmetrical plane model. 1. — dielectric, 2. — electrode

appear in both cases, or more accurately the loss of I^2R joule forming in the inner electrode flows through the dielectric and is transferred into the ambient. In Fig. 2 our plane and cylindrical models are asymmetrical, i.e. the electrodes are in contact with spaces of different ambient temperatures. Naturally, the combination of both cases may also occur.

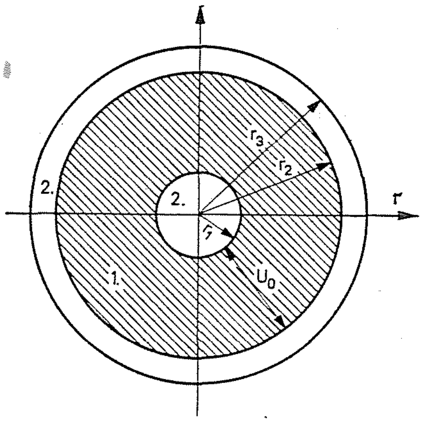

Fig. 2. Asymmetrical plane and cylindrical model. 1. — dielectric; 2. — electrode

By applying the KIRCHHOFF—FOURIER heat conduction differential equation to our models [4, 5] and assuming an exponential temperature dependence of the losses [7, 9] we have:

$$\frac{d^2\Theta}{dz^2} + B_1 \cdot e^{\Theta} = 0 \text{ for the plane} \tag{1}$$

and

$$\frac{d^2\Theta}{dR^2} + \frac{1}{R}\frac{d\Theta}{dR} + \frac{B_2}{R^2} \cdot e^{\Theta} = 0 \tag{2}$$

for the cylinder, where

$$B_1 = \frac{p_0' \cdot b \cdot U_0^2 \cdot e^{-\Theta_0}}{\lambda_1} \quad \text{and} \quad B_2 = \frac{p_0' \cdot b \cdot U_0^2 \cdot e^{-\Theta_0}}{\lambda_1 \cdot (\ln R_2)^2} \tag{3}$$

$$\Theta = b\vartheta, \quad z = \frac{x}{h}, \quad R = \frac{r}{r_1}\ . \tag{4}$$

The boundary conditions required for the solution of the equations are [4, 14]:

$$\Theta(z = 0) = \Theta_m \tag{5}$$

for the plane

$$\dot{\Theta}(z = 0) = -\frac{b \cdot h}{\lambda_1 \cdot F} \cdot Q_{in} \tag{6}$$

$$\Theta(R = 1) = \Theta_m \tag{7}$$

for the cylinder

$$\dot{\Theta}(R = 1) = -\frac{b \cdot r_1}{\lambda_1 \cdot F_1} \cdot Q_{in}\ . \tag{8}$$

The fundamental idea of the calculation process described in ref. [42] was that Eqs (1) and (2) are solved with the given boundary conditions and the B value, by a digital computer and the thermal breakdown voltage is established by the envelope curve of the obtained set of curves.

2. Zero additional heat flow, symmetrical model

If $Q_{in} = 0$, i.e. no heat quantity is flowing into the dielectric from the outside, only the losses forming in it is transferred to the ambient, then the analytical solution of Eqs (1) and (2) with the maximum temperature (ϑ_m) as a parameter may be given as:

for the plane

$$\Theta(z, \Theta_m) = \Theta_m - 2 \cdot \ln [\mathrm{ch}\, (\beta z)] \tag{9}$$

and for the cylinder

$$\Theta(R, \Theta_m) = \Theta_m - 2 \cdot \ln [\mathrm{ch}\, (\beta \ln R)] \tag{10}$$

where

$$\beta = \sqrt{\frac{B}{2} e^{\frac{1}{2}\Theta_m}} \quad . \tag{11}$$

The envelope curve of this set of curves of the temperature distribution $\Theta(z, \Theta_m)$ (with the parameter Θ_m) may be obtained by the elimination of Θ_m as [37]:

for the plane

$$\Theta_b(Z, B_1) = 2 \cdot \ln \left(\frac{\beta_\infty}{z} \sqrt{\frac{2}{B_1}} \right) - 2 \cdot \ln[\mathrm{ch}(\beta_\infty)] \tag{12}$$

and for the cylinder

$$\Theta_b(R, B_2) = 2 \cdot \ln \left(\frac{\beta_\infty}{\ln R} \sqrt{\frac{2}{B_2}} \right) - 2 \cdot \ln[\mathrm{ch}(\beta_\infty)] \tag{13}$$

where

$$\beta_\infty = 1.19967864$$

(see in ref. [32, 43]).

It can be seen that in the case of given B values the envelope curves are easily calculated. The horizontal axis, as shown in ref. [42] may also be calibrated for relative voltage values. Fig. 3 shows the envelope curves for our plane model with 6 different B_1 values. The dashed line gives the solution of (9) for the case of $\Theta_m = 6$ and $B_1 = 0.22$. Fig. 4 presents the same results for the cylindrical model.

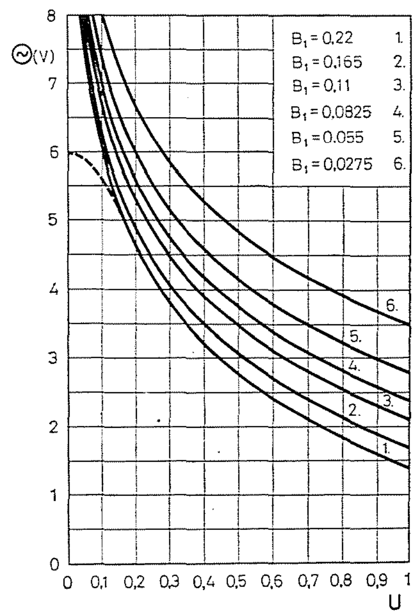

Fig. 3. Temperature distribution envelope curve of a plane dielectric versus the relative thermal breakdown voltage with the parameter B_1, $\dot{\Theta}(0) = 0$

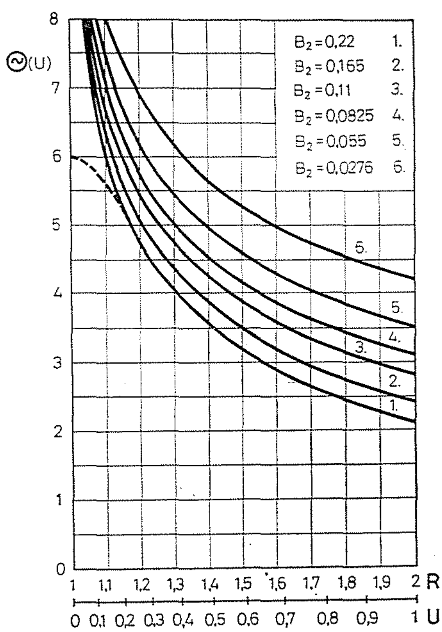

Fig. 4. Temperature distribution envelope curve of a cylindrical dielectric versus the relative thermal breapqown voltage with the parameter B_2, $\dot{\Theta}(1) = 0$

The voltage-temperature characteristic curve determining the thermal breakdown voltage can easily be constructed now in the knowledge of the set of curves and its envelope curve for a given relative ambient temperature, as shown in Figs 5 and 6 for various relative ambient temperatures, also showing the individual values of the relative thermal breakdown voltages. The thermal breakdown voltage may be expressed now in the following form:

$$U_{th} = v \cdot U_0 \ [V] \tag{14}$$

where U_0 is the value of the voltage figuring in expression B. The value of U_{th} can be proved to be independent of the assumed value of U_0 [43].

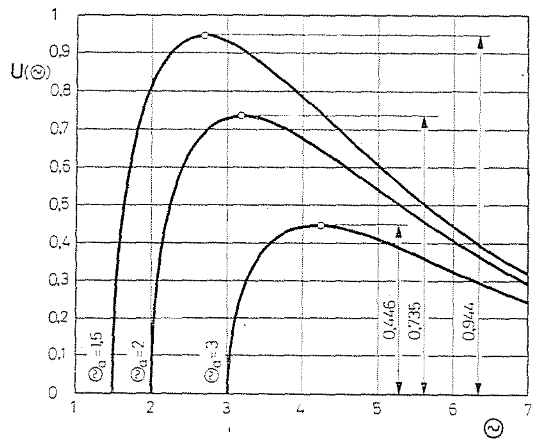

Fig. 5. Voltage-temperature characteristic curve of a plane dielectric for various relative ambient temperatures, $\dot{\Theta}(0) = 0$

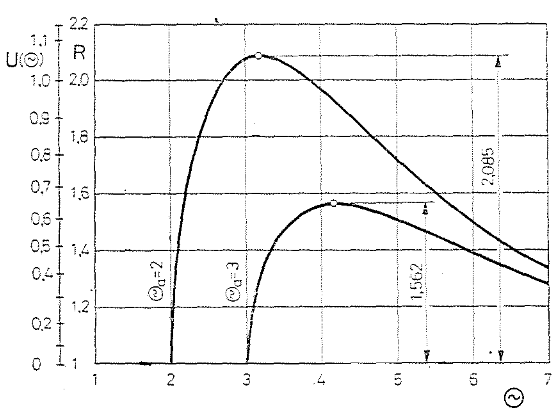

Fig. 6. Voltage-temperature characteristic curve of a cylindrical dielectric for various relative ambient temperatures, $\dot{\Theta}(1) = 0$

3. Additional heat flow allowance

If $Q_{in} \neq 0$, then its effect can easily be allowed for the boundary conditions of (6), (8), i.e. the value $\dot{\Theta}(0) < 0$. It can easily be admitted that the value of the thermal breakdown voltage is reduced by the additional heat flow, as it enhances the temperature drop in the dielectric. Figs 7 and 8 give

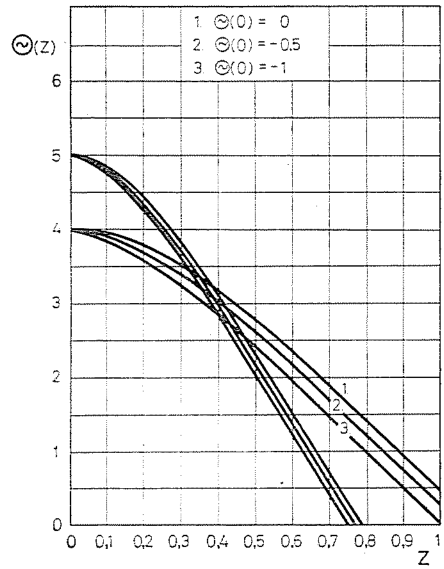

Fig. 7. Temperature distribution in the plane dielectric in the case of additional heat flow. Parameter: the temperature in point $z = 0$ (Θ_m)

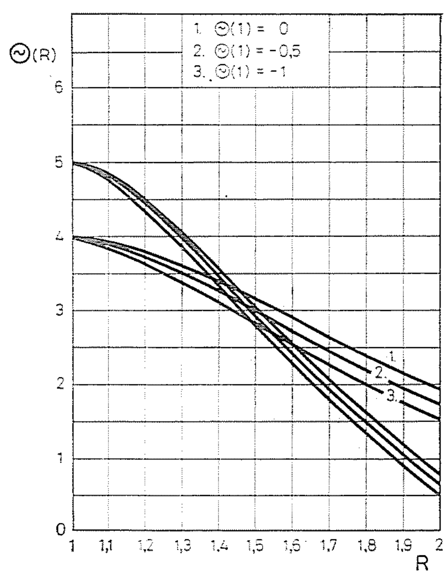

Fig. 8. Temperature distribution in the cylindrical dielectric in the case of additional heat flow. Parameter: the temperature in point $R = 1$ (Θ_m)

the solutions of (1) and (2) for three values of $\dot{\Theta}(0)$ in the plane and the cylindrical cases. For the construction of the envelope curve this should naturally be determined for more Θ_m values. The rate of increase of the temperature drop in the dielectric as well as the implied rate of decrease of the thermal breakdown voltage is evident from the figures.

4. Asymmetrically cooled dielectric

In certain cases different ambient temperatures may appear on both sides of the dielectric. Then the maximum temperature arises not in the symmetry axis (see Fig. 2), but nearer to the side of the higher ambient temperature. For reducing this asymmetrical case to two symmetrical cases we must know the location of Θ_m, i.e. the plane in which the heat flow is zero. Unfortunately this cannot be established analytically. This is why the value of the thermal breakdown voltage could not be determined by the previous processes (e.g. the FOK-functions). But with the aid of our sets of curves [i.e. by the solutions of Eqs (1) and (2)] the effect of the asymmetrical cooling can be allowed for rather easily. The process is as follows: We determine the functions shown in Figs 5 and 6, respectively (the voltage-temperature characteristics of the dielectric) for both ambient temperatures from the set of curves of parameter B determined by the data of the dielectric with $\dot{\Theta}(0) = 0$. By the superposition of both curves the relative thermal breakdown voltage is also easily obtained (as the thickness of the dielectric is $2h$ now, we obtain the thermal breakdown voltage for this thickness !). It is very notable that in the knowledge of the thermal breakdown voltage we can also establish how it is distributed in the dielectric, i.e. the location of Θ_m may be determined by their ratio. Figs 9 and 10 show the course of the construction in the case of our plane and cylindrical models for the relative ambient temperatures of $\Theta_{a1} = 2$ and $\Theta_{a2} = 3$, respectively. It is also evident from the figures that the resulting thermal breakdown voltages are established for two different ϑ_a values. The reason is that the dielectric cannot be divided into two uniform parts (of the thickness h each). On the basis of Fig. 9:

$$v(\Theta_{a1} = 2) = v_1 = 0.735$$

$$v(\Theta_{a2} = 3) = v_2 = 0.445$$

$$v(\Theta_{a1}, \Theta_{a2}) = v_{12} = 1.13 \neq 0.735 + 0.445.$$

The envelope curves already referring to the symmetrical case, similarly to those shown in Figs 3 and 4, may easily be plotted on the basis of characteristic curves $U - \Theta$ superpositioned in this way. For facilitating the calcula-

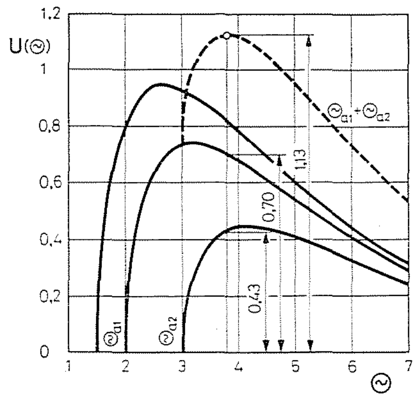

Fig. 9. Construction of the voltage-temperature characteristic curve for the plane dielectric in the case of asymmetrical cooling, $\dot{\Theta}(0) = 0$

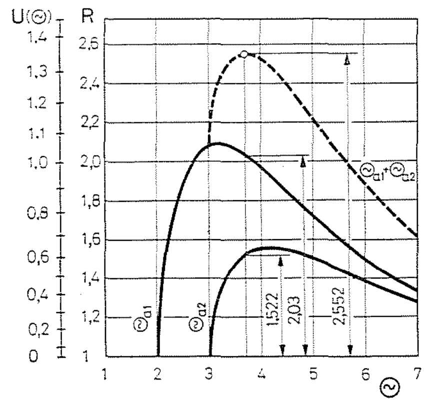

Fig. 10. Construction of the voltage-temperature characteristic curve for the cylindrical dielectric in the case of asymmetrical cooling, $\dot{\Theta}(1) = 0$

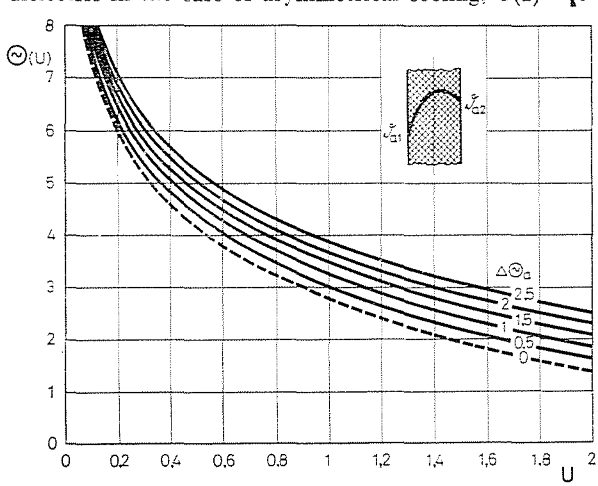

Fig. 11. Temperature distribution envelope curve for the plane dielectric versus the therma breakdown voltage in the case of asymmetrical cooling, $\Theta(0) = 0$

tion of the envelope curves we have prepared a program, given in the Appendix. Fig. 11 shows the envelope curves calculated by this program for a plane dielectric with the relative ambient temperature varying in the range of 0—2.5 in steps of $\Delta\Theta_a = 0.5$. The relative thermal breakdown voltage may be determined on the basis of the envelope curve in the manner described above.

5. Potential distortion allowance

We have assumed up till now a linear potential distribution in our calculations (or its equivalent: a homogeneous electric field distribution). But in practice the potential distribution is subject to distortions due to the temperature dependence of the electrical parameters and will deviate from the linear course. It can be proved that the relative potential distribution will have — on the basis of modelling the dielectric with parallel $R—C$ members — the following form [43]:
for the plane

$$\frac{U(z)}{U_0} = \frac{\sqrt{\left(\int\limits_0^z \frac{\mathrm{tg}\delta \cdot e^{\Theta(z)}\, dz}{1 + \mathrm{tg}\delta \cdot e^{2\Theta(z)}}\right)^2 + \left(\int\limits_0^z \frac{dz}{1 + \mathrm{tg}\delta \cdot e^{2\Theta(z)}}\right)^2}}{\sqrt{\left(\int\limits_0^1 \frac{\mathrm{tg}\delta \cdot e^{\Theta(z)}\, dz}{1 + \mathrm{tg}\delta \cdot e^{2\Theta(z)}}\right)^2 + \left(\int\limits_0^1 \frac{dz}{1 + \mathrm{tg}\delta \cdot e^{2\Theta(z)}}\right)^2}} \tag{15}$$

and for the cylinder

$$\frac{U(R)}{U_0} = \frac{\sqrt{\left(\int\limits_1^R \frac{1}{R} \cdot \frac{\mathrm{tg}\delta \cdot e^{\Theta(R)}\, dR}{1 + \mathrm{tg}\delta \cdot e^{2\Theta(R)}}\right)^2 + \left(\int\limits_1^R \frac{1}{R} \cdot \frac{dR}{1 + \mathrm{tg}\delta \cdot e^{2\Theta(R)}}\right)^2}}{\sqrt{\left(\int\limits_1^{R_2} \frac{1}{R} \cdot \frac{\mathrm{tg}\delta \cdot e^{\Theta(R)}\, dR}{1 + \mathrm{tg}\delta \cdot e^{2\Theta(R)}}\right)^2 + \left(\int\limits_1^{R_2} \frac{1}{R} \cdot \frac{dR}{1 + \mathrm{tg}\delta \cdot e^{2\Theta(R)}}\right)^2}}. \tag{16}$$

As the potential distribution is not linear, the horizontal scale z of the set of curves $\Theta = f(z)$ cannot be converted in a linear manner into the voltage scale. Although the value of the relative voltage arising at every z point might be determined in the knowledge of the potential distortion, yet as both sets of curves also depend on Θ_m, so the scale conversion $z \rightarrow v$ should be performed for every Θ_m individually. But in the knowledge of the potential distribution the scaling can also be performed in a way that the horizontal z values stay

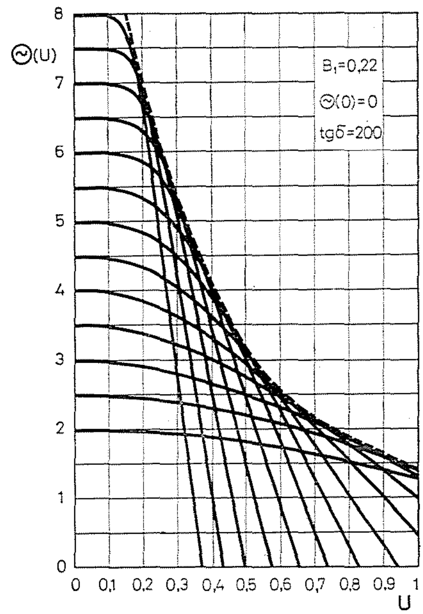

Fig. 12. Temperature distribution in the plane dielectric with the potential distortion taken into consideration with the parameter Θ_m

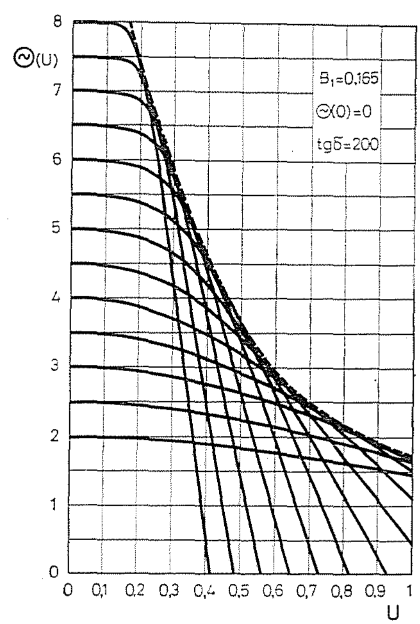

Fig. 13. Temperature distribution in the cylindrical dielectric with the potential distribution taken into consideration, with the parameter Θ_m

linear (permitting easy conversion into the v scale for the plane) and only the $\Theta(z)$ values belonging to them will vary.

Based on the solution of the differential equation (9) we have prepared a program, — given in the Appendix, — for the determination of the potential and field strength distortion, respectively, and for the performance of the necessary scale conversion. Figs 12 and 13 present the set of curves computed with the aid of this program, with the potential distortion also taken into consideration. A very remarkable fact is that in this case the thermal breakdown voltage increases moderately. The reason is that the higher electric load shifts, — due to the potential distortion, — towards the colder regions (the direction of the electrodes), while in the warmer regions the load decreases; consequently this effect contributes in a very interesting way to the increase of the thermal breakdown voltage.

In summarizing we should mention that the developed calculation process can easily be extended to other geometrical arrangements as well, or to the case of other than exponential temperature dependence and other boundary conditions, respectively.

Acknowledgement

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Summary

The paper contains some remarks concerning the calculation of thermal breakdown voltage of solid dielectrics with the aid of a digital computer. After a short description of the algorithm the effect of additional heat flow, and asymmetrical cooling, respectively, on the thermal breakdown voltage is considered. The results are presented by curves calculated in advance and finally the possibility of allowing for the potential distribution developing in the dielectric is dealt with. For calculating the allowances for the individual effects two programs are given.

APPENDIX

The following two programs were written in the ALGOL—60 language and were run on the computer type RAZDAN—3 of the University Computating Centre.

```
begin
irnteger i, j, k;
 real a, b, c, ta, tm, x, z;
 real array za [1 : 45, 1 : 17];
 comment The program calculates the relative thermal breakdown voltage of plane solid dielectric
cooled asymmetrically.
The maximum relative temperature of the dielectric is denoted by tm, the ambient relative
temperature is denoted by ta;
text tm       ta = 0.5          ta = 1.0          ta = 1.5;
text          ta = 2.0          ta = 2.5          ta = 3.0;
text          ta = 3.5          ta = 4.0          ta = 4.5;  lines 3;
```

```
i : = 0;  b : = 0.22;
for tm : = 1 step 1 until 9.01 do
 begin
   i : = i + 1;  j : = 0;
   a : = sqrt(2/b)*exp(−tm/2);
   for ta : = 0.5 step 0.5 until 8.51 do
    begin
      j : = j + 1;  x : = (tm−ta)/2;  c : = exp(x);
      if x < 0 then za[i, j] : = 0 else
      za[i, j] : = a*ln(c + sqrt(c ↑ 2 − 1) )
      end ta
end tm;
i : = 0;
for tm : = 1 step 0.1 until 9.01 do
begin
  i : = i + 1; output(tm : 1 : 2); spaces 3;
  for j : = 1 step 1 until 9 do
  begin
    z : = za[i, j]; output(z : 1 : 5); spaces 2
  end j;
  line
end tm;
lines 10;
for k : = 1 step 1 until 8 do
begin
  i : = 0; z : = k*0.5; textdelta t =;
  output(z : 2 : 1); lines 5;
  for tm : = 1 step 0.1 until 9.01 do
  begin
    i : = i + 1; output(tm : 1 : 2); spaces 3;
    for j : = 1 step 1 until 9 do
    begin
      z : = za[i, j] + za [i, j + k]; output(z : 1 : 5); spaces 2
    end j;
    line
  end tm;
  lines 5;
end k;
lines 5;
text The program was run on a computer type RAZDAN−3 of the University Computating
Centre.;
lines 5;
end end ⍊
```

```
begin
integer i, j, k;
real t,tm,ta,tb,z,beta,tg,b,nb,uv,u,h,aa;
real array re,im,us,der [0 : 40, 1 : 8];
comment The program calculates the relative thermal breakdown voltage of plane solid
dielectrics using t(z) = tm − 2*ln[ch(beta*z)] temperature distribution for a.c. voltage taking
the potential distribution
into consideration.;
procedure SIMP (a,b,f,eps,no,int);
value a,b,eps,no; integer no;
real a,b,eps,int; real procedure f;
begin
  integer j,k,n; real s1,s2,in,ie,v,h,delt,i;
  switch z : = z[j],z1,z2;
  k; = 1: delt : = ie : = s2 : = 0; n : = no; j : = 3;
  rep:
  s1 : = 0; h : = (b−a)/n;
```

```
  for i : = delt step 1 until n do
  begin
    v : = f(a + i*h);
    if i = 0 V i = n then v : = v/2:
    goto z[k]:
    z1: s1 : = s1 + v; j : = 3; goto la;
    z2: s2 : = s2 + v; j : = 2;
  la : end i;
  in : = (4*s1 + 2*s2)*h/(3*k);
  if k = 1 then begin k : = 2; delt : = 0.5 end else n : = 2*n;
  if abs( (in—ie)/in) < eps then begin
  s2 : = s1 + s2; ie : = in; goto rep end else
  int : = in + 1.0666666666*(in—ie)
end simp;
real procedure RE(z);
value z: real z;
begin
  real ch,tt;
  ch : = (exp(beta*z) + exp(—beta*z) )/2;
  tt : = tm—2*ln(ch):
  re : = tg*exp(tt)/(1 + (tg ↑ 2)*exp(2*tt) )
end re;
real procedure IM(z);
value z; real z;
begin
  real ch,tt;
  ch : = (exp(beta*z) + exp(—beta*z) )/2;
  tt : = tm—2*ln(ch);
  im : = 1/(1 + (tg ↑ 2)*exp(2*tt) )
end im;
input (nb,tg);
comment Expression (3) is denoted by nb, the power factor is denoted by tg, the relative
maximum dielectric temperature is denoted by tm.;
textb = ; output(nb : 1 : 6);
texttg = ; output(tg : 1 : 6); lines 5;
for j : = 1 step 1 until 8 do
begin
  re[0, j] : = im[0, j]: = us[0, j]: = der[0, j] : = 0
end j:
text Potential distribution:; lines 5;
k : = 0;
cim 3: k : = k + 1;
text   z             tm = 2.0            tm = 2.5;
text          tm = 3.0           tm = 3.5          tm = 4.0;
text          tm = 4.5           tm = 5.0          tm = 5.5;
if k = 2 then goto cim4 else if k = 3 then goto cim5;
lines 2; i : = 0;
for z : = 0.025 step 0.025 until 1.01 do
begin
  i : = i + 1; b : = z — 0.025; j; = 0;
  for tm : = 2.0 step 0.5 until 5.51 do
  begin
    j : = j + 1; beta : = sqrt(nb/2)*exp(tm/2);
    SIMP(b,z,re,0.000001,32,ta);
    SIMP(b,z,im,0.000001,32,tb):
    re[i, j]: = re[i—1,j] + ta;
    im[i, j] : = im[i—1,j] + tb
  end tm
end of calculations of integrals (15);
i: = 0;
for z : = 0.025 step 0.025 until 1.01 do
begin
```

```
   i : = i + 1; output (z : 1 : 3); spaces 3;
for j: = 1 step 1 until 8 do
   begin
      uv : = sqrt(re[40, j] ↑ 2 + im[40, j] ↑ 2);
      us[i, j]: =sqrt(re[i, j] ↑ 2 + im[i, j] ↑ 2)/uv;
      u : = us [i, j]; output (u : 3 : 5); space
   end j;
   line;
end of transcript of the potential distribution;
lines 10;
text Field distribution; lines 5; goto cim3;
cim4: lines 2;
h : = 1/(12*0.025);
for j : = 1 step 1 until 8 do
begin
   der[0,j]: = h*(-25*us[0,j] + 48*us[1, j] - 36*us[2, j] + 16*us[3,j] - 3*us[4, j]);
   der[1,j]: = h*(-3*us[0,j] - 10*us[1,j] + 18*us[2,j] - 6*us[3,j] + us[4,j]);
   der[39,j]: = h*(-us[36,j] + 6*us[37,j] - 18*us[38,j] + 10*us[39,j] + 3*us[40,j]);
   der[40,j]: = h*(3*us[36,j] - 16*us[37,j] + 36*us[38,j] - 48*us[39,j] + 25*us[40,j])
end j;
for i : = 2 step 1 until 38 do
  for j : = 1 step 1 until 8 do
  der[i,j]: = h*(us[i - 2,j] - 8*us[i - 1,j] + 8*us[i + 1,j] - us[i + 2,j])
comment end of the field distribution calculation;
for i: = 0 step 1 until 40 do
begin
  z: = i*0.025; output(z : 1 : 3); spaces 3;
  for j : = 1 step 1 until 8 do
  begin
     ta: = der[i,j]; output (ta : 3 : 5); space
  end j;
  line
end of transcript of the field distribution;
lines 10;
text The modified relative temperature distribution for the determinate of the relative thermal
breakdown voltage for a.c. voltage;
lines 5; goto cim3;
cim5: lines 2;
for i: = 0 step 1 until 40 do
begin
  z: = 0.025*i; output (z : 1 : 3); spaces 3; j: = 0;
  for tm: = 2.0 step 0.5 until 5.51 do
  begin
     j: = j + 1; ta : = us[i,j];
     beta: = sqrt(nb/2)*exp(tm/2);
     tb: = (exp(beta*ta) + exp(- beta*ta) )/2;
     t: = tm - 2*ln(tb); output(t : 3 : 5); space
  end tm;
  line
end i;
lines 5;
text The program was run on a computer type RAZDAN-3 of the University Computing
Centre.;
lines 5;
end end end ↓
```

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