DETERMINATION OF PERFORMANCE INDICES OF LINEAR CONTROL SYSTEMS WITH DEAD TIME

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The practical identification processes are often approximating — sometimes rather closely — the process to be controlled, by means of systems that can be described by first-order or more frequently by second-order lag transfer functions with dead time. So in planning control processes, the task mostly consists in selecting and adjusting the parameters of the controller to be used in the systems with dead time.

To design the control systems with dead time by classic methods encounters computation difficulties. But modern synthesis methods permit the synthesis of processes with dead time at the desired accuracy if a digital computer is available.

The analysis results are useful for designing control systems in case of less strict requirements. Numerical analyses in the frequency and the time domains permit quick selection of the compensating element meeting the prescribed performance indices at a satisfactory accuracy. Thus, numerical results of analyses in the frequency and the time domains of the control systems of various structures with dead time are rather useful.

The stability tests of linear control systems containing a first-order lag plant with dead time and the determination of their transfer processes have been dealt with by several papers [1, 2, 3, 4, 5, 6].

The results of frequency domain analyses of control systems described by linear, second-order lag transfer functions with dead time in the case of serial PID (and in the resultant specific cases of P, I, PI, PD) controllers in a single-loop control system (Fig. 1) have been presented in [7, 8, 9, 10, 11, 12, 13].

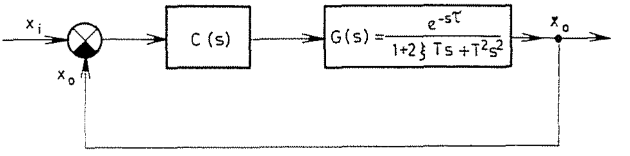

Fig. 1

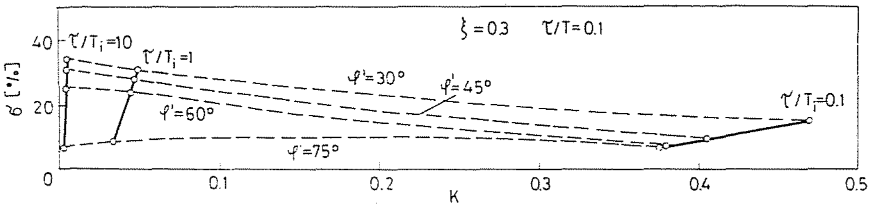

Fig. 2

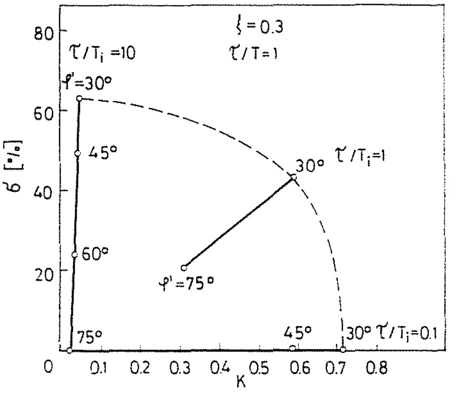

Fig. 3

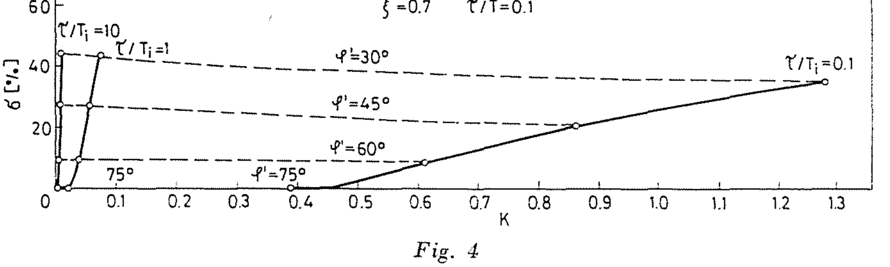

Fig. 4

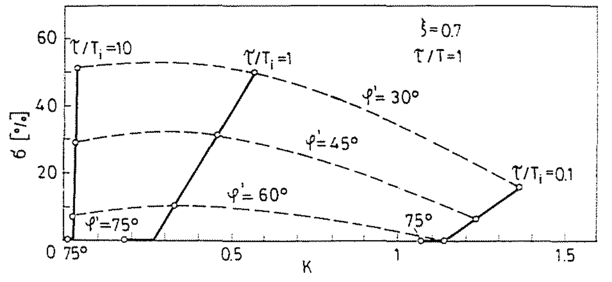

Fig. 5

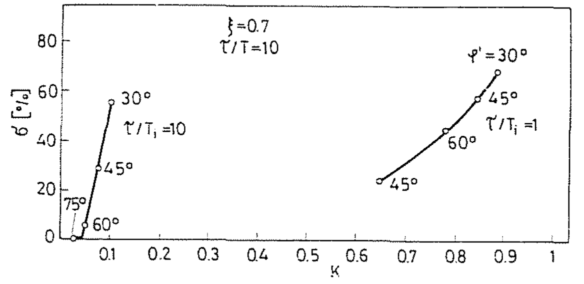

Fig. 6

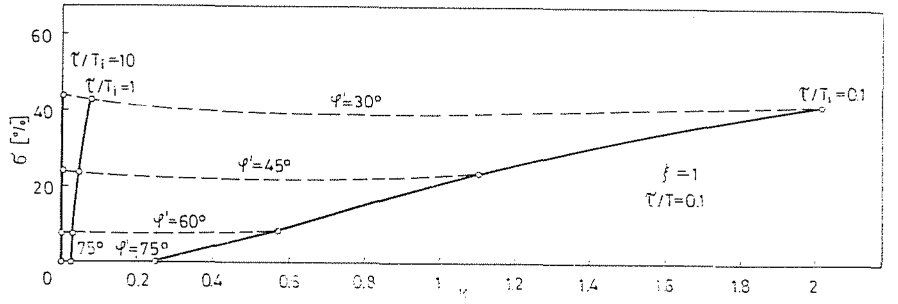

Fig. 7

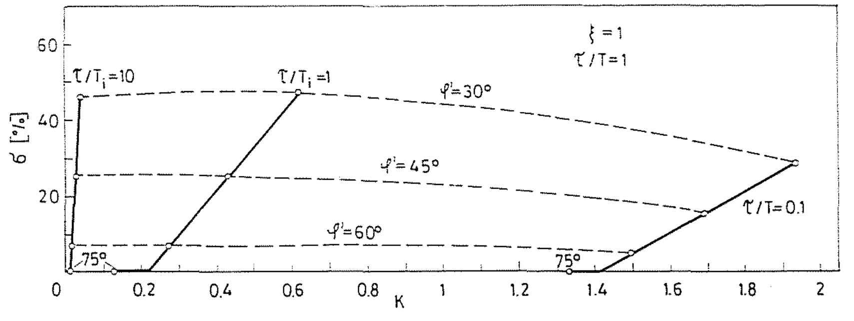

Fig. 8

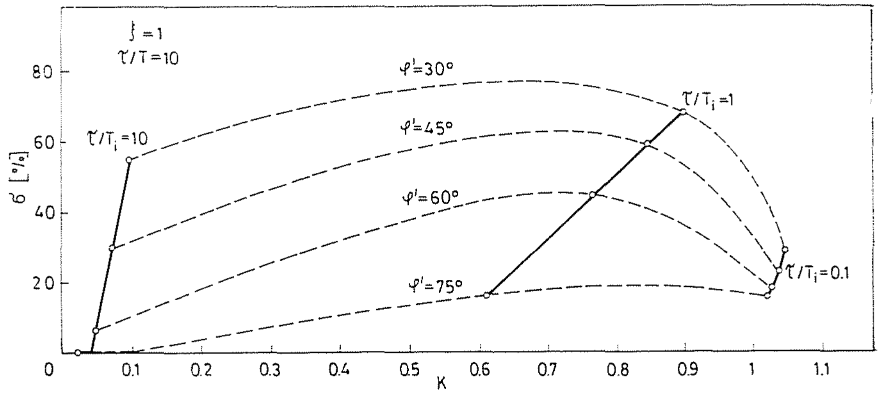

Fig. 9

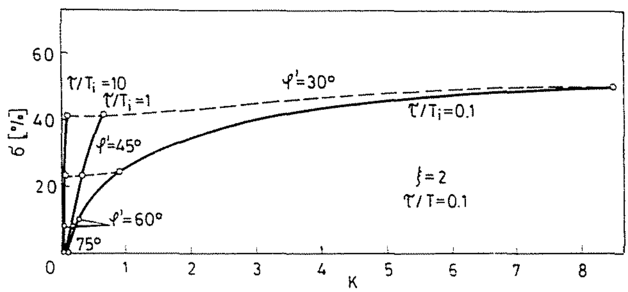

Fig. 10

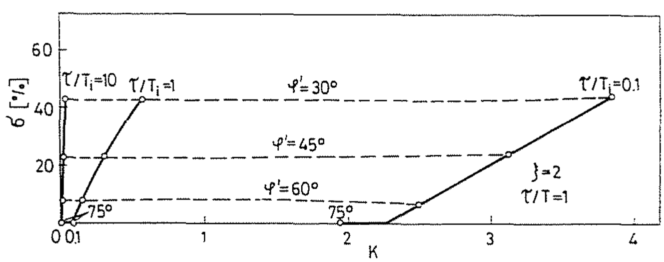

Fig. 11

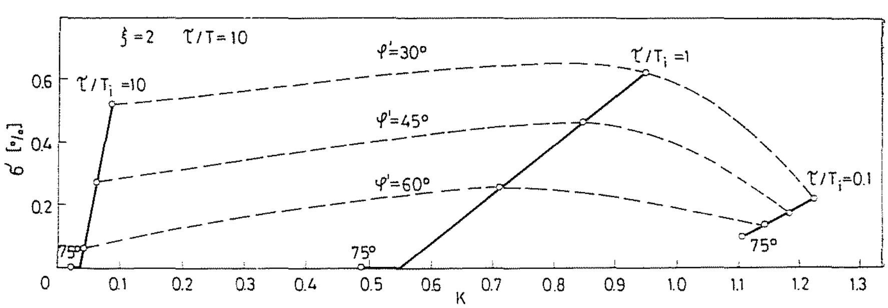

Fig. 12

The results of time domain analyses for a linear control system with dead time shown in Fig. 1, with selection of an I controller, were given in [15].

In the present paper diagrams determined for various parameter values are presented, giving the maximum overshoot σ and the control time t_c of the examined control system with dead time.

Proportional-plus-integral control

The transfer function of the plant in the linear control system with dead time to be analysed numerically is

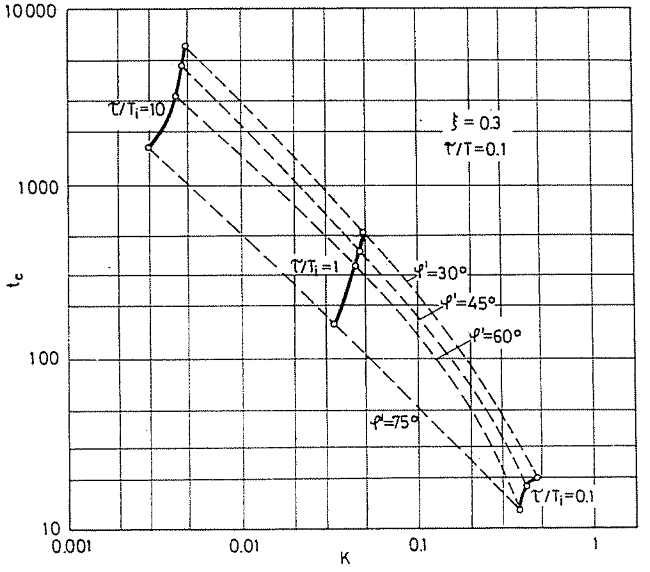

Fig. 13

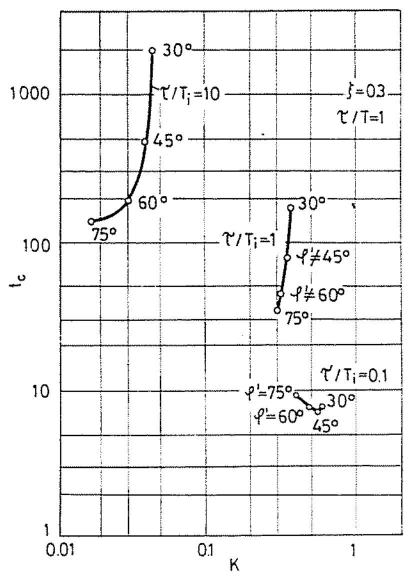

Fig. 14

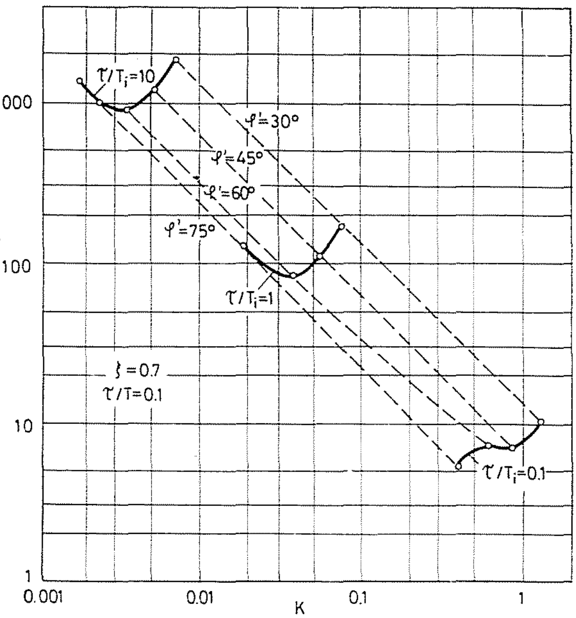
τ/T_i=10
φ'=30°
φ'=45°
φ'=60°
φ'=75°
τ/T_i=1
ξ=0.7
τ/T=0.1
τ/T_i=0.1
1000
100
10
1
t_c
0.001
0.01
0.1
1
K

Fig. 15

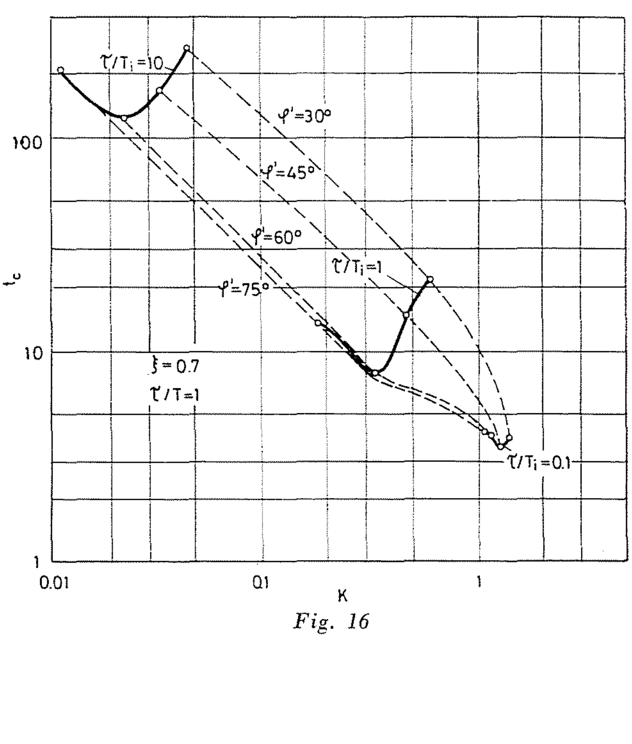
τ/T_i=10
φ'=30°
φ'=45°
φ'=60°
τ/T_i=1
φ'=75°
ξ=0.7
τ/T=1
τ/T_i=0.1
100
10
1
t_c
0.01
0.1
1
K

Fig. 16

$$C(s) = \frac{\exp(-s\,\tau)}{1 + 2\xi\,Ts + T^2\,s^2}$$

with τ — dead time,
T — time constant of the second-order lag,
ξ — damping factor.

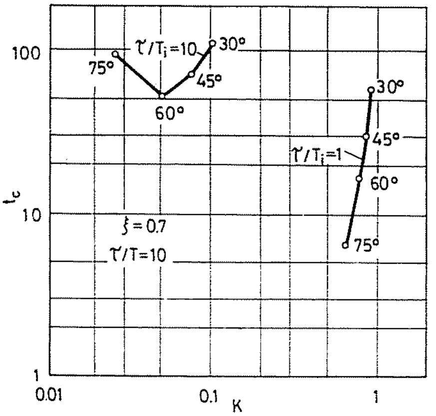

Fig. 17

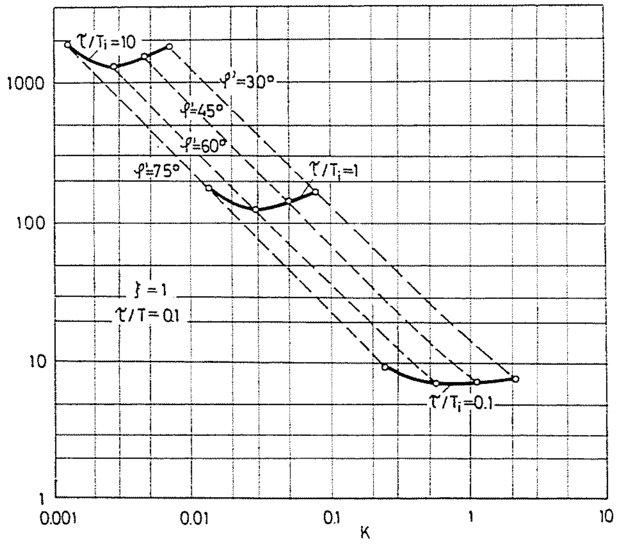

Fig. 18

The series controller is the proportional-plus-integral PI lag with a transfer function:

$$G(s) = K\left(1 + \frac{1}{sT_i}\right),$$

where K — loop gain,

T_i — integral time constant.

The unit step responses for the various parameters of the control system ($\xi = 0.3, 0.7, 1, 2$; $\tau/T = 0.1, 1, 10$ and $\tau/T_i = 0.1, 1, 10$) were determined by the digital simulation method using a computer [12]. The maximum overshoot and the control time were plotted in diagrams versus the loop gain, by connecting the points belonging to identical phase margin values φ' (Figs 2 to 12 and 13 to 23, respectively).

In some cases the σ-values determined by the digital computer were compared with the approximative values obtained from the M-$\varkappa$ curves [14]. The best approximation was obtained, — as it has been established also in the case of the integral control, — in the $45° \leqq \varphi' \leqq 60°$ range [13].

From $\sigma = \sigma$ (K) diagrams (Figs 2 to 12) the value of the maximum overshoot is seen to greatly depend on the time constants of the system. By varying the parameters of the controller in the range $0.1 \leqq \tau/T_i \leqq 10$, in the case of a given phase margin, the value of σ greatly depends on the time constants of the controller.

In plotting the control time diagrams, the time needed for the deviation of the unit step response from the final value to keep below ± 5 per cent was regarded as control time.

From the $t_c/T_i = f(K)$ diagrams (Figs 13 to 22) it may be read off that

a) for a given phase margin the value of t_c/T_i is decreasing with increasing loop gain,

b) for a given plant and fixed T_i, the maximum control time greatly varies as a function of the phase margin.

An important characteristic of the unit step response of a control — beside the maximum overshoot and the control time — is also the number N of deflections occurring up to steady state. For meeting the performance indices $N_{max} = 3$ is generally demanded. The σ and t_c diagrams determined for the actually studied control system do not yield, however, the N_{max} value. But in the knowledge of the unit step responses [14] it can be stated that with decreasing damping factor values the deflection tendency is increasing, as expected. When $\tau \ll T_i$, then for low ξ values the number of the deflections $N \gg 3$.

The above considerations show that if a PI controller is chosen for the compensation of the tested control system, no far-reaching conclusions can be drawn on the characteristics of the unit step response. But utilizing the frequency domain analysis results the presented diagrams may be of help in establishing the course of the unit step response of the control system of the given structure at a fair accuracy without calculations in the time domain.

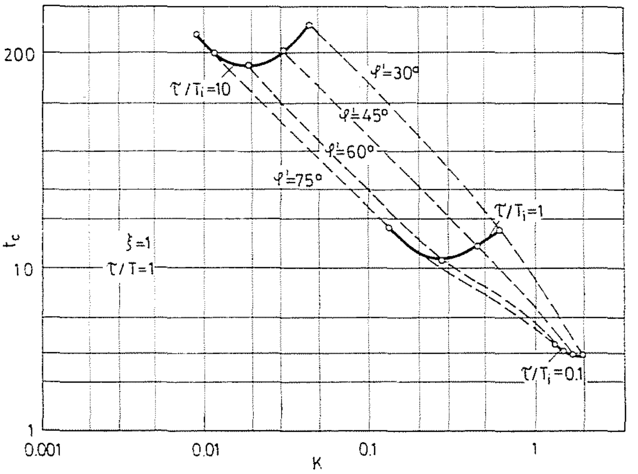

Fig. 19

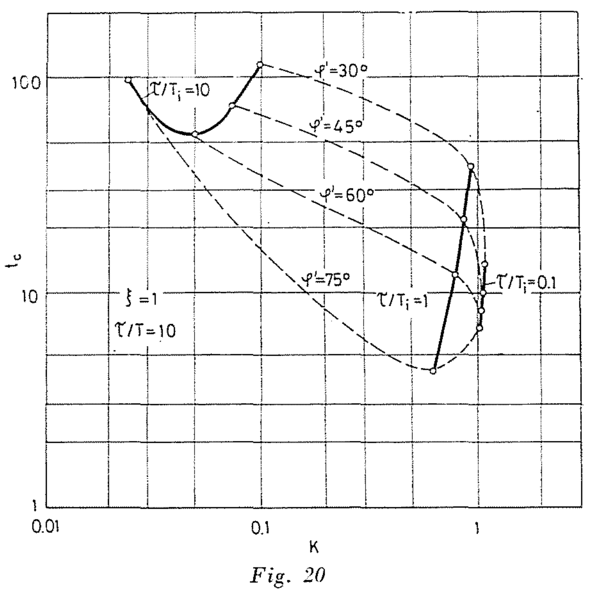

Fig. 20

Conclusions

The time domain analysis of the linear control system with dead time given in Fig. 1 suggests that in case of less strict requirements, the performance indices of control systems may be estimated from empirical relationships with a sufficient accuracy in the environment of the phase margin of 45° [14]. But for synthetizing control systems, subject to stricter requirements, correct results can only be obtained by computer analysis. If the behaviour of the plant can be characterized by a linear, second-order lag transfer function with

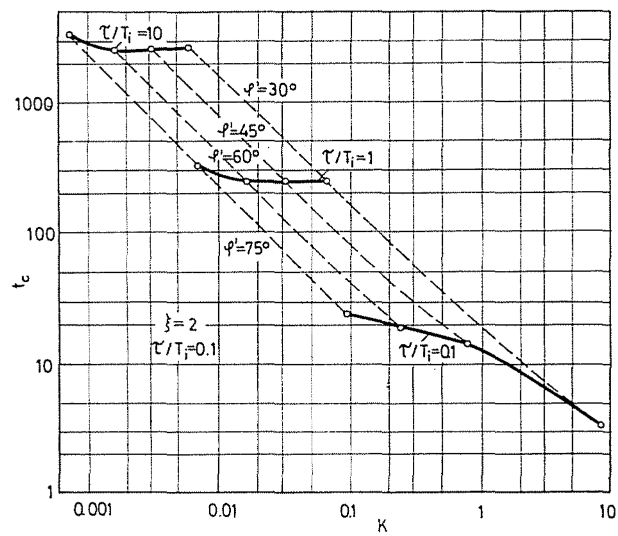

Fig. 21

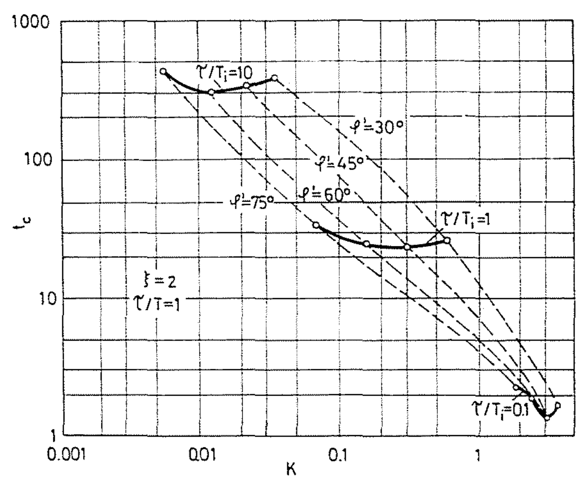

Fig. 22

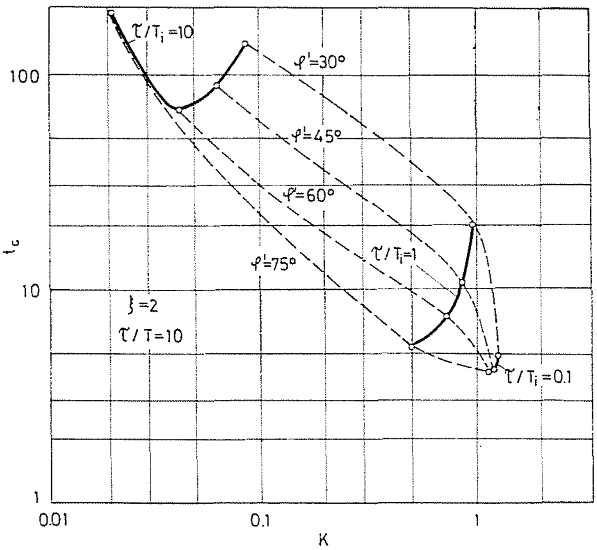

Fig. 23

dead time — and the practical identification processes are aimed in many cases at the estimation of the parameters of such structures — then the maximum overshoot and the control time values given in [15] and those determined in the present paper are useful in designing the control system with dead time corresponding to Fig. 1.

Summary

Diagrams of the maximum overshoot and the control time of a linear, one-loop control system with dead time versus the time constants of the system, determined by means of a digital computer are presented. The plant was regarded, — as it often occurs in control processes — as a second-order lag element with dead time and a PI element was chosen as a series connected controller.

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