SOME ALGORITHMIC MODELS FOR PRODUCTION LINES

By

F. Csáki, L. Keviczky, T. Kovács and J. Kocsis

Department of Automation, Technical University, Budapest

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Introduction

This paper deals with the mathematical modelling of production by a production line consisting of g working places (homogeneous machine groups) served by a conveyor, where w product sorts are produced and there are n_j $(j = 1, 2, \ldots, w)$ pieces in each product sort. The available data are: the preparation periods $\mathbf{E}$, the operation periods $\mathbf{A}$, the matrix $\mathbf{T}$ of the transportation periods and the assignment matrix $\mathbf{M}$ describing the technological sequence (see definitions further).

The above problem has been dealt with by various authors (e. g. [2], [3], [4]), but the published models either contain some essential restriction (no preparation period, operational sequence coincident with the sequence of the machines, etc.), or for reasons of concurrence they are incomplete, or they are extraordinarily complex and demanding in computation.

Our algorithms were developed for production lines whose creation generally an essential technological aim. So they are closed production lines, i.e. they are exempt of product interflow with other production lines, and of production loops, i.e. the same machine does not enter repeatedly in the sequence of the technological operations.

The production models of the production line

Let us investigate first the case where the speed of the conveyor is infinitely high. For formulating the production model some notations must be introduced. Be $\mathbf{n} = \{n_j\}$ the vector of $(w \times 1)$ containing the piece numbers of the products to be produced; $\mathbf{A} = \{a_{i,j}\}$ the matrix of the $(g \times w)$ of the operational periods; $\mathbf{E} = \{e_{i,j}\}$ the matrix of the same dimension of the preparation periods. Subscripts i and j denote the serial number of the involved machine and of the product respectively. Let us define an $\mathbf{M} = \{m_{i,j}\}$ assignment, or operational sequence matrix of dimension $(g \times w)$ whose element

$m_{i,j}$ denotes the serial number of the i-th machine in the technological sequence of the j-th product. $m_{i,j} = 0$ means that the involved machine is not operating, or its operation is absent from the manufacture of the j-th product. The matrix $\mathbf{M}$ permits to obtain matrices of operational sequence elements from the matrices of machine sequence elements (see further the matrices marked by *).

The algorithm for simulating the production by the production line can be constructed on the basis of Fig. 1, where — in addition to the initial data, — also some new quantities appear, with the following definitions. $\Delta^*_{i,j}$ is the cycle period of the i-th operation in manufacturing the j-th product; $ta^*_{i,j}$ is the throughput time of n_j pieces of the j-th product as counted from the first piece during the i-th operation; $tk^*_{i,j}$ is the starting point of the i-th operation from the start of manufacturing the first piece of the j-th product, as the origin of the coordinate system; $ts^*_{i,j}$ is the so-called beginning time of the i-th operation for the j-th product as obtained from the starting time methods by counting back the preparation periods (if the origin of our coordinate system is at $tk^*_{1,j}$, these can be negative as well) and finally $tb^*_{i,j}$ is the ending time of the i-th operation for the j-th product ($tb^*_{i,j} = 0,\ i = 1, 2, \ldots g$ by definition). With the newly introduced quantities and throughout the following the symbol $*$ means allowance for the subscript i in the operational sequence, while its missing refers to the sequence of the machines. Both sequences can be mutually transformed into each other with the help of the operational sequence matrix $\mathbf{M}$.

From Fig. 1 it is obvious, that in the case of manufacturing processes permitting intermittent operation as well, the introduction of the cycle time is advisable for calculating the individual characteristic quantities. This is the period, after that machining of a new workpiece can be begun with by way of the i-th operation for the j-th product. The $\Delta^*_{i,j}$ quantities can be calculated by the relationships

$$\left.\begin{aligned} \Delta^*_{1,j} &= a_{1,j} \quad \text{and} \\ \Delta^*_{i,j} &= \max\left\{\Delta^*_{i-1,j}\,;\, a_{i,j}\right\} \end{aligned}\right\}. \tag{1}$$

Accordingly, the cycle time is always determined by the maximum operation period.

After the introduction of the cycle time and with the help of the piece times and the preparation periods the relationships applying to the previously defined quantities can be written. The correctness of these relationships can be checked by Fig. 1. Thus:

$$ta^*_{i,j} = \sum_{k=1}^{i} a^*_{k,j} + (n_j - 1)\, a^*_{i,j} \tag{2}$$

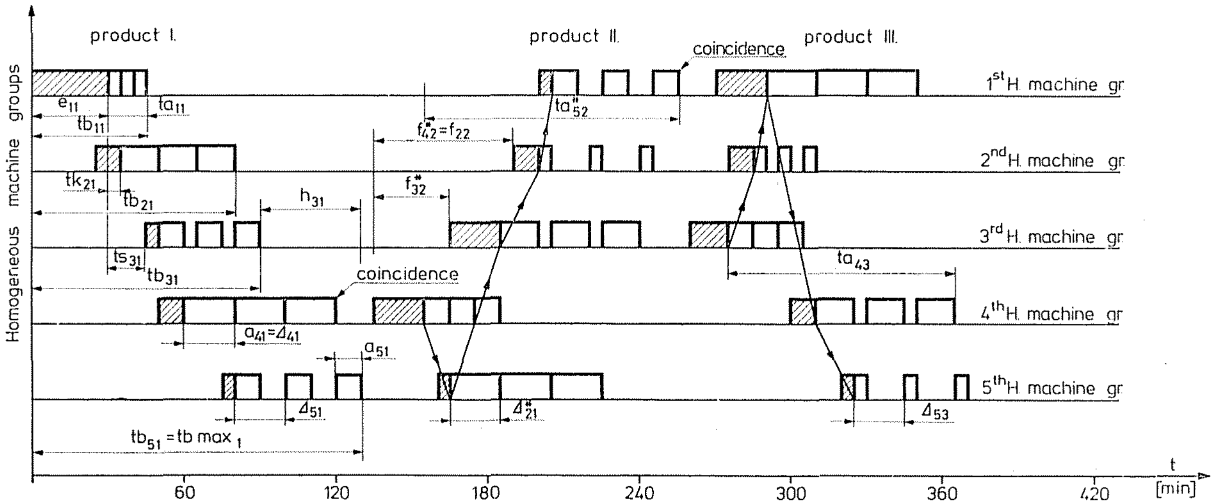
product I.
product II.
product III.
coincidence
1st H. machine gr
2nd H. machine gr
3rd H. machine gr
4th H. machine gr
5th H. machine gr
Homogeneous machine groups
e11
ta11
tb11
tk21
tb21
h31
ts31
tb31
f*42 = f22
f*32
ta*52
ta43
a41 = Δ41
a51
Δ51
Δ*21
Δ53
tb51 = tb max 1
60
120
180
240
300
360
420
t
[min]

Fig. 1

and

$$tk^*_{i,j} = \sum_{k=1}^{i-1} a^*_{k,j} \tag{3}$$

further

$$ts^*_{i,j} = tk^*_{i,j} - e^*_{i,j} = - e^*_{i,j} + \sum_{k=1}^{i-1} a^*_{i,j}\,. \tag{4}$$

The relationships derived so far are suitable for simulating in itself the production of the individual products. In as much as the product sort w is produced in series containing n_j pieces each, following each other in some sequence, then the separate production processes need "adjustment". This can be effected by introducing the concept of "junction". Junction is brought about by sliding the separate diagrams of the part-figures together until contact. The junction will appear at that machine where the quantity

$$d_{i,j} = h_{i,j-1} + f_{i,j} \tag{5}$$

assumes its minimum. Here

$$h_{i,j-1} = tb\,\max_{j-1} - tb_{i,j-1} \tag{6}$$

and

$$f_{i,j} = ts_{i,j} - ts\,\min_j \tag{7}$$

where

$$tb\,\max^*_j = \max_{i=1,\ldots,g} \{tb^*_{i,j}\} \tag{8}$$

$$ts\,\min^*_j = \min_{i=1,\ldots,g} \{ts^*_{i,j}\}\,. \tag{9}$$

We draw attention repeatedly to the meaning of *. Also in the above formulae its absence refers to transition to the sequence of the machines.

"Junction index" will be called the machine-sequential serial number, where two neighbouring $(j-1, j)$ series are contacting. Let us denote it by u_j; then

$$d_{u_j,j} = \min_{i=1,\ldots,g} \{d_{i,j}\}\,. \tag{10}$$

In knowledge of the junction indices, the total throughput times for a given product sequence can be calculated. Namely, on the basis of Fig. 1;

$$tb\,\max_1 = ta^*_{g,j} \tag{11}$$

and

$$tb_{ij} = tb_{u_i j-1} + e_{u_j,j} - \sum_{k=1}^{u_j^*-1} a^*_{k,j} + ta_{i,j} =$$

$$= tb_{u_i j-1} + e_{u_i j} - tk^*_{u_i j} + ta_{i,j}. \qquad (12)$$

This latter recurrent relationship permits to calculate the ending times and, similarly as in (11), the series of the j-th product is thrown off the conveyor by the end of the period $tb^*_{g,j}$.

Here, in simulating intermittent production with infinite conveyor speed, it was also presupposed that the product does not return to one and the same homogeneous machine group. But one, or more homogeneous machine groups are allowed not to take part in the production of some product. This is denoted, — as mentioned previously, — by the corresponding zero-value elements of the matrix **M**. In this case:

$$tb_{i,j} = tb_{i,j-1}. \qquad (13)$$

Let us investigate now the possibility of simulating continuous production. The production process based on this principle is shown in Fig. 2. Here too, the corresponding algorithms can be constructed by the observation of the figure. In order to avoid repetitions, only the deviations encountered in the calculation of the throughput times, the starting and the ending times

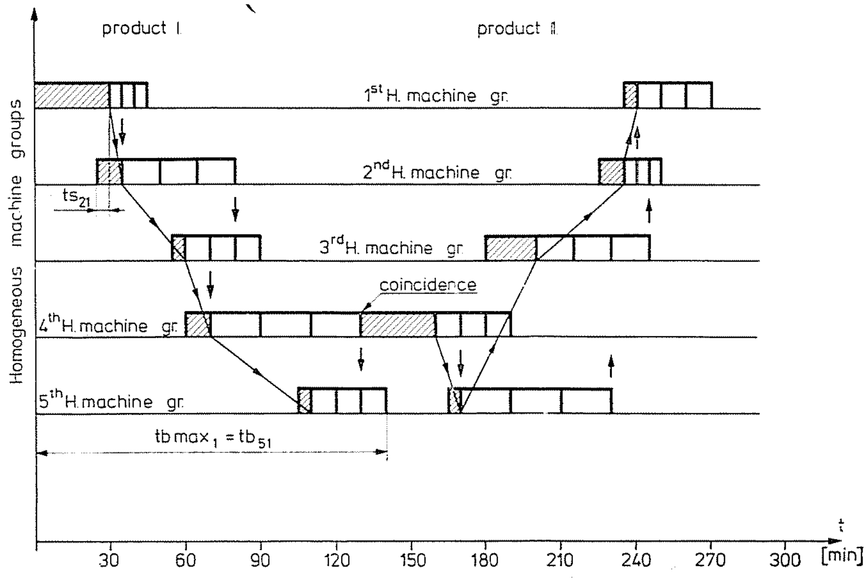

Fig. 2

will be indicated, using the previous definitions. In particular formulae (2), (3) and (4) are replaced by the following relationships:

$$\left.\begin{aligned} & ta^*_{0,j} = 0\,; \quad a^*_{0,j} = 0 \\ & ta^*_{i,j} = \begin{cases} ta^*_{i-1,j} - (n_j - 1)\, a^*_{i-1,j} + n_j\, a^*_{i,j}, & \text{for } a^*_{i,j} \geq a^*_{i-1,j} \\ ta^*_{i-1,j} + a^*_{i,j} & , \text{for } a^*_{i,j} < a^*_{i-1,j} \end{cases} \end{aligned}\right\} \tag{14}$$

$$tk^*_{i,j} = ta^*_{i,j} - n_j\, a^*_{i,j} \tag{15}$$

$$ts^*_{i,j} = tk^*_{i,j} - e^*_{i,j}. \tag{16}$$

If the transportation times $t^*_{i,j}$ between the machines performing the i-th and $(i+1)$-th operations in manufacturing the j-th product are known in the form of matrix $\mathbf{T}^* = \{t^*_{i,j}\}$ of dimension $(g \times w)$ (the last row of $\mathbf{T}^*$ consists of zeros), then the effect of the transportation times of the product arriving by and taken off the conveyor can be relatively simply allowed for in modifying formulae (2), (3), (4), (14), (15) and (16). With the above assumption the cycle time is not influenced by the transportation period. Assuming intermittent production the throughput time, the starting and the ending times can be calculated by the following relationships:

$$ta^*_{i,j} = \sum_{k=1}^{i-1} (a^*_{k,j} + t^*_{k,j}) + a^*_{i,j} + (n_j - 1)\, \Delta^*_{i,j} \tag{17}$$

$$tk^*_{i,j} = \sum_{k=1}^{i-1} (a^*_{k,j} + t^*_{k,j}) \tag{18}$$

$$ts^*_{k,j} = tk^*_{i,j} - e^*_{i,j}. \tag{19}$$

Also the formulae applying to the continuous production can be modified in a similar way:

$$ta^*_{0,j} = 0; \quad a^*_{0,j} = 0; \quad t^*_{c,j} = 0$$

$$ta^*_{i,j} = \begin{cases} ta^*_{i-1,j} - (n_j - 1)\, a^*_{i-1,j} + t^*_{i-1,j} + n_j\, a^*_{i,j}, & \text{for} \quad a^*_{i,j} \geq a^*_{i-1,j} \\ ta^*_{i-1,j} + a^*_{i,j} + t^*_{i-1,j} & , \text{for} \quad a^*_{i,j} < a^*_{i-1,j} \end{cases} \tag{20}$$

$$tk^*_{i,j} = ta^*_{i,j} - n_j\, a^*_{i,j} \tag{21}$$

$$ts^*_{i,j} = tk^*_{i,j} - e^*_{i,j}. \tag{22}$$

Concerning the evaluation of the developed algorithms it must be said that even the last two cases represent a rather high degree of abstraction a compared to the operation of the real production line. According to our experi ence models from among the presented that simulating the intermittent operation can be utilized best, assuming infinite conveyor speed. Namely in this case the characteristic times of the theoretically possible ideal (fastest) production are obtained. Comparing them with the real times results in the relation between theoretical and real preparation and particularly, operational periods. After a few corrections, and using the algorithm the corresponding standard periods (operational periods, piece-times) can be determined.

Then the initial matrices can be made more exact on the basis of site measurements. For instance matrices of the preparation and the operational periods are modified after some q-th series of measurements according to:

$$\begin{aligned} \mathbf{E}[q] &= \mathbf{E}[q-1] + \frac{1}{q}\left(\mathbf{E}[q] - \mathbf{E}[q-1]\right) \\ \mathbf{A}[q] &= \mathbf{A}[q-1] + \frac{1}{q}\left(\mathbf{A}[q] - \mathbf{A}[q-1]\right). \end{aligned} \tag{23}$$

The relationships (22) may be regarded as a sequential average formation (stochastic approximation) method.

Determination of the optimum product sequence

As optimum sequence will be regarded the one implying minimum total throughput time $tb^*_{g,w}$. This can be calculated by (12) and for low product numbers ($w < 6$) its minimum can be found by counting the possible $w!$ permutations of the products. Naturally this way is not viable in the case of high product numbers.

In manufacturing products consecutively, a time-saving $p_{j-1,j}$ is obtained by creating the phenomenon of "junction" when going over from the $(j-1)$-th to the j-th product, against the case of waiting until the last piece of the $(j-1)$-th product comes of the conveyor. The value of $p_{j-1,j}$ can be expressed by our previous quantities as

$$p_{j-1j,} = \min_{i=1,\ldots,g} \{d_{i,j}\} = d_{u_j,j} \tag{24}$$

Be the matrix $\mathbf{P}$ of dimension $(w \times w)$ composed of elements, $p_{j-1,j}$ i.e. with element $p_{r,s}$ representing the time saving at finishing the r-th and beginning the s-th product. The diagonal elements of $\mathbf{P}$ are indifferent by meaning.

The optimization is intended to select from among the sequential vectors

$$\mathbf{s}_{s_1} = [s_1, s_2, \ldots, s_w]^{\mathrm{T}} \qquad s_i \neq s_j \tag{25}$$

the one, for which the quantity

$$Q_{s_1} = p_{s_1, s_2} + p_{s_2, s_3} + \ldots + p_{s_{w-1}, s_w} \tag{26}$$

is maximum, that is the problem is to determine the optimum permutation

$$\mathbf{s}^* = \max_{s_1} \{Q_{s_1}\}. \tag{27}$$

This problem is a well-known fundamental type of discrete programming which can be treated in various ways. It can be investigated as a travelling salesman problem and solved as Hamiltonian ways of the maximum length [1]. Various solutions are known, for the problem but most of them are very complex and become especially cumbersome, when combined with the degeneration problem [7].

An algorithm supplying a simplified, sub-optimum solution will be presented. Accordingly the value of (26) will be calculate with the initial values of $s_1 = 1, 2, \ldots, w$ for the permutations generated in the following way:

$$p_{s_{k-1}, s_k} = \max_j \{p_{s_{k-1}, j}\} \tag{28}$$

$$j = s_1, s_2, \ldots, s_{k-1}.$$

Then $\mathbf{s}^*$ will be selected according to (27). The algorithm gives no global optimum, but it is easily programmed and the solution gives usually a very favourable result.

An illustrative example

For illustrating the above, let us present a simple example. Be $w = 3$, $g = 5$ and $\mathbf{n} = [3, 3, 3]^{\mathrm{T}}$, further

$$\mathbf{E} = \begin{bmatrix} 30 & 5 & 20 \\ 10 & 10 & 10 \\ 5 & 20 & 15 \\ 10 & 30 & 10 \\ 5 & 5 & 5 \end{bmatrix}; \mathbf{A} = \begin{bmatrix} 5 & 10 & 20 \\ 15 & 5 & 5 \\ 10 & 15 & 10 \\ 20 & 10 & 15 \\ 10 & 20 & 5 \end{bmatrix}$$

Assuming intermittent production, and infinite conveyor speed, relationships (2), (3), (4) and (12) yield:

$$\{ta^*_{i,j}\} = \begin{bmatrix} 15 & 30 & 30 \\ 50 & 70 & 35 \\ 60 & 85 & 75 \\ 90 & 90 & 90 \\ 100 & 100 & 95 \end{bmatrix} ; \{tk^*_{i,j}\} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 10 & 10 \\ 20 & 30 & 15 \\ 30 & 45 & 35 \\ 50 & 50 & 50 \end{bmatrix}$$

$$\{ts^*_{i,j}\} = \begin{bmatrix} -30 & -30 & -15 \\ -5 & 5 & 0 \\ 15 & 10 & -5 \\ 20 & 35 & 25 \\ 45 & 45 & 45 \end{bmatrix} ; \{tb^*_{i,j}\} = \begin{bmatrix} 45 & 250 & 330 \\ 80 & 240 & 290 \\ 90 & 235 & 285 \\ 120 & 180 & 345 \\ 130 & 220 & 350 \end{bmatrix}$$

The junction indices are:

$$u_1 = 1; \quad u_2 = 4; \quad u_3 = 1$$
$$u_1^* = 1; \quad u_2^* = 1; \quad u_3^* = 3$$

The simulation referred to the sequence shown in Fig. 1. Determining the matrix **P**

$$\mathbf{P} = \begin{bmatrix} - & 10 & 40 \\ 0 & - & 10 \\ 20 & 5 & - \end{bmatrix}$$

the optimum starting sequence according to the algorithm (28) is:

$$1 - 3 - 2 .$$

Conclusions

Simple algorithms, easy to be programmed, were presented for simulating the production by closed, homogeneous production lines. The algorithms can be associated with a procedure determining a sub-optimum starting sequence and finally in this way useful advice can be obtained for the formulation of optimum control decisions.

The procedures were programmed for a SIEMENS 4004/40 computer in the FORTRAN language. The computations were tested by simulating a real plant production in the range between $w = 6-60$ and $g = 20-30$. The most important conclusions drawn from the simulation are related — in addition to the optimum sequence — to the demonstration of the irrealistic operationa times and the study of their modification.

Summary

This paper deals with the simulation of the production of closed, homogeneous production lines in the complex case, where the preparation periods of the products on the individual machines and the sequence of the machines applied for the machining of the individual products are different.

Simple algorithms, suitable for easy physical interpretation and computer-programming are presented.

Also the approximative determination of the product sequence ensuring minimum throughput time is given. The computer programs prepared on the basis of the developed algorithms might provide considerable assistance in the preparation of better founded decisions for production management.

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Prof. Dr. Frigyes CSÁKI
dr. László KEVICZKY
dr. Tivadar KOVÁCS
dr. János KOCSIS

1502, Budapest, P.O.B. 91. Hungary