A GRAPH THEORY APPROACH OF DECOMPOSITION TECHNIQUES IN POWER SYSTEM CALCULATION

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1. Introduction

Decomposition techniques is an effective means of solving various types of power system calculation problems by digital computers. The (numerous) existing methods for the implementation of decomposition are widely dealt with by several authors; first of all G. KRON has to be mentioned, who has laid the theoretical foundations of what he called "the piece-wise" solution of large networks [1]. It is to be emphasized, however, that KRON has worked out his fundamental theory of "tearing" without making use of the theorems of graph theory, applied at the same time with great success by several authors for the solution of the same type of problems [2], [3].

The main purpose of this paper is to provide a graph theory foundation of the well-known decomposition techniques. These latter are applied for the piece-wise calculation of big interconnected systems, featured by being composed of several subsystems, all of which are strongly meshed in themselves, but have relatively few interties among them. These techniques are based essentially upon the graph theory concept of "multiterminal graph elements" [3] and upon the method called "multiterminal representation of power systems" developed by KESAVAN and PAI [6].

2. The main statements of multiterminal representation method

In the following a brief summary is given about the fundamental concepts of multiterminal representation, basis of the decomposition techniques dealt with hereafter. The knowledge of the main theorems of graph theory and network calculation is assumed throughout this paper.

An interconnected power system may be regarded as being composed of several subsystems, which are interconnected at terminal nodes i.e. substation buses. The basic point of interest is to find mathematical equivalents describing exactly the behaviour of these subsystems when viewed externally, from their nodes.

Such an equivalent consists of a set of equations interrelating voltage and current values, which can be measured independently with respect to every pair of nodes of the subsystem in question. It can be proved that the graph corresponding to this independent set of measurements is a tree. This graph will be referred to in the following as terminal graph, while the equations mentioned above as terminal equations — both constituting basic concepts of the method of multiterminal representation.

The theory of this method is established through Tellegen's theorem, a simple form of which is presented here without proof:

If $V(t)$ and $I(t)$ are the voltage and current matrices associated with the oriented graph G of an arbitrary lumped network, then:

$$V(t) \cdot I(t) = 0 \tag{1}$$

provided that the entries $V(t)$ and $I(t)$ correspond to the same ordering of elements in G.

In their paper mentioned above, KESAVAN and PAI are expounding in detail — making use of Eq. (1) — the method of finding the terminal tree and the corresponding terminal equations, which describe externally the behaviour of the studied system in various types of network calculations. From the power system engineer's point of view, two types of studies are of special importance, namely:

a) load-flow calculations — by which the so-called slack-bus is used as reference; its voltage is specified with respect to the (otherwise isolated) ground bus;

b) short-circuit studies — in this case the network is regarded with ground bus included and used as reference vertex in the graph.

It is proved in this paper that the terminal equations of any power system containing n buses are composed of the well-known nodal voltage equations, which can be written in matrix form as follows:

$$U_n = \mathbf{Z}_n \cdot I_n \tag{2}$$

where

U_n is the vector of nodal voltages
I_n is the vector of nodal current injections
$\mathbf{Z}_n$ is the nodal impedance matrix.

The order of the above matrix equation and also the form of the respective terminal graph depends on whether type a) or b) of system studies is required.

In case a) one has to choose one of the buses as slack bus, and the row belonging to it in the **A** (incidence) matrix is deleted — taking it thereby as reference vertex — in building the nodal admittance equations. The order of the nodal admittance matrix is then $n - 1$, and being nonsingular it can be inverted to give Z_n. The terminal graph is — as shown above — a tree, the most advantageous form of which for our purposes being a *Lagrange-type* tree, with the slack bus as common vertex (see Fig. 1).

In case b) the ground bus is taken as reference vertex — the row of which is deleted in matrix A — consequently the order of the nonsingular admittance matrix and that of its inverse is n. The terminal graph is again a Lagrangian-tree, shown in Fig. 2.

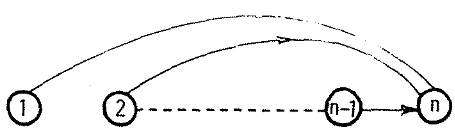

Fig. 1

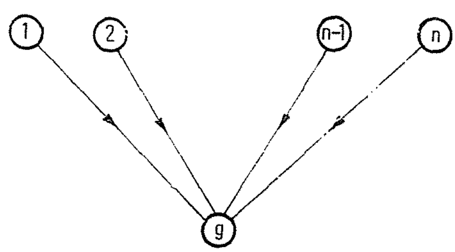

Fig. 2

3. The graph of a power system composed of subsystems

An interconnected power system is composed of very many nodes and branches. Since by applying the nodal method of network analysis in the classical way, the size of the system admittance matrix to be inverted grows proportionally with the node number, the required computer storage capacity and computation time may be prohibitive for the available digital computer at a given stage of system development. In this case, decomposition technique can yield a significant help in solving the problem. The common ground of nearly all of the decomposition techniques — dealt with in the following — is based on the fact that the subsystems of an interconnected power system can be ragarded from graph theory aspects as multiterminal graph components [6]. Their terminal characteristics are defined exactly by the terminal equations referred to in the preceding paragraph; in the same time they are topologically characterised by their terminal graphs.

Taking into account the above statements one can easily realize that by combining the methods of multiterminal representation and of multiterminal graph elements a much simpler topological representation of interconnected systems can be derived than the system graph drawn in the usual way. The principle of the procedure is demonstrated in Figs 3/a and b.

In Fig. 3/a the classical graph of a system composed of three subsystems is shown, each of them having four nodes. The graph is constructed for load-flow studies — which constitute the overwhelming majority of calculations occurring in p.s. operation practice — i.e. ground bus is omitted. In all of the subsystems the node marked with circle is taken as slack bus (and reference).

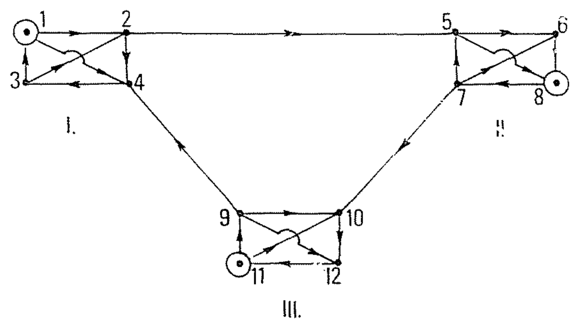

Fig. 3a

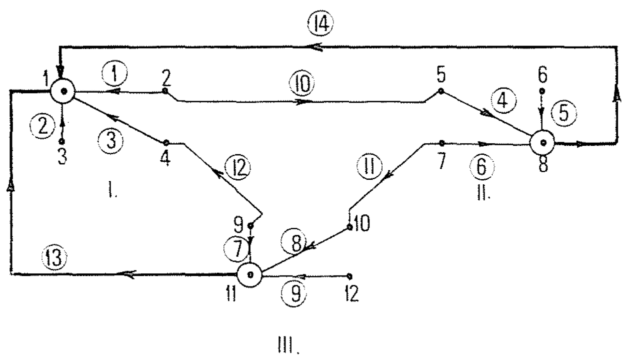

Fig. 3b

On the other hand, Fig. 3/b shows the simplified system graph obtained in substituting each subsystem graph by the appropriate terminal graph (similarly as in Fig. 1). The very fact that the voltage of each subsystem slack node has to be specified with regard to the common — but omitted — ground bus, can be accounted for by inserting ideal voltage sources between them, which represent their respective — specified — voltage differences. Considering the fact that in load flow studies the slack bus takes on the active power balance of the system in question, each of the subsystems will have its active power balanced by its own slack bus in our case. That is by no means a restriction, because in the operation practice of interconnected power systems every subsystem is responsible for its own active (and also reactive) power balance.

The voltages of the slack buses can be specified by the aid of former calculation results and operation routine.

The graph elements belonging to the ideal voltage sources between subsystem slack buses mentioned above are represented by heavy lines in Fig. 3/b.

For this special type of graph the following notation will be used hereafter: "decomposition graph". It is shown in the literature [3] that any kind of network composed of multiterminal components can be handled by the same principle as networks of two terminal components. On this basis the network equations of the decomposed system can be deduced followingly:

The component terminal equation on the ground of Eq. (2), omitting the subscripts n and written in hypermatrix form, is as follows:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathrm{U}_{g13} \\ \mathrm{U}_{g14} \end{bmatrix} + \begin{bmatrix} U_{\mathrm{I}} \\ U_{\mathrm{II}} \\ U_{\mathrm{III}} \\ \mathrm{U}_{10} \\ \mathrm{U}_{11} \\ \mathrm{U}_{12} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{\mathrm{I}} & Q & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{Z}_{\mathrm{II}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{Z}_{\mathrm{III}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{Z}_{10} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathrm{Z}_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathrm{Z}_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} .$$

$$\left[\begin{bmatrix} I_{\mathrm{I}} \\ I_{\mathrm{II}} \\ I_{\mathrm{III}} \\ \mathrm{I}_{10} \\ \mathrm{I}_{11} \\ \mathrm{I}_{12} \\ 0 \\ 0 \end{bmatrix} + I_i \right] \qquad (3)$$

or in simpler form:

$$U_g + U = \mathbf{Z} \cdot (I + I_i) \,. \qquad (3/a)$$

Where subscripts I ... III refer to subnetwork matrices, namely

$U_{\mathrm{I}} \ldots U_{\mathrm{III}}$ the nodal voltage vectors
$I_{\mathrm{I}} \ldots I_{\mathrm{III}}$ the nodal current injection vectors
I_i vector of (nodal) currents coming from the interconnections
and $\mathbf{Z}_{\mathrm{I}} \ldots \mathbf{Z}_{\mathrm{III}}$ the nodal imp. matrixes, obtained by inverting the respective admittance matrices;

$U_l I_l$ and Z_l are the branch voltage, current and imp. values, respectivel of the interconnection lines.

To describe the topological relations of the decomposition graph, the loop-set matrix is chosen. It has the following form:

$$\mathbf{B} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}. \tag{4}$$

(It will be easily realized that for this type of graphs the mesh method of analysis is more convenient than the nodal one, the volume of calculations and computer storage place requirement being much less with the former. This will be clear from the equations developed further on.)

The number of independent loops is equal to that of the chords. Meanwhile it is easily realized that each of the interties constitutes a chord. This can be stated in a general form:

In building the decomposition graph as described above, every intersystem-connection constitutes a chord.

According to the loop-law of Kirchoff written in matrix form for the network belonging to the graph in Fig. 3/b:

$$\mathbf{B}U = 0 = \mathbf{B} \cdot \mathbf{Z} \cdot I + \mathbf{B} \cdot \mathbf{Z} \cdot I_l - \mathbf{B}U_g . \tag{5}$$

In the same time according to the network theory, the following holds:

$$I = \mathbf{B}_t \cdot i_h \tag{6}$$

where in our case:

$$i_h = \begin{bmatrix} i_{h_1} \\ i_{h_2} \\ i_{h_3} \end{bmatrix} : \text{the loop current vector}.$$

Combining Eqs (5) and (6) we obtain:

$$I_l = -\mathbf{B}_t(\mathbf{B} \cdot \mathbf{Z} \cdot \mathbf{B}_t)^{-1} \cdot \mathbf{B} \cdot (\mathbf{Z} \cdot I - U_g) \tag{7}$$

and after substitution into Eq. (3/a):

$$U = -\mathbf{Z}[\mathbf{B}_t(\mathbf{B} \cdot \mathbf{Z} \cdot \mathbf{B}_t)^{-1} \cdot \mathbf{B}(\mathbf{Z} \cdot I - U_g) - I] - U_g \tag{8}$$

or, in more convenient form:

$$U = (\mathbf{Z} - \Delta\mathbf{Z}) \cdot I + (\Delta\mathbf{Z} \cdot \mathbf{Z}^{-1} - \mathbf{E}) \cdot U_g = (\mathbf{Z} - \Delta\mathbf{Z}) \cdot I + \mathbf{K} \cdot U_g \tag{9/a}$$

where:

$$\Delta\mathbf{Z} = \mathbf{Z} \cdot \mathbf{B}_t(\mathbf{B} \cdot \mathbf{Z} \cdot \mathbf{B}_t)^{-1} \cdot \mathbf{B} \cdot \mathbf{Z}, \mathbf{K} = \Delta\mathbf{Z} \cdot \mathbf{Z}^{-1} - \mathbf{E} \tag{9/b}$$

($\mathbf{E}$ denotes the identity matrix.)

Which means that one can take into consideration the effect of subsystem-interconnections simply by correcting the original block-diagonal formed nodal impedance matrix of the total system without interconnections (shown in Eq. (3)) with an additional imp. matrix $\Delta\mathbf{Z}$. The value of $\mathbf{K}$ is also unchanged during the iteration process.

The calculation of $\Delta\mathbf{Z}$ and $\mathbf{K}$ requires very few computation time and storage place, as the triple product $\mathbf{B} \cdot \mathbf{Z} \cdot \mathbf{B}_t$ is a small matrix, the order of which equals the generally low number of interconnections (according to Eq. (4)). With this corrected imp. matrix the load flow calculation can be concluded quite in the usual way.

In the special case, if the subsystem slack-bus voltages are equivalent in absolute value with zero angle deviations among them, the U_g vector in Eq. (9/a) vanishes and therefore we arrive at the following simpler form:

$$U = (\mathbf{Z} - \Delta\mathbf{Z}) \cdot I\,. \tag{9/c}$$

If the network study is of the short circuit type, then the terminal graph shown in Fig. 2 can be used and in building the decomposition graph the ground bus is taken for reference vertex, otherwise the method explained in connection with Fig. 3 can be applied.

4. Computational aspects of the described method

The "*decomposition graph*" technique described above can advantageously be applied for solving the most frequently encountered power system calculation problems. The form of the resulting equations is rather simple and therefore easily applicable for programming purposes. They can be used most effectively for solving the problems of power system types, easy to cut into several parts with rather few interconnections. For the decomposition logic the same algorithms apply as with diacoptics, the logic of building the loop set matrix $\mathbf{B}$ is also very simple and it can be stored in compact form in modern computers. By adequate organisation of the matrix operations of Eq. (9/b) a great economy in working storage place and computation time can be achieved.

Summary

The author attempts to establish a very simple and clearly arranged relation between graph theory and decomposition technique of network calculation overall applied nowadays. After introducing the concept of the so-called "decomposition graph" he gives the respective basic formulae and some aspects of computer applicability.

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