# SELF-CONSISTENT MODELLING OF TRANSFERRED ELECTRON DEVICES 

By<br>L. Zombory, Z. Kôhalmi and G. Veszely<br>Department of Theoretical Electricity<br>Technical University, Budapest<br>Received May 9, 1972<br>Presented by Prof. Dr. G. Fodor

A self-consistent modelling of the circuits consisting transferred electron device (TED) is proposed. The TED model is shown on the right hand side of Fig. 1. It is an improved form of the known models applied for non selfconsistent modelling [1-5].


Fig. 1. The equivalent circuit of a TEO

The operating period without domain is represented by the position 1 of the switch $S$. In this case the $R C_{0}$ block represents the low field region of the TED only. The capacitance $C_{0}$ is assumed to be constant and the characteristics $i_{0}\left(u_{0}\right)$ of the nonlinear conductor is determined by the D.C. characteristics of the TED below threshold. [1-5] These parameters can be measured easily.

Position 2 of the switch S represents the operating period during which a domain is present in the TED. To describe the behaviour of the domain another $R C_{d}$ block is connected in series with the $R C_{0}$ block representing the low-field region. The nonlinear capacitance $C_{d}\left(u_{d}\right)$ corresponds to the dy-
namic capacity of the domain assuming a fully depleted domain wall. Thus, we get $[2,3]$ :

$$
\begin{equation*}
C_{d}=A \sqrt{\frac{e n_{0} \varepsilon}{2}} \frac{1}{\sqrt{u_{d}}}, \tag{1}
\end{equation*}
$$

where $A$ is the cross-sectional area of the sample, $e$ is the electronic charge, $n_{0}$ is the carrier concentration and $\varepsilon$ is the permittivity of GaAs.
$i_{d}\left(u_{d}\right)$ given by an empirical formula represents the dynamic currentvoltage characteristics of the domain. To determine $C_{d}\left(u_{d}\right)$ and $i_{d}\left(u_{d}\right)$ at microwave frequencies by measurements is not possible as yet. Results published only for $0,3 \mathrm{~mm}$ long samples [6] - are somewhat contradictory to the simplest lumped parameter models [7]. A new method to get the function $i_{d}\left(u_{d}\right)$ is proposed below.

As it has been mentioned, the position of the switch $S$ depends upon the presence of the domain in the TED. Therefore its operation is controlled by the following conditions:

1. The switch takes up position 2 if $u_{0} \geq U_{t h}$, where $U_{t h}$ is the threshold voltage,
2. the switch takes up position 1 if $u_{0} \div u_{d}<U_{s}$, where $U_{s}$ is the sustaining voltage,
or $t \geq t_{0}+T_{t}$, where $t_{0}$ is the time at starting the domain and $T_{t}$ is the transit time.

The lowest possible value of $U_{s}$ is determined from the implicit formula

$$
\begin{equation*}
\left|\frac{d i_{0}}{d u_{0}}\right|_{u_{\mathrm{a}} \div u_{d}=U_{s}}=\left|\frac{d i_{d}}{d u_{d}}\right|_{u_{\mathrm{s}}+u_{d}=U_{z}} \tag{2}
\end{equation*}
$$

Our actual investigations have been concerned with a modified controlling condition $u_{d}<U_{s d}$ where $U_{s d}$ is a properly chosen (nearly zero) sustaining domain voltage. It seems to be more exact from physical point of view, but reduces the number of free parameters and makes the correct fitting of the model to a particular device difficult.

The transit time $T_{t}$ is considered to be constant and measured under ohmic load.
$C_{p}$ is the package capacitance.
The proposed model suits to describe

- the finite time of the domain formation and that of the transient response, taking the delaying effect of the accumulated charge into account;
- the vanishing of the domain due to loss of voltage (quenched domain mode) and its dissolution at the anode (transit time mode) as well.

The above described TED model can be used in any particular imbedding circuit. In order to analyse the resulting nonlinear circuit the state equation
can be written as follows:

$$
\begin{equation*}
\dot{\mathbf{y}}=\mathbf{A}_{1,2}(\mathbf{y}) \mathbf{y}+\mathbf{b}_{1,2}\left(\mathbf{y}, \mathrm{U}_{B}\right) \tag{3}
\end{equation*}
$$

where $\mathbf{y}$ is the column vector of the state variables, $\dot{\mathbf{y}}$ is its time-derivative. $\mathbf{A}_{1}$ and $\mathbf{A}_{2}$ are the state matrices while $\mathbf{b}_{1}$ and $\mathbf{b}_{2}$ are the "constant" vectors depending on the switch position.

As it is obvious from Equ. (3), two alternating initial value problems are to be solved with respect to both positions of the switch S . To assure the continuity of the voltage at the part of the TED model we may assume that in position 1

$$
u_{0}^{(2)}(t)=u_{0}^{(1)}(t) \quad \text { and } u_{d}(t)=0
$$

while in position 2

$$
u_{0}^{(1)}(t)=u_{0}^{(2)}(t)+u_{d}(t)
$$

are satisfied formally at any moment. In this way the TED voltage does not change by any change-over of the switch.

The total charge contained by the "activated"circuit after the changeover $2 \rightarrow 1$ of the switch differs from that just before the change-over. But the arbitrarily neglected charge practically does not alter the power in the circuit in a remarkable manner.

When the domain disappears and therefore the switch turns over from $2 \rightarrow 1$ due to the transit time but the voltage is still higher than $U_{t h}$ the next switching to the position 2 may be artificially delayed by an estimated domain annihilation time.

For the particular example in Fig. 1, Equ. (3) is, in particular:

1. without domain

$$
\begin{gather*}
\dot{y}=\mathbf{A}_{1} \mathbf{y}+\mathbf{b}_{1}  \tag{3a}\\
\mathbf{A}_{1}=\left[\begin{array}{cc}
-\frac{1}{\left(C_{0}+C\right) R} & \mathbf{y}=\left[\begin{array}{l}
u_{0} \\
i_{L}
\end{array}\right] \\
\frac{1}{L} & 0
\end{array}\right] \\
\mathbf{b}_{1}=\left[\begin{array}{c}
-\frac{1}{C_{0}+C} \\
C_{0}+C \\
-\frac{U_{B}}{\left(C_{0}+C\right) R}+\frac{C}{C_{0}+C} \frac{d U_{B}}{d t} \\
L
\end{array}\right]
\end{gather*}
$$

2. with domain

$$
\begin{gather*}
\dot{\mathbf{y}}=\mathbf{A}_{2} \mathbf{y}+\mathbf{b}_{2}  \tag{3b}\\
\mathbf{A}_{2}=\left[\begin{array}{ccc}
-\frac{C_{d}\left(u_{d}\right)}{\Delta R} & -\frac{C_{d}\left(u_{d}\right)}{\Delta} & -\frac{C_{d}\left(u_{d}\right)}{\Delta R} \\
\frac{1}{L} & 0 & \frac{1}{L} \\
i_{L} \\
u_{d}
\end{array}\right] \\
-\frac{C_{0}}{\Delta R} \\
\mathbf{b}_{\mathbf{2}}=\left[\begin{array}{c}
\frac{C_{0}}{4} \\
\frac{C_{d}\left(u_{d}\right)}{\Delta R} U_{B}+\frac{C}{\Delta R}\left[i_{d}\left(u_{d}\right)-i_{0}\left(u_{0}\right)+C_{d}\left(u_{d}\right) \frac{d U_{B}}{d t}\right]-\frac{i_{0}\left(u_{0}\right) C_{d}\left(u_{d}\right)}{\Delta} \\
\frac{C_{0}}{\Delta R} U_{B}+\frac{C}{\Delta}\left[i_{0}\left(u_{0}\right)-i_{d}\left(u_{d}\right)+C_{0} \frac{d U_{B}}{d t}\right]-\frac{i_{d}\left(u_{d}\right) C_{0}}{4}
\end{array}\right]
\end{gather*}
$$

where

$$
\Delta=C_{0} C_{d}\left(u_{d}\right)+C C_{0}+C C_{d}\left(u_{d}\right) .
$$

Qualitative operation of the TEO computer model
At the beginning, to avoid unnecessary contradictions, a full zero initial condition is chosen. The bias voltage grows continuously from zero up to its final value $U_{B}$. Since the voltage is low, at the beginning the switch takes up position 1. In this state Equ. 3a is solved by the Hamming-procedure [8]. When $u_{0}$ reaches $U_{t h}$ the switch turns to position 2 and the $R C_{d}$ block representing the domain becomes activated.

Providing some particular sets of parameters of the passive circuit, the domain arrives harmless at the anode and there dissolves (transit time mode, delayed domain mode). This dissolution is represented by the change-over $2 \rightarrow 1$ of the switch S .

If the domain vanishes while in transit because the TED voltage drops below the sustaining value $U_{s}$, the switch changes its position as well (quenched domain mode).

As it has been mentioned the solution is composed of a series of individual stages with respect to each change-over of the switch. Each of them is a solution of an individual initial-value problem (Cauchy-problem). Their coupling is
governed by the above mentioned rules. The flow chart of the total program is given in Fig. 2.

As it has been mentioned before, the drifting of domains through the sample is represented by the position of the switch. There are, however, transient states where the voltage of the TED oscillates very fast between $U_{s}$ and $U_{\text {ih }}$. These "abortive" domains are generated because of the zero domain


Fig. 2. Flow chart of the program
falltime, or may be attributed to the arbitrary determination of $U_{s}$ on the basis of Equ. (2).

Nevertheless the computed time-functions of the state variables shown in Fig. 3 are similar to some recorded curves $[9,10]$ if the average of the ripple of the abortive domains is taken.

The complete computer program performs the Fourier analysis of the obtained periodical time functions after having established the cycle time, counting the switch change-overs, the power, efficiency and diode impedance on the required harmonics.


Fig. 3. Calculated state variables vs. time in quenched domain mode

The function $i_{d}\left(u_{d}\right)$ may be obtained by the following method. The time averages of current and voltage define a point of the D.C. characteristics on the TED. Varying $U_{B}$ the whole D.C. characteristics can be determined. Inverting this process by an iterative chain, a very important feature appears. The hard-to-measure dynamic characteristics $i_{d}\left(u_{d}\right)$ can be determined from the D.C. measurements on the oscillating diode.

Research is being done to extend the validity of the model to other modes including the hybrid one, to simplify reckoning with the finite domain falltime, the effects of the inhomogeneous impurity concentration and of the contact region. Examples with different cavities and loads (including the analysis of logic circuits) are being computed.

The operational frequency-range of the different modes, the dependence of the output parameters on the operating frequency of the TED and on the passive circuit parameters are hoped to become available. Thus, by means of the suggested self-consistent lumped model the optimisation of circuits containing TED-s is likely to be feasible.

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## Summary

A lumped non-linear model of transferred electron devices is proposed to simulate transit time and quenched domain mode oscillators in a self-consistent manner. A procedure to analyze such a circuit and some remarks on the numerical results are given.

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Dr. László Zombory
Zsolt Köralyi
1502. Budapest, P.O.B. 91. Hungary

Dr. Gyula Veszely

